Coupled lateral bending–torsional vibration sensitivity of atomic force microscope cantilever

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Abstract

We study the influence of the contact stiffness and the ration between cantilever and tip lengths on the resonance frequencies and sensitivities of lateral cantilever modes. We derive expressions to determine both the effective resonance frequency and the mode sensitivity of an atomic force microscope (AFM) rectangular cantilever.

Once the contact stiffness is given, the resonance frequency and the sensitivity of the vibration modes can be obtained from the expression. The results show that each mode has a different resonant frequency to variations in contact stiffness and each frequency increased until it eventually reached a constant value at very high contact stiffness. The low-order vibration modes are more sensitive to vibration than the high-order mode when the contact stiffness is low. However, the situation is reversed when the lateral contact stiffness became higher. Furthermore, increasing the ratio of tip length to cantilever length increases the vibration frequency and the sensitivity of AFM cantilever.

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1. Introduction

The atomic force microscope (AFM) has widely used for imaging the surface topography of conductors and insulators on an atomic scale [1–5]. It is well known that a cantilever with a sharp pyramidal or conical tip at the free end plays an important role in AFM measurements. When a tip scans across a sample surface, it induces a dynamic interaction force between the tip and the surface. The dynamic behavior is complicated and precise analysis is difficult, but it can influence the resolution of the surface image. Therefore, the dynamic vibration analysis is significant enough to warrant further study.

In the last few years, many researchers have had a growing interest in studying the dynamic responses of the AFM cantilever [6–12]. For example, Wu et al. [9] studied the effects of tip length and normal and lateral contact stiffness on the flexural vibration responses of the AFM rectangular cantilever. Schäffer and Fuchs [10] presented a theoretical investigation of the sensitivity with which normal vibration modes of AFM cantilevers are detected using the optical beam deflection method. Recently, Song and Bhushan [13] performed forced vibration analysis of the AFM cantilever, considering both the torsional and lateral bending deflections of the cantilever. They found that the lateral bending of the cantilever can not be ignored if the lateral tip–sample interaction is relatively strong compared to the torsional or lateral bending stiffness of the cantilever. The torsional and lateral bending stiffness of a cantilever are typically two orders of magnitude larger than vertical bending. Therefore, they can be used to measure stiff and hard samples.

The resonant frequency of the cantilever can influence the imaging rate in the operating process; and the sensitivity can influence the image contrast. Therefore, the study of the resonant frequency and the sensitivity of an AFM cantilever are significant. In this paper, the

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resonant frequency and the sensitivity of coupled lateral bending–torsional vibration modes of an AFM rectangular cantilever was analyzed and a closed-form expression was derived. According to the expression, the resonant frequency and the sensitivity for each vibrational mode can be obtained.

2. Analysis

The AFM cantilever, shown in Fig. 1, is a rectangular cross section beam with width \( w \), thickness \( h \) and length \( L \). \( X \) is the coordinate along the longitudinal direction of the cantilever, \( t \) is time, \( \Phi(X, t) \) is rotation angle around the \( X \)-axis and \( V(X, t) \) is the lateral displacement along the \( Y \)-axis. One end of the cantilever, at \( X = 0 \), is clamped, while the other end, at \( X = L \), has a pyramidal tip with length \( H \). In torsional resonance mode, the tip interacts with the sample by a spring \( k \) for lateral interaction. The lateral tip–sample interaction results in a torque and a lateral force exerted on the cantilever, which cause the torsion vibration and lateral bending vibration, respectively. The torsion and lateral bending of the cantilever are governed by the following partial differential equations [13,14]:

\[
GJ \frac{\partial^2 \Phi(X, t)}{\partial X^2} = \rho I_p \frac{\partial^2 \Phi(X, t)}{\partial t^2},
\]

\[
EI_z \frac{\partial^4 V(X, t)}{\partial X^4} + \rho A \frac{\partial^2 V(X, t)}{\partial t^2} = 0,
\]

