JOURNAL OF MATERIALS PROCESSING TECHNOLOGY 209 (2009) 2940–2945

journal homepage: www.elsevier.com/locate/jmatprotec

Analytical solution of flexural vibration responses on nanoscale processing using atomic force microscopy

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ARTICLE INFO

Article history:
Received 29 March 2007
Received in revised form
25 June 2008
Accepted 28 June 2008

Keywords:
Modal superposition method
Flexural vibration problem
Nanomachining process
Tip holder
Atomic force microscope (AFM)

ABSTRACT

An analytical solution is developed to deal with the flexural vibration problem during a nanomachining process which involves an atomic force microscope (AFM) cantilever. The modal superposition method is employed to analyze the response of an AFM subjected to a cutting force with an excitation force of an arbitrarily chosen frequency. The cutting forces were transformed into distributed transversal and bending loading, and were applied to the end region of the AFM by means of the tip holder. The effects of transverse stress and bending stress were adopted to solve the dynamic model. Based on the result, applying a cutting force with an excitation force near the high-order modal frequencies and using a wide tip holder are recommended when nanoscale processing using AFM is performed.

1. Introduction

In this investigation, the solution of the vibration response in nanoscale processing using atomic force microscopy is obtained by using the modal superposition method. The atomic force microscope (AFM) was developed for producing high-resolution images of surface structures of both wear zone (Pramanik et al., 2003) and machined surface topography (Zhou et al., 2006). In addition, the AFM can be used as a cutting tool for nanolithography work (Chang et al., 2004) and as a nanoindentation tester for evaluating mechanical properties (Miyahara et al., 1999). In recent years, AFM has also been applied to nanolithography, as it possesses higher resolution capabilities than conventional optical and electron-beam lithography (Majumdar et al., 1995), in micro/nanoelectromechanical systems (MEMS/NEMS) (Notargiacomo et al., 1999). With the development of new nanodevices, it has become increasingly difficult to conduct the fabrication processes using conventional lithography. However, AFM-based mechanical lithography has been demonstrated as a very useful technique for machining diverse nanostructures, such as semiconductors, metals, and soft materials. Hu et al. (1998) used mechanical AFM lithography on ultrathin Ti film, while Fang and Chang (2003) carried out several scribing experiments to study the machining characterizations of the nanolithography process using AFM, and conducted a surface analysis of nanomachining Al material by AFM. They found that the AFM cantilever should not be too soft and it should have a high resonant frequency in order to minimize sensitivity to vibration noise from buildings and to have a large cutting bandwidth. Although the dynamic vibration response is complicated and precise analysis is difficult, it can influence the precision of machining in the operating process, and therefore requires further investigation. The vibration behavior depends on the excitation forces, which are composed of the transverse stress

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0924-0136/$ – see front matter
doi:10.1016/j.jmatprotec.2008.06.059

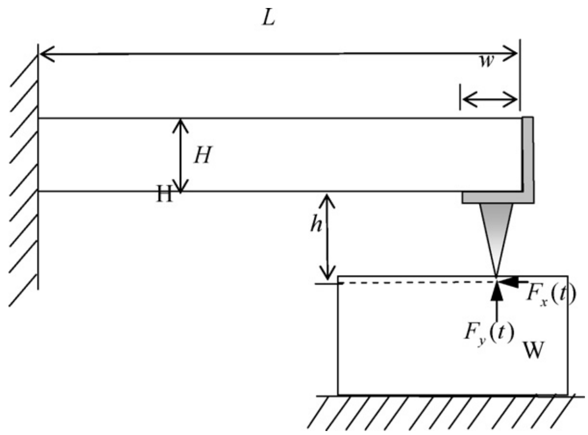

Fig. 1 – Schematic diagram of an AFM cantilever nanomachining a sample.

