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The influences of recycle on a double-pass laminar counterflow concentric-tube heat exchangers with sinusoidal wall fluxes ☆

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ABSTRACT

The effects of external recycle at the ends on double-pass laminar countercurrent heat exchangers with sinusoidal heat flux distribution are investigated analytically by setting a general solution to separate the original boundary value problem into a partial differential equation, which is solved by Frobenius method, and an ordinary differential equation. Analytical results show that recycle effects enhance the heat-transfer efficiency due to that the desirable effect of forced-convection increment has more influence than the undesirable effect of the heat-transfer driving-force decrement, and hence the forced-convection increment by increasing the recycle ratio leads to improved device performance in heat-transfer rate as compared with that in the single-pass operation (without an impermeable sheet inserted).

1. Introduction

A new flow pattern for a fluid through a concentric circular tube is considered in this study in order to improve the heat-transfer efficiency. An impermeable sheet is inserted for conducting double-pass operations with external recycle at both ends (with negligible entrance and end effects and with fully developed flow right at the entrance assumed). The heat-transfer efficiency enhancement and Nusselt numbers compared with that in an open conduit are also delineated. The simple case of single-pass operations of the negligible axial conduction in cylindrical or parallel-plate geometries is the classical Graetz problem [1–4] with a well-known solution while the laminar convective heat-transfer problems extended to multi-stream or multiphase systems by conjugating the governing equations in each phase (or each stream) are called conjugated Graetz problems [5–7]. The resultant conjugated partial differential equations with the Neumann boundary condition of which the heat fluxes at the walls were specified are solved analytically by using the eigenfunction expansion technique in a simple form [8–12]. Moreover, the mathematical models of periodic heating systems with sinusoidal temperature distribution [13] or sinusoidal wall heat flux [14,15] were also developed by many researchers. Such a simplifying mathematical formulation as developed in the present study will be an important contribution to the analysis and solution of the conjugated Graetz problem of mutual conditions at the boundaries.

The availability of the recycle-effect concept in designing separation processes and reactor equipments with external or internal recycle has been developed and widely used in loop reactors [16], air-lift reactor [17], draft-tube bubble column [18] and thermal diffusion column [19]. The recycle ratio and the position of an impermeable sheet, which is inserted to conducting double-pass operations with external recycle, are the operating parameter and designing parameter, respectively, to be suitably selected for increasing heat-transfer efficiency enhancement in the heat exchangers of the present study.

There are two purposes in the present study: first, to develop a theoretical mathematical model of double-pass laminar counterflow concentric-tube heat exchangers with external recycle under sinusoidal wall fluxes in the designing of the cooling tubes in nuclear reactors and to solve the resultant conjugated partial differential equations analytically by setting a general solution to separate the original boundary value problem into a partial differential equation, which is solved by the Frobenius method, and an ordinary differential equation; second, to investigate the heat-transfer efficiency improvement of such double-pass heat exchangers under sinusoidal wall fluxes with the recycle ratio and impermeable sheet-position as parameters.

2. Temperature distributions

A new double-pass concentric-tube heat exchanger with external recycle under sinusoidal heat fluxes, as shown in Fig. 1, was investigated. The length and the inner diameter of the double-pass concentric-tube heat exchanger are L and $2R$, respectively. The fluid flowing channel was divided into two subchannels by inserting an impermeable barrier. The thickness of the inner (subchannel a) and annular tube (subchannel b) are $2\kappa R$ and $2(1-\kappa)R$, respectively. This new double-pass concentric-tube heat exchanger with external recycle under sinusoidal heat fluxes is an extension of our previous work [20] and two flow patterns are discussed. The case of the working fluid feeding into inner subchannel

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Nomenclature

a_n	coefficient in Eq. (22)
b_n	coefficient in Eq. (23)
B	constant, defined in Eq. (13)
Gz	Graetz Number, $4V/\alpha\pi L$
h	heat-transfer coefficient kW/m^2K
I_h	heat-transfer improvement based on single-pass devices, defined by Eq. (44)
k	heat conductivity coefficient of the wall, kW/mK
L	conduit length, m
$Nu(\xi)$	local Nusselt number, defined by Eq. (40)
$\overline{Nu}$	average Nusselt number, defined by Eq. (43)
q''	heat flux on the wall, J/m^2s
R	radius of outer tube, m
r	radial coordinate, m
R_1	radius of inner tube, m
T	temperature, K
u	velocity distribution of fluid, m/s
V	input volume flow rate of conduit, m^3/s
W_1	constant, defined in Eq. (11)
W_2	constant, defined in Eq. (12)
z	longitudinal coordinate, m

