

適應性模式追蹤 PID 控制器之頻域性能研究
A Study on the Frequency Domain Performance of the Adaptive Model-following PID Controller

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摘要

本文主要係將電腦輔助頻域分析法應用於適應性模式追蹤 PID 控制器，首先將超穩定理論應用於適應性模式追蹤 PID 控制器，接著討論利用 Matlab/Simulink 軟體的電腦輔助頻域分析法在適應性模式追蹤 PID 控制器之應用。本文主要模擬分析 PID 控制器在有/無適應性模式追蹤控制的效應時之階級響應,而在頻域上之模式追蹤性能分析顯示,PID 控制器的頻寬可由合適的適應性調整信號加以改善。
關鍵詞：模式追蹤控制，適應性控制，頻域分析

Abstract

In this paper, the hyperstability theory applied to the adaptive model-following PID controller is first presented. Second, a new method of frequency domain analysis with the Matlab/Simulink software on the adaptive model-following PID controller is presented, From the simulation results, the step responses of the PID controller with and without the adaptive model-following control effects are discussed. The model-following performances on the frequency domain also show the bandwidth of the PID controller can be improved by a suitable adaptation signal.
Keywords: model-following control, adaptive control, frequency domain analysis

I. INTRODUCTION

The need for accurate control systems for industrial applications has produced great research efforts for applying advanced control theory. The adaptive model- following control(AMFC) theory is one of the principal approaches in advanced control. The AMFC technique has been widely described in the literatures[1-4]. The purpose of AMFC is to force the dynamic response of a plant to follow the response of a reference model by a correction mechanism within itself. There are several approaches to the design of the AMFC system: (1) the Lyaponov function method, (2) the variable structure system theory, (3) hyperstability theory, (4) the robust control theory and (5) the fuzzy control theory. The third approach is used in this paper to develop the AMFC system. In 1989, Shih and Sheu[5] have shown that this approach is useful for the position control of a servo-hydraulic cylinder under the variable loading effect. In 1991, Wei and others.[6] proposed the adaptive model-following PID control using hyperstability theory for nonlinear plant tracking. In 2004, Wei and others.[7] first used the Matlab/Simulink software to simulate the tracking performance of the model- following PID controller. In 2005, Tzou and others.[8] applied the adaptive model-following with PID compensator on the autopilot of motorcycles.

Using the Popov hyperstability theory to design the AMFC system is carried out either through adjustment of the boundaries in a controller or through synthesis of the control input[9]. From the engineering viewpoint, synthesizing the control input is feasiable for on-line computer implementation because the signal produced from the adaptation mechanism is easily superimposed on the other control signals and does not affect the original hardware. The most commonly used control algorithms in industry are the three mode PID controllers. Therefore, the technique by which to combine the conventional PID signal and the signal genernated from the AMFC system will be explored in this paper.

Frequency response methods were developed by Nyquist and Bode in the 1930s[10]. The idea of frequency response is the sinusoidal inputs to a linear system form a sinusoidal responses of the same frequency in the steady

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state. Even though these responses are of the same frequency as the input, they differ in amplitude and phase angle from the input. These differences are a function of frequency. When the system is not a linear system, for example: it is a nonlinear system or adaptive control system, the sinusoidal transfer function cannot be gained. In such cases, the frequency response of the system, from input to output, can be gained by using a sinusoidal force at the input to the system and measure the output steady state sinusoidal amplitude and phase angle.

In this paper, the frequency response of the adaptive model-following PID controller will be researched with the Matlab/Simulink software. The size and phase bandwidth of the adaptive model-following PID controller in an adaptive system are not shown in any previous paper and will be first discussed in this paper. From the simulation results, the model following performances on the frequency domain and the step responses of the PID controller with and without the adaptive model-following control effects will be discussed.

II. ADAPTIVE MODEL-FOLLOWING PID CONTROLLER

Adaptive control is usually classified as self-tuning control and model reference adaptive control as shown in Figs. 1 and 2. Intuitively, model reference adaptive control can directly satisfy the conventional control theory's need, such as rise time, overshoot, and damping coefficient. The theory of model reference adaptive control is the most popular method applied on the tracking control.