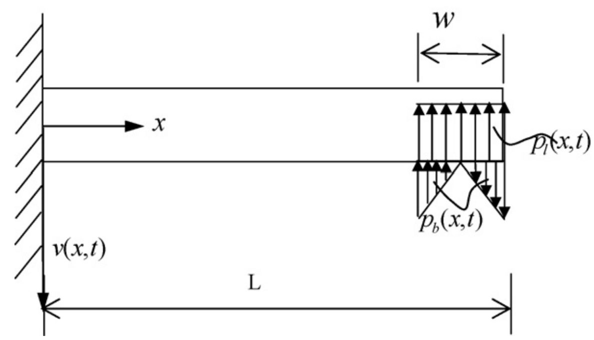

Fig. 2 – Schematic diagram of excitations acting on the AFM.

and bending stress, which act on the tip holder and are transmitted from the cutting force.

In the last few years, there has being growing interest in the dynamic responses of the AFM cantilever (Rabe et al., 1996). Ashhab et al. (1999) analyzed the dynamic behavior of a microcantilever-sample system in order to control the cantilever-sample interaction and avoid the possibility of chaos. Based on the calculation of the contact radius and radiation impedance, Yaralioglu et al. (2000) proposed an algorithm to calculate the contact stiffness between an AFM tip and a layered material as a function of layer thickness. Nevertheless, it seems that distributed transversal and bending loading, applied to the end region of AFM by means of tip the holder, appear to be absent from the literature.

In this paper, the flexural vibration responses of a rectangular AFM cantilever subjected to a cutting force which can be arbitrarily chosen are studied analytically by using the modal superposition method. The effects of transverse stress and bending stress are adopted to solve the dynamic model. The results show that the bending effect resulting from the horizontal cutting force has a significant, if not first order, effect on the vibration response. Meanwhile, the magnitude of the response decreases with increasing tip holder width when the excitation forces are about to approach the high-order mode. Based on these results, the use of a cutting force with an excitation force near the high-order modal frequencies and a wide tip holder are recommended for nanoscale processing using AFM.

2. Analysis

In this paper, AFM cantilever moves down vertically with small amplitude (1–5 nm) when the cantilever tip processes a sample surface in contact mode. Therefore, the linear model can be used to describe the tip-sample interaction. The atomic force microscope cantilever, shown in Fig. 1, is a small elastic beam with a length L, thickness H, width a, and a tip holder with a width of w and tip length h. x is the coordinate along the cantilever and $y(x,t)$ is the vertical deflection in x-direction as shown in Fig. 2. One end of the cantilever, at $x=0$, is clamped, while the another end, at $x=L$, has a conical tip.

When the machining is in progress, the tip makes contact with the specimen, resulting in a vertical reaction force, $F_y(t)$, and a horizontal reaction force, $F_x(t)$, which are both functions of time t. Assuming that the reaction forces are acted on the tip end, the product of the horizontal force and the tip length can form bending stress on the bottom surface of cantilever. The cutting system can be modeled as a flexural vibration motion of the cantilever. The motion is a function of mode shape and natural frequency, and its transverse displacement is dependent on time and the spatial coordinate x (Chang and Chu, 2003; Wu et al., 2004). When the cutting forces are applied, the loads transmitted from the tip holder act on the end of the AFM, and can be modeled as the two parts shown in Fig. 2, termed transverse excitation $p_l(x,t)$ and bending excitation $p_b(x,t)$, respectively.

Suppose transverse excitation is uniformly distributed on the bottom surface of the AFM, it can be written as:

$$p_l(x,t) = \frac{F_y(t)}{w}, \qquad L-w \le x \le L \tag{1}$$

The relationship between $F_x(t)$ and $F_y(t)$ can be expressed as $F_x = (2\cos\theta/\pi)F_y$ for a cone shape cantilever tip, where θ is the half-conic angle. The bending excitations, which resulted from the horizontal cutting force $F_x(t)$, are acted on the bottom surface of the AFM within the region from $L-w$ to L, and can be written as:

$$p_b(x,t) = \left[\frac{12h(2L-w-2x)\cos\theta}{\pi w^3}\right] F_y, \qquad L-w \le x \le L \tag{2}$$