Greek symbols

α	thermal diffusivity of fluid, m^2/s
β	constant
δ	impermeable barrier thickness, m
ϕ	dimensionless temperature, $k(T-T_i)/q''_0R$
γ_{1a}, γ_{2a}	integration constants in Eq. (35)
γ_{1b}, γ_{2b}	integration constants in Eq. (36)
η	radial coordinate, r/R
κ	impermeable barrier location, defined in Eq. (13)
λ	constant
θ_{0a}	constant in Eq. (14)
θ_{0b}	constant in Eq. (15)
$\theta_{1a}, \theta_{2a}, \theta_{3a}$	functions of η, defined in Eq. (14)
$\theta_{1b}, \theta_{2b}, \theta_{3b}$	functions of η, defined in Eq. (15)
ξ	longitudinal coordinate, $z/(LGz)$

Subscripts

a	subchannel *a*
b	subchannel b
F	at the outlet
i	at the inlet
0	in a single-pass device without recycle
w	at the wall surface

firstly with volumetric flow rate V and a recycle fluid flowing in outer subchannel with volumetric flow rate MV is called flow pattern A, as shown in Fig. 1(a), while flow pattern B is the case of the working fluid feeding into outer subchannel firstly with volumetric flow rate V and a recycle fluid flowing in inner subchannel with volumetric flow rate MV, as shown in Fig. 1(b). In both flow patterns, the fluid is heated by the outer wall with sinusoidal heat fluxes, $q''_w(z) = q''_0[1+\sin(\beta z)]$.

Based on following assumptions: constant physical properties of fluid; fully-developed laminar flow in the entire length in the inner and annular subchannels; neglecting the entrance length and the end effects; ignoring the longitudinal heat conduction and the thermal resistance of the impermeable barrier, the energy balance equations of a double-pass heat exchanger with sinusoidal heat fluxes can be formulated as

$$\frac{u_a(\eta)R^2}{GzL\alpha}\frac{\partial\phi_a(\eta,\xi)}{\partial\xi} = \frac{1}{\eta}\frac{\partial}{\partial\eta}\left(\eta\frac{\partial\phi_a(\eta,\xi)}{\partial\eta}\right) \quad (1)$$

$$\frac{u_b(\eta)R^2}{GzL\alpha}\frac{\partial\phi_b(\eta,\xi)}{\partial\xi} = \frac{1}{\eta}\frac{\partial}{\partial\eta}\left(\eta\frac{\partial\phi_b(\eta,\xi)}{\partial\eta}\right) \quad (2)$$

and the corresponding boundary conditions are

$$\frac{\partial\phi_a(0,\xi)}{\partial\eta} = 0 \quad (3)$$

$$\frac{\partial\phi_b(1,\xi)}{\partial\eta} = 1 + \sin(B\xi) \quad (4)$$

$$\frac{\partial\phi_a(\kappa,\xi)}{\partial\eta} = \frac{\partial\phi_b(\kappa,\xi)}{\partial\eta} \quad (5)$$

$$\phi_a(\kappa,\xi) = \phi_b(\kappa,\xi) \quad (6)$$

The velocity distributions in subcannels *a* and *b*, say u_a and u_b, are different in two flow patterns. In flow pattern A, the u_a and u_b, respectively, are

$$u_a(\eta) = \frac{2(M+1)V}{\pi(\kappa R)^2}\left(1-\left(\frac{\eta}{\kappa}\right)^2\right) 0 \le \eta_a \le \kappa \quad (7)$$

$$u_b(\eta) = -\frac{2MV}{[\pi R^2 - \pi(\kappa R)^2]W_1}\left[1-(\eta)^2 + W_2 \ln\eta\right] \kappa \le \eta_b \le 1 \quad (8)$$