The AMFC control theory is one of the principal approaches in model reference adaptive control. The purpose of AMFC is to use the adaptation mechanism to force the dynamic response of a plant to follow the response of a reference model when the plant is under external load or disturbance. The benefits of AMFC are there is no need to adjust the boundaries in a controller on-line. Instead, synthesizing control signal to perform the control purpose is used.

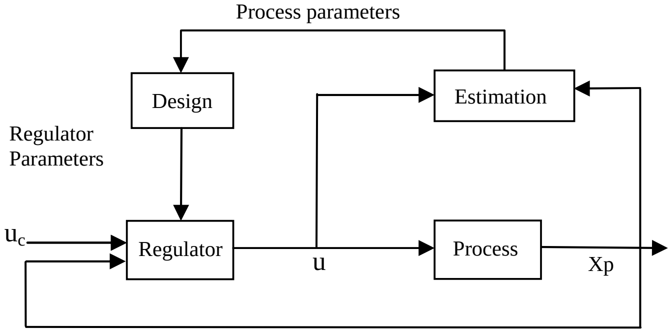

Fig. 1 Self-tuning control

The hyperstability approach is used in this paper to develop the adaptive model-following PID controller [6]. The main configuration of AMFCsystem includes AMFC system with parameters adaptation (shown in Fig. 3), AMFC system with synthesis of control signal (shown in Fig. 4) and AMFC system with synthesis of control signal and external disturbance (shown in Fig. 5).

The configuration shown in Fig. 5, a plant is described in the following state equation:

$$\dot{X}_P = A_p X_P + B_P U_P + H(X_P, U_P, t) \qquad (1)$$

where $H(X_P, U_P, t)$ is used to represent the sum of the modeling errors and the unmeasured disturbances acting on the plant[4].

The potential choices for the reference model in the design of the AMFC system are of many varieties. The reference model in this paper is a closed loop control system with a PID compensator. This provided a useful guide for selecting a reference model. In this paper, a new control algorithm is proposed whereby the extra loop produces

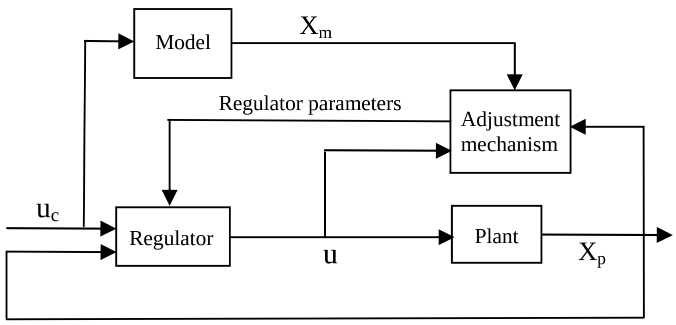

Fig. 2 Model reference adaptive control

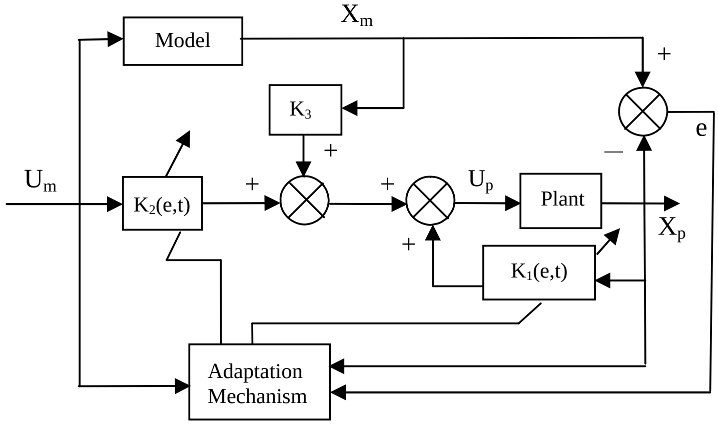

Fig. 3 AMFC system with parameters adaptation

a correction signal and is superimposed on the conventional PID control signal to ensure the performance of the adaptive model-following control. When the extra loop signal is null, the AMFC system is reduced to the conventional PID control system.