By summing the above two excitations, the total transverse excitation $P_t(x,t)$ can be expressed as:

$$p_t(x,t) = C(x)F_y(t), \qquad L-w \le x \le L \tag{3}$$

where

$$C(x) = \frac{1+12h(2L-w)\cos\theta/\pi w^2}{w} - \frac{24h\cos\theta}{\pi w^3}x \tag{4}$$

The mode-superposition analysis of a distributed-parameter system is entirely equivalent to that of a discrete-coordinate system once the mode shapes and frequencies have been determined because in both cases the amplitudes of the modal-response components are used as generalized coordinates in defining the response of the structure. In principle, an infinite number of these coordinates are available for a distributed-parameter system, since it has an infinite number of modes of vibration. Practically, however, only those modal components which provide significant contributions to the response need be considered. (Ray and Joseph, 1993; William, 1998).

Considering a prismatic member with uniform properties along its length, the partial differential equation of motion for the elementary case of beam flexure can be written as:

$$\frac{\partial^2}{\partial x^2}\left[EI\frac{\partial^2 v(x,t)}{\partial x^2}\right]+\bar{m}\frac{\partial^2 v(x,t)}{\partial t^2}=p(x,t) \quad (5)$$

where $v(x,t)$ is the transversal displacement response, $p(x,t)$ is the transversal excitation, EI is the flexure stiffness, and $\bar{m}$ is the mass per unit length. The essential operation of the mode-superposition analysis is the transformation from the geometric displacement coordinates to the modal-amplitude or normal coordinates. For a one-dimensional system, this transformation is expressed as:

$$v(x,t)=\sum_{n=1}^{\infty}\phi_n(x)Y_n(t)=\sum_{n=1}^{\infty}q_n(x,t) \quad (6)$$

where $q_n(x,t)$ is the response contribution of the n-th mode, $Y_n(t)$ is normal coordinate, and $\phi_n(x)$ is the n-th mode shape of the AFM given by (Ray and Joseph, 1993; William, 1998):

$$\phi_n(x)=\left[\cos\alpha_n x-\cosh\alpha_n x-\frac{(\cos\alpha_n L+\cosh\alpha_n L)}{(\sin\alpha_n L+\sinh\alpha_n L)}(\sin\alpha_n x+\sinh\alpha_n x)\right] \quad (7)$$

and

$$\alpha_n L \doteq \begin{cases} 1.857, & n=1 \\ 4.694, & n=2 \\ 7.855, & n=3 \\ \frac{\pi}{2}(2n-1), & n=4,5,6,\cdots \end{cases} \quad (8)$$

Eq. (7) simply states that any physically permissible displacement pattern can be modeled by superposing appropriate amplitudes of the vibration mode shapes for the structure. Substituting Eq. (6) into Eq. (5) and using orthogonality conditions gives

$$M_n\ddot{Y}_n(t)+\omega_n^2 M_n Y_n(t)=P_n(t) \quad (9)$$

where ω_n is the n-th mode natural frequency of the AFM from (Ray and Joseph, 1993; William, 1998)

$$\omega_n=(\alpha_n L)^2\sqrt{\frac{EI}{\bar{m}L^4}} \quad (10)$$

M_n and p_n are the generalized mass and generalized load of the n-th mode, respectively, given by