and the u_a and u_b, in flow pattern B, respectively, are

$$u_a(\eta) = \frac{-2MV}{\pi(\kappa R)^2}\left(1-\left(\frac{\eta}{\kappa}\right)^2\right) 0 \le \eta_a \le \kappa \quad (9)$$

$$u_b(\eta) = \frac{2(M+1)V}{[\pi R^2 - \pi(\kappa R)^2]W_1}\left[1-(\eta)^2 + W_2 \ln\eta\right] \kappa \le \eta_b \le 1 \quad (10)$$

where W_1 and W_2 are

$$W_1 = \left[\frac{1-\kappa^4}{1-\kappa^2} - \frac{1-\kappa^2}{ln\frac{1}{\kappa}}\right] \quad (11)$$

and

$$W_2 = \left(\frac{1-\kappa^2}{ln1/\kappa}\right) \quad (12)$$

The dimensionless groups in Eqs. (1)–(12) are defined as

$$\eta = \frac{r}{R},\ \xi = \frac{z}{GzL},\ \phi_a = \frac{k(T_a - T_i)}{q''_0R},\ \phi_b = \frac{k(T_b - T_i)}{q''_0R},$$
$$Gz = \frac{4V}{\alpha\pi L},\ \kappa = \frac{R_1}{R},\ B = \beta GzL \quad (13)$$

The general solution of the dimensionless temperature distribution of the laminar double-pass counterflow concentric-tube heat exchangers with sinusoidal wall fluxes can be expressed as follows: [20,21]

$$\phi_a(\eta,\xi) = \theta_{0a}\xi + \theta_{1a}(\eta) + \theta_{2a}(\eta)\sin(B\xi) + \theta_{3a}(\eta)\cos(B\xi) \quad (14)$$

$$\phi_b(\eta,\xi) = \theta_{0b}\left(\frac{1}{Gz} - \xi\right) + \theta_{1b}(\eta) + \theta_{2b}(\eta)\sin(B\xi) + \theta_{3b}(\eta)\cos(B\xi) \tag{15}$$

where the θ_{0a} and θ_{0b} are the undetermined constants, and the θ_{1a}, θ_{2a}, θ_{3a}, θ_{1b}, θ_{2b} and θ_{3b} are the functions of η to be determined. According to the same mathematical treatment in our previous work [20], two sets of boundary value problems were obtained by substituting Eqs. (14) and (15) into the governing equations, Eqs. (1) and (2), and the boundary conditions, Eqs. (3)–(6). The boundary value problem for solving $\theta_{2a}(\eta)$, $\theta_{3a}(\eta)$, $\theta_{2b}(\eta)$ and $\theta_{3b}(\eta)$ is

$$\frac{d}{d\eta}\left(\eta\frac{\partial\psi_a(\eta)}{\partial\eta}\right) - \frac{u_a(\eta)BR^2\eta}{GzL\alpha}\psi_a(\eta)i = 0 \tag{16}$$

$$\frac{d}{d\eta}\left(\eta\frac{\partial\psi_b(\eta)}{\partial\eta}\right) - \frac{u_b(\eta)BR^2\eta}{GzL\alpha}\psi_b(\eta)i = 0 \tag{17}$$

$$\frac{d\psi_a(0)}{d\eta} = 0 \tag{18}$$

$$\frac{d\psi_b(1)}{d\eta} = 1 \tag{19}$$

$$\frac{d\psi_a(\kappa)}{d\eta} = \frac{d\psi_b(\kappa)}{d\eta} \tag{20}$$

$$\psi_a(\kappa) = \psi_b(\kappa) \tag{21}$$

where the complex functions are $\psi_a(\eta) = \theta_{2a}(\eta) + \theta_{3a}(\eta)i$ and $\psi_b(\eta) = \theta_{2b}(\eta) + \theta_{3b}(\eta)i$. The $\psi_a(\eta)$ and $\psi_b(\eta)$ can be solved by applying the Frobenius method as follows

$$\psi_a(\eta) = \sum_{n=0}^{\infty} a_n\eta^n,\ n \ge 0 \tag{22}$$

$$\psi_b(\eta) = \sum_{n=0}^{\infty} b_n\eta^n,\ n \ge 0 \tag{23}$$

The recursive relation form of coefficients a_n and b_n in flow pattern A, respectively, are