where \( E \) and \( G \) are Young’s modulus and shear modulus, \( \rho \) is the mass density, \( A \) is the cross-section area, \( J \) is the torsional constant, and \( I_p \) is the polar area moment of the inertia. For a rectangular cantilever, the torsional constant and the polar area moment of the inertia can be expressed as follows: \( J \approx (1/3)wh^3[1.0 - 0.63(h/w) + 0.052(h/w)^3] \) and \( I_p = (1/12)(hw^3 + wh^3) \), where \( w \) and \( h \) are the width and thickness of the cross-section, respectively. The corresponding boundary conditions are

\[
\Phi(0, t) = 0,
\]

\[
GJ \frac{\partial \Phi(L, t)}{\partial X} = -kH(H\Phi(L, t) + V(L, t)),
\]

\[
V(0, t) = 0,
\]

\[
\frac{\partial V(0, t)}{\partial X} = 0,
\]

\[
EI_z \frac{\partial^2 V(L, t)}{\partial X^2} = 0,
\]

\[
EI_z \frac{\partial^3 V(L, t)}{\partial X^3} = k(H\Phi(L, t) + V(L, t)),
\]

where \( L \) is the length of cantilever and \( H \) is the length of tip. The boundary condition of the cantilever at \( X = 0 \) is assumed fixed, then the boundary conditions given by Eqs. (3), (5), and (6) correspond to conditions of zero twist angle, zero lateral displacement, and zero lateral slope, respectively. A lateral force and a torque at \( X = L \) are exerted on the cantilever, resulting in it to deflect in a combination of torsion and lateral bending. The boundary conditions given by Eqs. (4), (7), and (8) correspond to the torque, the lateral bending moment and the lateral force between the beam and the linear lateral tip–sample stiffness, respectively, at \( X = L \).

We seek harmonic solution of the form as

\[
V(X, t) = v(X)e^{i\omega t}, \quad \Phi(X, t) = \theta(X)e^{i\omega t},
\]

where \( \omega \) is the radian frequency.

The dimensionless variables are defined as

\[
x = X/L, \quad v = V/L, \quad b = H/L,
\]

\[
p^2 = \frac{\omega^2 \rho I_p L^2}{GJ},
\]

\[
s = \frac{GJ}{EI_z}, \quad r = \frac{AL^2}{I_p},
\]

\[
y^4 = srp^2,
\]

\[
\beta = \frac{kLH^2}{GJ}.
\]

Substituting the harmonic solution of the form given in Eq. (9) into Eqs. (1)–(8) and using the dimensionless variables given in Eqs. (10). The govern equations and the associated boundary conditions can be simplified to the following dimensionless differential equations and boundary conditions:

\[
\frac{\partial^2 \theta(x)}{\partial x^2} = -p^2 \theta(x),
\]

Fig. 1. Schematic diagram of an AFM tip–cantilever due to a combination of torsion and lateral bending vibration.
According to dimensionless variables $p^2 = \sigma^2 \rho I_p L^2 / GJ$, given by Eq. (10), the frequency is obtained and given by

$$ f = \frac{p}{2\pi} \sqrt{\frac{GJ}{\rho I_p L^2}}. $$

The sensitivity can be obtained from the above frequency equation because the frequency can be measured. Differentiation of Eq. (21) with respect to $\beta$ yields

$$ \frac{df}{d\beta} = -\frac{C}{p} \frac{\partial C / \partial \beta}{\partial C / \partial p}. $$

The relationship between the dimensionless frequency $f$ and the dimensionless normal contact stiffness along the sample $\beta$ can be expressed as

$$ \frac{\partial f}{\partial \beta} = \frac{\partial f}{\partial p} \frac{\partial p}{\partial \beta}. $$

After calculation and simplification, we can obtain the following expression:

$$ \frac{df}{d\beta} = \frac{1}{2\pi} \sqrt{\frac{GJ}{\rho I_p L^2}} \frac{\partial p}{\partial \beta}. $$