$$\omega_n=(\alpha_n L)^2\sqrt{\frac{EI}{\bar{m}L^4}} \quad (11)$$

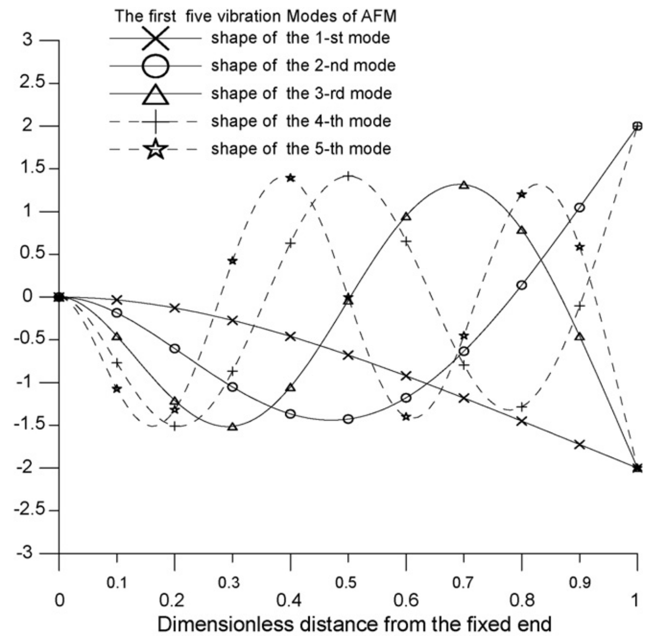

Fig. 3 – The shape of the first five vibration modes for an AFM cantilever.

$$P_n(t)=\int_0^L \phi_n(x)p_t(x,t)\,dx \quad (12)$$

By using Eqs. (3), (4) and (12) can be rewritten as

$$P_n(t)=c_n F_y(t) \quad (13)$$

where

$$c_n=\int_0^L \phi_n(x)\left(\frac{1+12h(2L-w)\cos\theta/\pi w^2}{w}-\frac{24h\cos\theta}{\pi w^3}x\right)dx \quad (14)$$

Then the Normal-Coordinate Response Equation, which is exactly the same equation considered for the discrete-parameter case, can be solved.

$$\ddot{Y}_n(t)+\omega_n^2 Y_n(t)=\frac{c_n}{M_n}F_y(t) \quad (15)$$

The Duhamel solution of this equation is

$$Y_n(t)=\frac{c_n}{M_n\omega_n}\int_0^t F_y(\tau)\sin\omega_n(t-\tau)\,d\tau \quad (16)$$

Assuming a zero initial condition, with $v(x,0)=0$, $\dot{v}(x,0)=0$, and provided that cutting force $F_y(t)$ is a series of harmonics, $F_y(t)$ can be written as:

$$F_y(t)=\sum_{i=1}^{m}F_i\sin(\omega_i t) \quad (17)$$

When the j-th excitation frequency ω_j is equal to the n-th natural frequency ω_n, the solution of the generalized coordi-

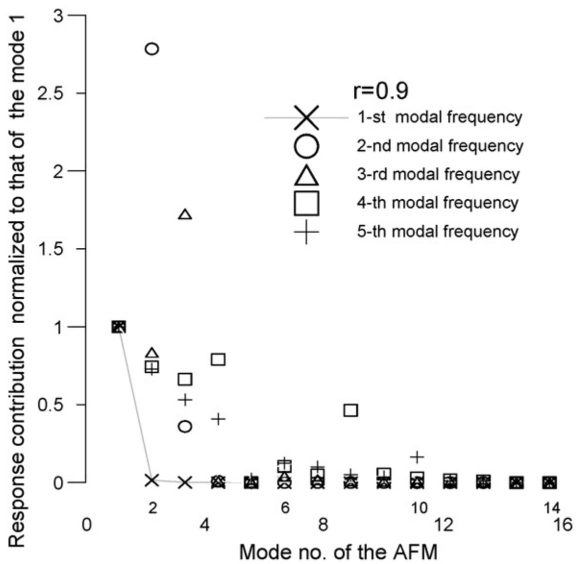

Fig. 4 – The response contribution of each mode for an AFM cantilever subjected to various excitation frequencies with $r = 0.9$ at $t = 0.1$ s and $x = L$.

nate can be written as:

$$Y_n(t) = \left[\frac{c_n}{2\omega_n\omega_j M_n}F_j - \frac{c_n}{\omega_n M_n}\sum_{i=1, i\neq j}^{m}\frac{F_i\omega_i}{\omega_n^2-\omega_i{}^2}\right]\sin(\omega_n t) + \frac{c_n}{M_n}\left(\sum_{i=1, i\neq j}^{m}F_i\frac{\sin(\omega_i t)}{\omega_n^2-\omega_i{}^2}\right) - \frac{c_n}{2\omega_j M_n}F_j t\cos(\omega_j t) \quad (18)$$