The reference model is a closed loop system that is compensated by the PID controller and is written as follows:

$$\dot{X}_m = A_m X_m + B_m U_S \tag{2}$$

where U_S is the control input of the reference model, which is given as

$$U_s = K_p(U_m - K_b X_m) + K_i I_m + K_d D_m + B_i \tag{3}$$

where $I_m = \int_0^t (U_m - K_b X_m) d\tau$, $D_m = \frac{d}{dt}(U_m - K_b X_m)$

where K_p, K_i, K_d are chosen by the designer to obtain the

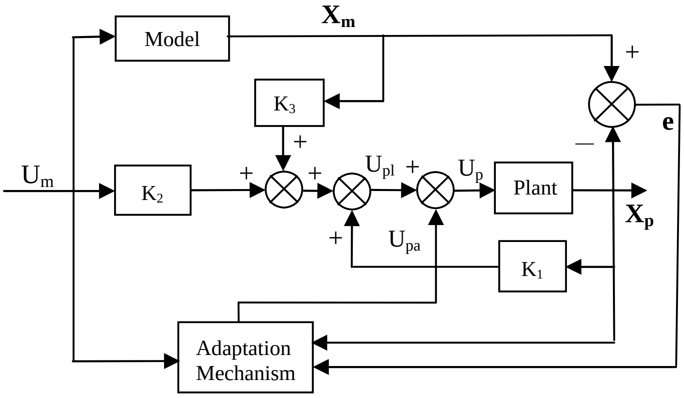

Fig. 4 AMFC system with synthesis of control signals

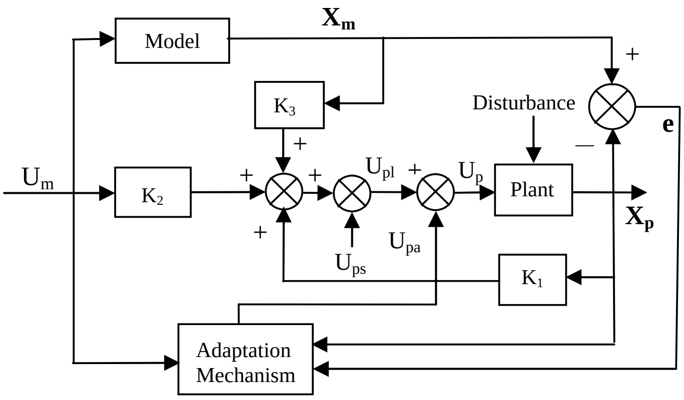

Fig. 5 AMFC system with synthesis of control signal and external disturbance

desired reponse of the plant. The objective of the controller is to guarantee that the error $e=X_m - X_p$ between the plant and the model states tends asymptotically to zero, i.e., $\lim_{t\to\infty} e(t) = 0$. From Eqs.(1), (2) and (3), the error dynamics satisfies the following equation:

$$\begin{aligned}\dot{e} = \dot{X}_m - \dot{X}_p &= (A_m - B_m K_p K_b)e + (A_m - A_P - B_m K_p K_b)X_p \\ &+ B_m K_p U_m + B_m.(K_i I_m + K_d D_m) + B_m B_i - B_p U_p - H(X_p, U_p, t)\end{aligned} \tag{4}$$