A dimensionless form of the flexural sensitivity is given by

$$ \sigma = \frac{df / d\beta}{(1/2\pi) \sqrt{GJ / \rho I_p L^2}} = \frac{\partial p}{\partial \beta}. $$

$$ = \left[ b^2 \gamma^2 (1 + \cos(\gamma) \cosh(\gamma)) \sin(p) 
+ p \cos(p) (\cos(\gamma) \sin(\gamma) - \cos(\gamma) \sinh(\gamma)) \right] 
/ \left[ b^2 \gamma^3 (1 + \cos(\gamma) \cosh(\gamma)) (1 + \beta) \cos(p) - p \sin(p) \right] 
+ s \beta (\cos(p) - p \sin(p)) (\cosh(\gamma) \sin(\gamma) - \cos(\gamma) \sinh(\gamma))]. $$

From the above equation, the corresponding dimensionless sensitivities for each mode can be calculated.

### 3. Results and discussions

In this analysis, resonant frequency and sensitivity of an AFM cantilever were expressed as functions of material parameters and contact stiffness. In order to know the effect of relative parameters on the vibration responses, we considered the geometric and material parameters as follows: $E = 150$ GPa, $G = 64$ GPa, $\rho = 2300$ kg/m$^3$, $L = 200$ μm, $w = 40$ μm, $h = 5$ μm, $H = 15$ μm. The resonance frequency and the sensitivity of AFM cantilever can be influenced by the lateral contact stiffness at the boundary. Once the contact stiffness is given, the resonance frequency of the vibration modes can be found from solving the vibration problem. Based on the coupled lateral bending–torsional vibration analysis, the dimensionless frequencies of the first five modes of flexural vibration were shown in Fig. 2. As $\beta$ increased, the frequency increased until it eventually reached a constant value at very high...
contact stiffness. Each mode has a different resonant frequency to variations in contact stiffness and the first mode was the lowest frequency. The dimensionless sensitivities of the first five modes as a function of lateral contact stiffness are depicted in Fig. 3. From the figure, it can be seen that the probe is sensitive to changes in the contact stiffness. When the lateral contact stiffness is low, the low-order vibration modes are more sensitive to vibration than the high-order mode and the first mode was the most sensitive. However, the situation is reversed when the lateral contact stiffness became higher.

Fig. 4 depicts the dimensionless frequency of mode 1 of AFM cantilever scanning a sample surface as functions of the contact stiffness and the ratio of tip length to cantilever length. When the ratio of the tip length to the cantilever length increases, the moment induced by the lateral tip–sample interaction force increases and that makes the cantilever stiffer. Therefore, from this figure it can be seen that increasing the ratio of tip length to cantilever length increases the vibration frequency. The trend is more apparent, particularly when the contact stiffness became larger. In addition, it can be found that the frequency rapidly increased as $\beta$ increased.

Fig. 5 shows the dimensionless sensitivity of mode 1 of AFM cantilever scanning a sample surface as functions of the contact stiffness and the ratio of tip length to cantilever length. The effects of the contact stiffness and the ratio of tip length to cantilever length on the modal sensitivity are similar to those on the vibrational frequency. This is because the modal sensitivity of cantilever is obtained according to the changes in its frequency.

4. Conclusions

In this paper, the resonant frequency and the sensitivity of lateral vibration for an atomic force microscope (AFM) cantilever due to the coupled lateral bending–torsional vibration has been analyzed. According to the analysis, the results showed that the frequency increased until it eventually reached a constant value at very high contact stiffness as the lateral contact stiffness $\beta$ increased. Each mode has a different resonant frequency and sensitivity to variations in contact stiffness. When $\beta$ was low, the low-order vibration modes were more sensitive to vibration than the high-order mode and the first mode was the most sensitive. However, the situation was reversed when $\beta$
became higher. In addition, increasing the ratio of tip length to cantilever length increased the vibration frequency and sensitivity. The trend was more apparent, particularly when the contact stiffness became larger.

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References