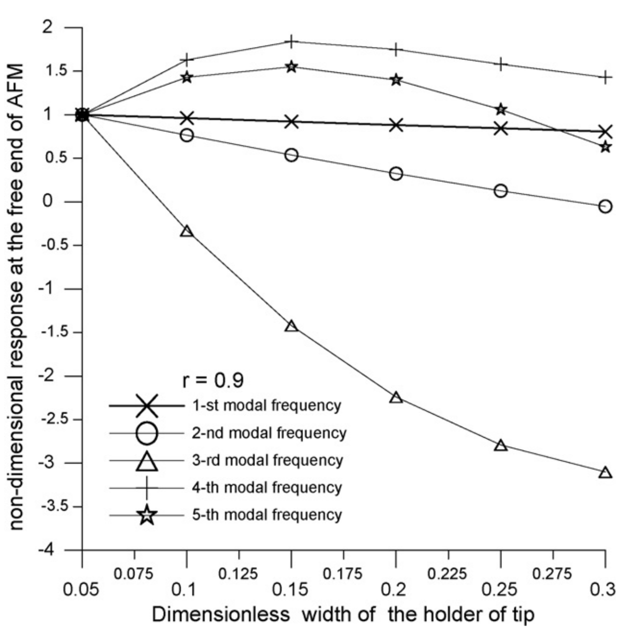

Fig. 5 – The effect of various tip holder widths w on the response of the end point for the first five modes of $r\omega_n$.

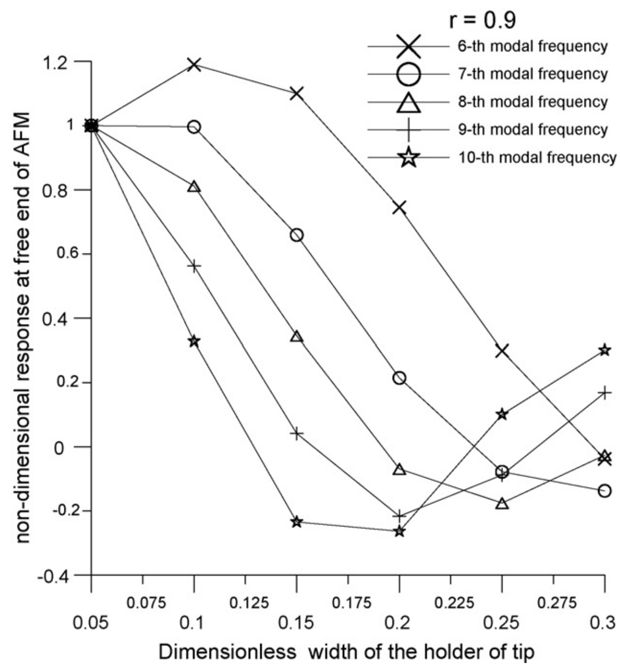

Fig. 6 – The effect of various tip holder widths w on the response of the end point for the high-order modes of $r\omega_n$.

3. Results and discussion

The main purpose of this study is to analyze flexural vibration responses in nanoscale processing using atomic force microscopy. The modal superposition method was employed to analyze the response of an AFM subjected to an arbitrarily chosen cutting force. The cutting force was transformed into distributed loading and applied to the end of the AFM by means of a tip holder, and the effects of transverse stress, and bending stress were used to solve the dynamic model. In order to demonstrate the validity of the analytical solution, numerical computations were performed. The geometric and material parameters considered were as follows:

$E = 170$ Gpa, $\bar{m} = 0.2936$ km/μm, $L = 125$ μm, $a = 30$ μm, $H = 4.2$ μm, $h = 5$ μm, $\theta = 30°$ and $h = 5$ μm. $m = 3$, $F_i = 1000/(2i-1)\times 10^{-9}$ and $\omega_i = (2i-1)\times r\omega_n$ were set as the simulated values of the excitation frequency of the vertical cutting force. Thus, $F_y(t)$ is taken as:

$$F_y(t) = 1000\left(\sin r\omega_n t + \frac{1}{3}\sin 3\times r\omega_n t + \frac{1}{5}\sin 5\times r\omega_n t\right)(nN) \quad (19)$$

where r is the frequency ratio. In this study, r is set to 0.9, and a non-dimensional response was used to normalized the static response, defined as $F_1L^3/(3EI)$. The shapes, $\phi(x)$, of the first five vibration modes for an AFM cantilever are pictured in Fig. 3.