$$a_{2n} = \frac{(M+1)Bi}{8\kappa^2}\frac{1}{n^2}\left(a_{2n-2} - \frac{1}{\kappa^2}a_{2n-4}\right),\ n \ge 1 \tag{24}$$

and

$$\begin{aligned} b_1 &= 0,\\ b_n &= \frac{-MBi}{2W_1(1-\kappa^2)}\frac{1}{n^2}\left[\left(1-\frac{3}{2}W_2\right)b_{n-2}\right.\\ &\quad\left. + 2W_2b_{n-3} - \left(1+\frac{1}{2}W_2\right)b_{n-4}\right],\ n \ge 2, \end{aligned} \tag{25}$$

and those in flow pattern B, respectively, are

$$a_{2n} = \frac{-MBi}{8\kappa^2}\frac{1}{n^2}\left(a_{2n-2} - \frac{1}{\kappa^2}a_{2n-4}\right),\ n \ge 1 \tag{26}$$

and

$$\begin{aligned} b_1 &= 0,\\ b_n &= \frac{(M+1)Bi}{2W_1(1-\kappa^2)}\frac{1}{n^2}\left[\left(1-\frac{3}{2}W_2\right)b_{n-2}\right.\\ &\quad\left. + 2W_2b_{n-3} - \left(1+\frac{1}{2}W_2\right)b_{n-4}\right],\ n \ge 2, \end{aligned} \tag{27}$$

Moreover, the boundary value problem for solving θ_{0a}, $\theta_{1a}(\eta)$, θ_{0b} and $\theta_{1b}(\eta)$ is

$$\frac{d}{d\eta}\left(\eta\frac{d\theta_{1a}(\eta)}{d\eta}\right) - \frac{u_a(\eta)R^2\eta}{GzL\alpha}\theta_{0a} = 0 \tag{28}$$

$$\frac{d}{d\eta}\left(\eta\frac{d\theta_{1b}(\eta)}{d\eta}\right) + \frac{u_b(\eta)R^2\eta}{GzL\alpha}\theta_{0b} = 0 \tag{29}$$

$$\frac{d\theta_{1a}(0)}{d\eta} = 0 \tag{30}$$

$$\frac{d\theta_{1b}(1)}{d\eta} = 1 \tag{31}$$

$$\frac{d\theta_{1a}(\kappa)}{d\eta} = \frac{d\theta_{1b}(\kappa)}{d\eta} \tag{32}$$

$$\theta_{0a} = -\theta_{0b} \tag{33}$$

$$\theta_{1a}(\kappa) = \frac{\theta_{0b}}{Gz} + \theta_{1b}(\kappa) \tag{34}$$

Then, integrating Eqs. (28) and (29) twice with respect to η for $\theta_{1a}(\eta)$ and $\theta_{1b}(\eta)$ yields

$$\theta_{1a} = \frac{(M+1)\theta_{0a}}{2\kappa^2}\left[\frac{1}{4}\eta^2 - \frac{\eta^4}{16\kappa^2} + \gamma_{1a}ln\eta + \gamma_{2a}\right] \tag{35}$$

and

$$\theta_{1b} = \frac{M\theta_{0b}}{2W_1(1-\kappa^2)}\left[\frac{1}{4}\eta^2 - \frac{1}{16}\eta^4 + \frac{W_2}{4}\eta^2[ln\eta - 1] + \gamma_{1b}ln\eta + \gamma_{2b}\right], \tag{36}$$

for flow pattern A and

$$\theta_{1a} = \frac{M\theta_{0a}}{2\kappa^2}\left[\frac{1}{4}\eta^2 - \frac{\eta^4}{16\kappa^2} + \gamma_{1a}ln\eta + \gamma_{2a}\right] \tag{37}$$

and

$$\theta_{1b} = \frac{(M+1)\theta_{0b}}{2W_1(1-\kappa^2)}\left[\frac{1}{4}\eta^2 - \frac{1}{16}\eta^4 + \frac{W_2}{4}\eta^2[ln\eta - 1] + \gamma_{1b}ln\eta + \gamma_{2b}\right], \tag{38}$$