The control input U_P in Eq.(4) is chosen as

$$U_p = U_{p1} + U_{pa} \tag{5}$$

$$U_{p1} = K_1 X_p + K_2 U_m + K_3 X_m + U_{ps} + B_i \tag{6}$$

$$U_{pa} = DK_p(e,t) X_p + DK_u(e,t) U_m \tag{7}$$

In this paper, $K_3 = 0$ is chosen that the linear part of the control system is called as an "implicit model-following control system"[9]. The equality $X_p = X_m$ can be achieved when $U_{pa} = 0$ in Eq.(5) and $H(X_p, U_p, t) = 0$ in Eq.(4). Thus, substituting Eq.(6) into Eq.(4), the following equations must be satisfied for perfect model following condition:

$$K_1 = K_p K_b, K_2 = K_p, U_{ps} = K_i I_p + K_d D_p \tag{8}$$

where I_p and D_p are defined as:

$$I_p = \int_0^t (U_m - K_b \cdot X_p) d\tau, \; D_p = \frac{d}{dt}(U_m - K_b X_p)$$

Thus, Eq.(6) is rewritten as follows:

$$U_{p1} = K_p(U_m - K_b X_p) + K_i I_p + K_d D_p + B_i \tag{9}$$

where U_{p1} is the linear part of the control system which is satisfied for the perfect model following condition. Thus, the control signal U_{p1} can be chosen as the conventional PID control signal [6]. In this case, the plant and the reference model will have the same structure.

If the system ouput makes the error asymptotically to zero, the Lyapunov equation and Popov integral inequality must be satisfied[6]. The correction signals for $DK_p(e,t)$ and $DK_u(e,t)$ are assumed to have the structure:

$$DK_p(e,t) = \int_0^t F v(GX_p)^T d\tau + F' v(GX_p)^T + DK_p(0) \tag{10}$$

$$DK_u(e,t) = \int_0^t M v(NU_m)^T d\tau + M' v(NU_m)^T + DK_u(0) \tag{11}$$

$DK_p(e,t)$ is the adaptation mechanism according to output signal, and $DK_u(e,t)$ is the adaptation mechanism according to input signal, Both adaptation mechanisms use proportional–plus–integral (PI) algorithm, in which F and M are integal constants, and $F'$and M' proportional constants. Control parameters in Eqs.(10) and (11) are chosen as :

$$G = 1, N = 1, F = f_{12}, F' = f_{11}, M = f_{21}, M' = f_{22}$$

Thus, the simplified adaptation mechanism is chosen as:

$$U_{pa} = (f_{12}\int_0^t vX_p d\tau + f_{11}vX_p)X_p + (f_{21}\int_0^t vU_m d\tau + f_{22}vU_m)U_m \quad (12)$$

The first part of Eq.(12) is the adaptation mechanism according to output signal X_p, and the second part of Eq.(12) is the adaptation mechanism according to input signal U_m, in which f_{12} and f_{21} are integal constants, and f_{11} and f_{22} are proportional constants.

Therefore, the plant input signal (U_p) is the summation of the PID (U_{p1}) and correction (U_{pa}) signals. In general, correction signal is minor, and must be less than PID signal $(U_{p1} > U_{pa})$. When the value of U_{pa} is null, the control loop becomes a conventional PID control loop. The controller developed based on hyperstability theory has two functions of PID and AMFC, and is called Adaptive Model-Following PID Controller. The simulation program for the Adaptive Model-Following PID Controller is written in MATLAB/Simulink software, and is shown in Figure 6. The system identification function of propose SIMULINK program is built by authors as simply just a function for simulation.

III. COMPUTER-AIDED METHOD OF FREQUENCY DOMAIN ANALYSIS

The frequency response of a control system whose sinusoidal transfer function is *G(jω)* or *Y(jω)/X(jω)*. The magnitude bandwidth of the frequency response is defined as the frequency at which the magnitude response curve is 3dB down from its value at zero frequency or the value of ｜*G(jω)*｜is equal to 0.707. The phase bandwidth of the frequency response is defined as the frequency at which the phase response curve is the value of ∠*G(jω)* equal to -90°. In such cases, the frequency response of the system, from input to output, can be gained using a sinusoidal force at the input to the system and measure the output steady state sinusoidal amplitude and phase angle. Repeating this process at several frequencies yields data for a frequency response plot. The amplitude response is the ratio