Firstly, in order to investigate the effects of each mode on the transversal response, the response contribution of each mode normalized by the first mode for an AFM cantilever subjected to various excitation frequencies at $t = 1$ sc and $x = L$ are plotted in Fig. 4. As expected, only 10 modal components need to be considered, with the first three modals being the most influential to the response. Fig. 4 reveals that the response

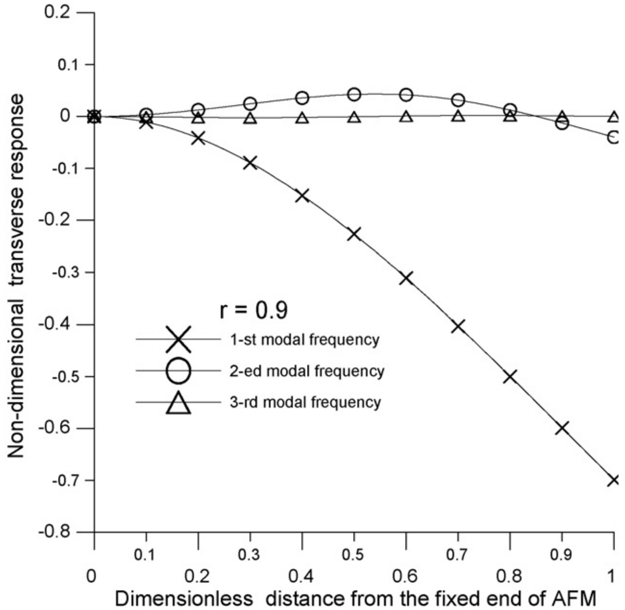

Fig. 7 – The response for an AFM cantilever subjected to the first three modes of $r\omega_n$.

is also strongly affected by the response contribution of n-mode, q_n when excitation frequencies are close to the natural frequency ω_n. This relationship is valid until the excitation frequencies are close to other high-order modes of the natural frequency. Figs. 5 and 6 respectively show the effects of various tip holder widths, w, on the response of the end point for the first five modes of $r\omega_n$ and those of the higher modes. In these two figures, the width is normalized by the length of the AFM, and the response for different widths of the tip holder is normalized by that of the narrowest tip holder, which was 0.05 L. From Fig. 5, it is seen that except for the first mode of excitation $r\omega_1$, the width of the tip holder has a significant effect on the response at the end point, especially when the excitation frequencies are near $r\omega_3$ and $r\omega_4$. This is because the average slope deviation near the exerting region of the tip holder for the first mode shape is relatively small compared with those of the third and the fourth modes. Moreover, the results shown in Fig. 6 reveal that the response at the free end decreases as the width of the tip holder increases when the exciting forces close to the high-order mode of $r\omega_n$. By assuming the zero initial condition, Figs. 7 and 8 show, respectively, the transverse response at $t = 0.1$ s and the free end point for an AFM cantilever subjected to the first three modes of $r\omega_n$ and that of the high-order modes. Figs. 9 and 10 show the resonance