for flow pattern B, respectively. The two undetermined constants θ_{0a} and θ_{0b}, and the four integrating constants γ_{1a}, γ_{2a}, γ_{1b} and γ_{2b} were calculated by the boundary conditions, Eqs. (30)–(34), and the overall energy balance

$$\phi_F = \int_0^{\frac{1}{Gz}} 8[1 + \sin(B\xi)]d\xi = 8\left[\frac{1}{Gz} - \frac{1}{B}\left(\cos\left(\frac{B}{Gz}\right) - 1\right)\right] \tag{39}$$

where the ϕ_F is the average outlet temperature. After determining all functions of θ_{0a}, θ_{0b}, θ_{1a}, θ_{1b}, θ_{2a}, θ_{2b}, θ_{3a} and θ_{3b} into the ϕ_a and ϕ_b, the complete solutions of the temperature distribution in a double-pass concentric-tube heat exchanger with external recycle were obtained.

3. Heat-transfer efficiency improvement

The heat-transfer efficiency of a double-pass concentric-tube heat exchanger with external recycle can be determined by the local Nusselt number as

$$Nu(\xi) = \frac{hD_e}{k} \tag{40}$$

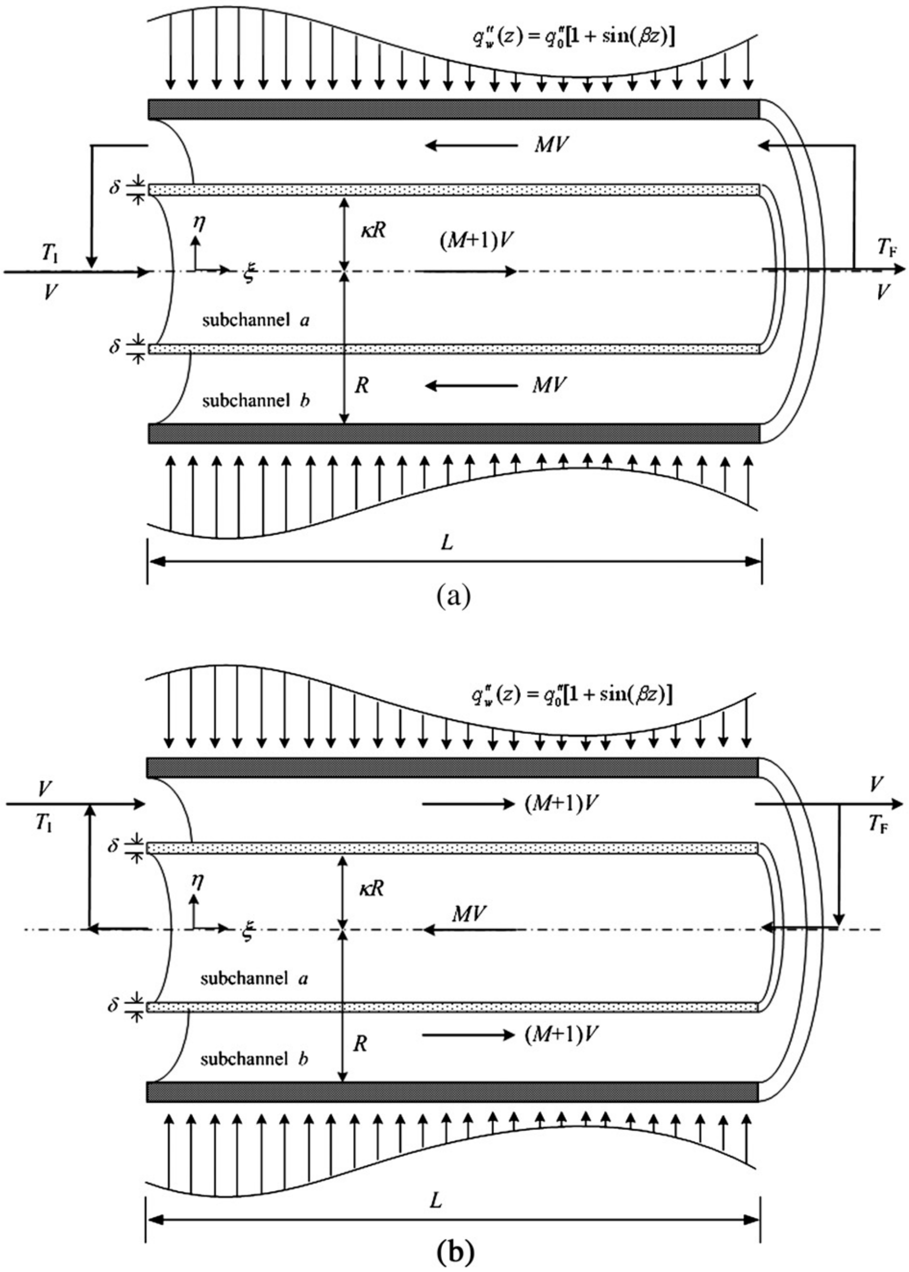

Fig. 1. Double-pass parallel-plate heat exchanger with recycle.

where k is the heat conductivity coefficient of the wall, D_e is the equivalent diameter of the conduit, $D_e = 2R$, and h is the heat-transfer coefficient defined as

$$h = \frac{k}{R} \frac{q''_w(\xi)}{q''_0 \phi_w(1,\xi)} = \frac{k}{R} \frac{1 + \sin(B\xi)}{\phi_w(1,\xi)} \tag{41}$$

Substituting Eq. (41) into Eq. (40) yields

$$Nu(\xi) = \frac{2[1 + \sin(B\xi)]}{\phi_w(1,\xi)} \tag{42}$$

Moreover, the average Nusselt number of a double-pass concentric-tube heat exchanger with external recycle was determined by

$$\overline{Nu} = Gz \int_0^{1/Gz} Nu(\xi) d\xi = Gz \int_0^{1/Gz} \frac{2[1 + \sin(B\xi)]}{\phi_w(1,\xi)} d\xi \tag{43}$$

The heat-transfer efficiency improvement by employing a double-pass operation was defined as the percent increase in heat transfer based on that in a single-pass device with the same working dimensions and operating parameters

$$I_h = \frac{\overline{Nu} - \overline{Nu_0}}{\overline{Nu_0}} (\%) \tag{44}$$

where $\overline{\mathrm{Nu}_0}$ is the average Nusselt numbers of the single-pass device [20].

4. Results and discussions

The material chosen for a heat exchanger depends on the working temperature. Unfortunately, the wall temperature of a heat exchanger with uniform or sinusoidal wall fluxes is difficult to estimate for an

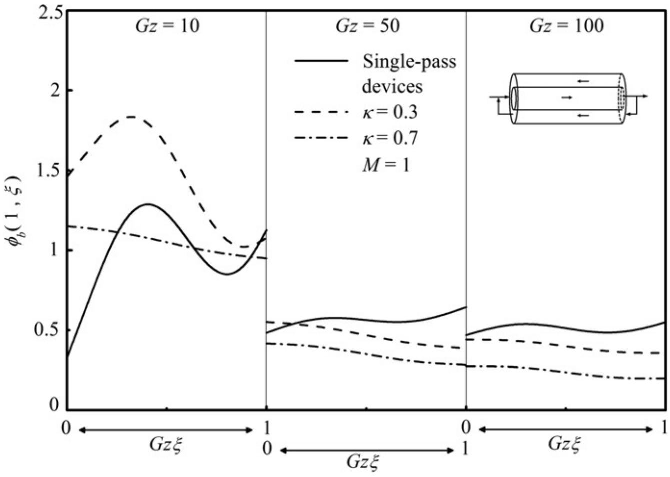

Fig. 2. Dimensionless wall temperature vs. $Gz\xi$ with κ and Gz as parameters; flow pattern A.