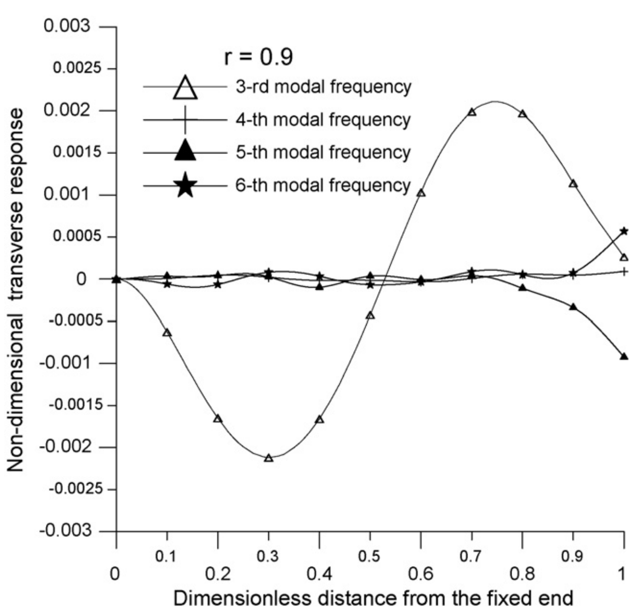

Fig. 8 – The response for an AFM cantilever subjected to the high-order modes of $r\omega_n$.

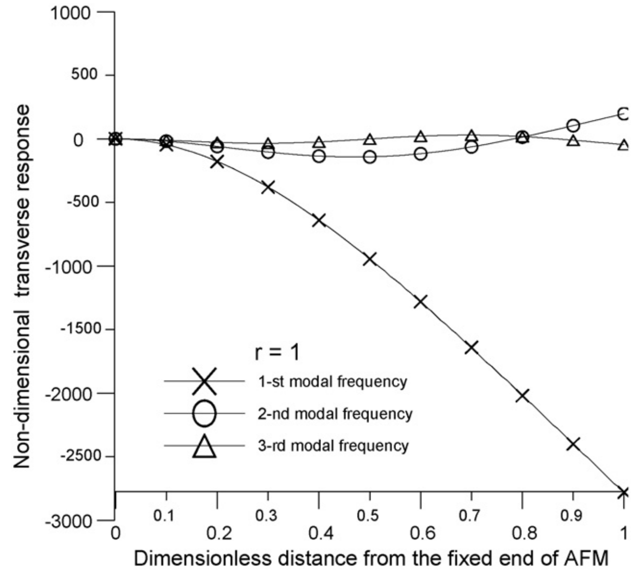

Fig. 9 – The resonance response for an AFM cantilever subjected to harmonics cutting forces which are composed of the first three modes of the natural frequencies.

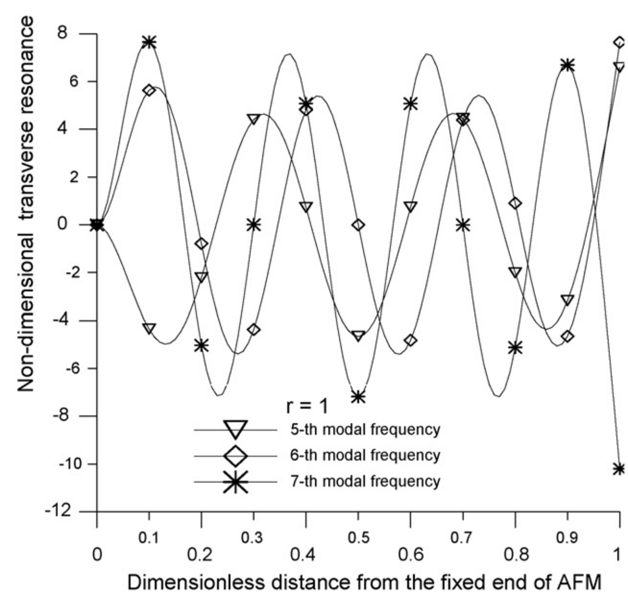

Fig. 10 – The resonance response for an AFM cantilever subjected to harmonics cutting forces which are composed of the high-order modes of the natural frequencies.

response at $t = 0.1$ s for an AFM cantilever subjected to the first three modes of ω_n and that of the high-order modes, respectively. It was found that the vibration magnitudes abruptly become large in the resonance response, and the deformation curve is similar to the mode shape under the low-order modal frequency. Resonance effects are not apparent under the high-order modal frequency.