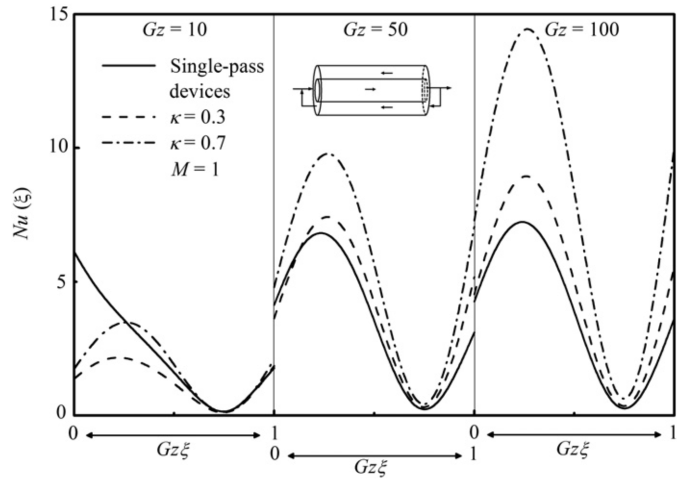

Fig. 4. Local Nusselt number vs. $Gz\xi$ with κ and Gz as parameters; flow pattern A.

engineer before designing. Therefore, the theoretical dimensionless wall temperatures of the double-pass heat exchangers under sinusoidal wall fluxes with external recycle were determined by solving the mathematical model analytically. The calculated results are illustrated in Figs. 2 and 3 for flow patterns A and B, respectively. The dimensionless wall temperature of flow pattern B is lower than that of flow pattern A by comparing with Figs. 2 and 3, which can be attributed to the cold feed flows into annular tube firstly in flow pattern B. The wall temperature distribution, due to the sinusoidal wall fluxes, is a sinusoidal curve and it increases along the working fluid flowing direction, as shown in Figs. 2 and 3. A larger Graetz number Gz means the higher working fluid volumetric flow rate or shorter conduit length resulting in short residence time of working fluid in conduit. Therefore, the dimensionless wall temperature decreases and more uniform with increasing Gz in both flow patterns, as demonstrated in Figs. 2 and 3. The effects of the impermeable barrier location κ on the dimensionless wall temperature are also shown in Figs. 2 and 3. A larger value of κ refers to the more narrow annular tube resulting in higher average velocity in annulus tube $\bar{v}_b$ and higher force heat convection rate of working fluid. Therefore, the dimensionless wall temperature decreases with increasing κ, as confirmed by Figs. 2 and 3.

The local Nusselt number distributions of the double- and single-pass concentric-tube heat exchangers are shown in Figs. 4 and 5 for flow patterns A and B, respectively. The Nusselt number is a dimensionless quantity of heat-transfer coefficient and provides a measure of convection heat transfer occurring on device wall surface. The local Nusselt number is inversely proportion to the dimensionless wall temperature in this study, as defined in Eq. (42). Therefore, the local Nusselt number increases with the increasing Graetz number Gz and the impermeable barrier location κ. The local Nusselt numbers of double-pass heat exchangers with external recycle in both flow patterns A and B are higher than that of a single-pass device for $Gz > 10$, as observed from Figs. 4 and 5. The heat-transfer efficiency improvement I_h of a double-pass device with external recycle is defined by Eq. (44). Table 1 shows the calculated results of the heat-transfer efficiency improvement I_h with recycle ratio M as a parameter. The heat-transfer efficiency improvement I_h increases with increasing Graetz number in both flow patterns, as indicated in Table 1. The minus signs in Table 1 represent that the concentric-tube heat exchanger performs lower heat-transfer efficiency than that of a single-pass device. The influences of recycle ratio M on the heat-transfer efficiency improvement I_h of two flow patterns are different. In flow pattern A, the heat-transfer efficiency improvement I_h increases with increasing recycle ratio M but the heat-transfer efficiency improvement I_h increases with decreasing recycle ratio M.

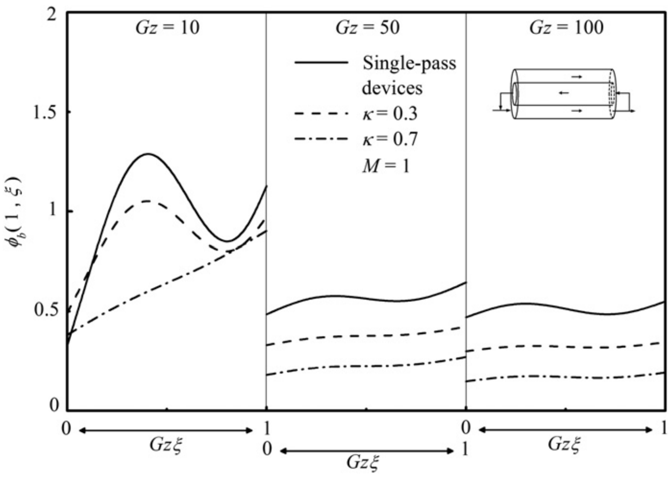

Fig. 3. Dimensionless wall temperature vs. $Gz\xi$ with κ and Gz as parameters; flow pattern B.