4. Conclusions

The modal superposition method was successfully applied to an AFM-based nanomachining process to determine flexural vibration responses. The analytical solution can be employed to evaluate frequencies of a cantilever with arbitrary tip holder widths and various cutting forces, including near or equal to the modal frequencies. As expected, only a finite number of modal components, which contribute most to the response, need be considered. The contributions from other modes, in contrast, are too trivial to be accounted for. In a machining process, the cutting force acts its excitation frequency on an AFM from the modal frequencies, especially for the first modal frequency. The magnitude of the response decreases as the width of tip holder increases when the excitation forces approach the high-order mode. This means that a wide tip holder seems to be suitable for excitation frequencies near the high-order mode. Based on there results, using a cutting force with an excitation force near the high-order modal frequencies and a wide tip holder are recommended for nanoscale processing involving AFM.

Acknowledgements

This work was supported by the National Science Council, Taiwan, Republic of China, under grant numbers NSC-93-2212-E-168-018 and NSC 95-2622-E-168-015-CC3.

REFERENCES

Ashhab, M., Salapaka, M.V., Dahleh, M., Mezic, I., 1999. Dynamical analysis and control of microcantilevers. Automatica 35, 1663–1670.

Chang, W.J., Chu, S.S., 2003. Analytical solution of flexural vibration response on taped atomic force microscope cantilevers. Phys. Lett. A 309, 133–137.

Chang, W.J., Fang, T.H., Weng, C.I., 2004. Inverse determination of the cutting force on nanoscale processing using atomic force microscopy. Nanotechnology 15, 427–430.

Fang, T.H., Chang, W.J., 2003. Effect of AFM-based nanomachining process on aluminum surface. J. Phys. Chem. Solids 64, 913–918.

Hu, S., Hamidi, A., Altrneyer, S., Koster, T., Spangenberg, B., Kurz, H., 1998. Fabrication of silicon and metal nanowires and dots using mechanical atomic force lithography. J. Vac. Sci. Technol. B 16, 2822–2824.

Majumdar, A., Lai, J., Chandrachood, M., Nakabeppu, O., Wu, Y., Shi, Z., 1995. Thermal imaging by atomic force microscopy using thermocouple cantilever probes. Rev. Sci. Instrum. 66, 3584.

Miyahara, K., Nagashima, N., Ohmura, T., Matsuoka, S., 1999. Evaluation of mechanical properties scale using AFM-based nanoindentation tester. Nanostruct. Mater. 12, 1049–1052.

Notargiacomo, A., Foglietti, V., Cianci, E., Capellini, G., Adami, M., Faraci, P., Evangelisti, F., Nicolin, C., 1999. Atomic force microscopy lithography as a nanodevice development technique. Nanotechnology 10, 458–463.

Pramanik, A., Neo, K.S., Rahman, M., Li, X.P., Sawa, M., Maeda, Y., 2003. Cutting performance of diamond tools during ultra-precision turning of electroless-nickel plated die materials. J. Mater. Process. Technol. 140, 308–313.

Rabe, U., Janser, K., Arnold, W., 1996. Vibrations of free and surface-coupled atomic force microscope cantilevers: theory and experiment. Rev. Sci. Instrum. 67, 3281.

Ray, W., Joseph, P., 1993. Dynamic of Structure. McGraw-Hill, Inc.

William, T., 1998. Theory of Vibrations with Application. Prentice Hall.

Wu, T.S., Chang, W.J., Hsu, J.C., 2004. Effect of tip length and normal and lateral contact stiffness on the flexural vibration response of atomic force microscope cantilevers. Microelectronic Eng. 71, 15–20.

Yaralioglu, G.G., Degertekin, F.L., Crozier, K.B., Quate, C.F., 2000. Contact stiffness of layered materials for ultrasonic atomic force microscopy. J. Appl. Phys. 87, 7491.

Zhou, M., Ngoi, B.K.A., Yusoff, M.N., Wang, X.J., 2006. Tool wear and surface finish in diamond cutting of optical glass. J. Mater. Process. Technol. 174, 29–33.