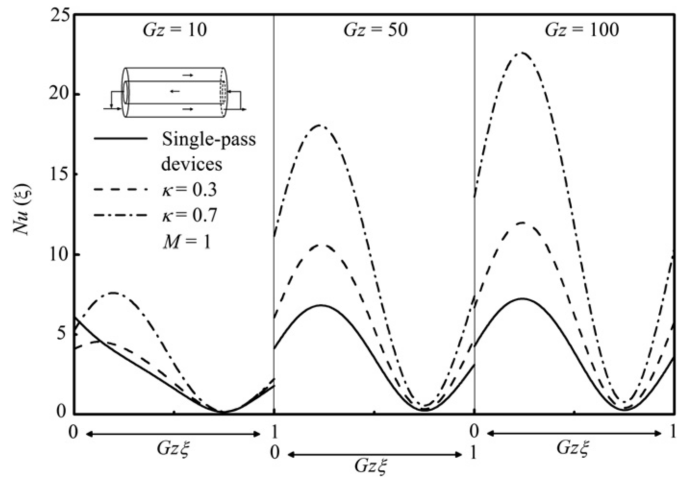

Fig. 5. Local Nusselt number vs. $Gz\xi$ with κ and Gz as parameters; flow pattern B.

Table 1
Heat-transfer efficiency improvement with recycle ratio M as a parameter; $\Delta=0.5$.

I_h	Flow pattern A		
	$M=1$	$M=3$	$M=5$
$Gz=1$	−0.49	−0.56	−0.65
$Gz=5$	−0.48	−0.39	−0.38
$Gz=10$	−0.33	−0.20	−0.17
$Gz=50$	0.20	0.41	0.47
$Gz=100$	0.56	0.75	0.81
I_h	Flow pattern B		
	$M=1$	$M=3$	$M=5$
$Gz=1$	−0.24	−0.45	−0.54
$Gz=5$	0.45	−0.08	−0.18
$Gz=10$	0.61	0.16	0.06
$Gz=50$	0.97	0.75	0.68
$Gz=100$	1.16	1.01	0.97

Comparing both flow patterns, one can find that the flow pattern B always performs better than flow pattern A.

5. Conclusions

The recycle concept and double-pass design were introduced to improve the heat-transfer efficiency of the heat exchangers under sinusoidal wall fluxes in this study. The theoretical mathematical model of the double-pass laminar counterflow concentric-tube heat exchangers with sinusoidal wall fluxes has been developed based on the energy balance. Two flow patterns were discussed and the temperature distributions of fluids in the conduit were obtained by solving the mathematical model. A general solution form was set to separate the original boundary value problem into a partial differential equation and an ordinary differential equation. The analytical solutions can be obtained by following the same mathematical treatment of previous work [20]. The calculating results of the dimensionless wall temperature and the local Nusselt numbers with the Graetz numbers Gz and impermeable barrier location κ as parameters were illustrated in this study. The dimensionless wall temperature decreases with the increasing Graetz numbers Gz and impermeable barrier location κ and the local Nusselt number increases with the increasing Graetz numbers Gz and impermeable barrier location κ. The influences of recycle ratio on the heat-transfer efficiency improvement I_h are shown in Table 1. The results indicate that the I_h increases with recycle ratio in flow pattern A but it decreases with recycle ratio in flow pattern B. Comparing to single-pass devices, the double-pass concentric-tube heat exchangers with sinusoidal wall fluxes perform higher heat-transfer efficiency especially for larger Graetz numbers. Furthermore, the flow pattern B achieves a good device performance than flow pattern A by comparing flow patterns A and B.

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