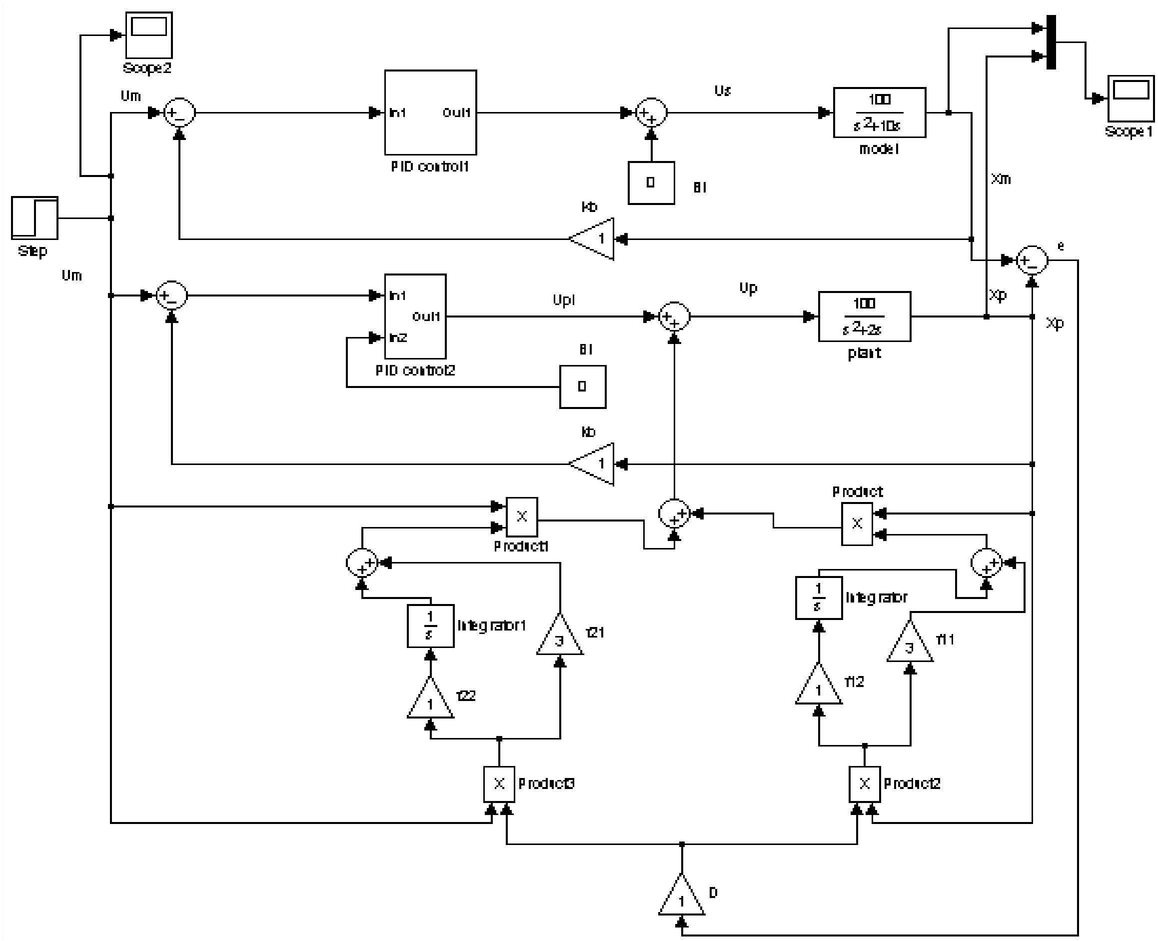

Fig. 6 Simulation program for AMFC system

of the ouput sinusoid's magnitude to the input sinusoid's magnitude. The phase response is the difference in phase angle between the output and the input sinusoids.

In this paper, the frequency response of the adaptive model-following PID controller will be examined with the Matlab/Simulink software. Wei and others.[11,12] have used the Matlab/Simulink software to get the frequency responses of the hydraulic servo-system in an injection molding machine and the autopilot of motorcycles. Their results show that these systems bandwidth can be adequately predicted by the computer-aided method of frequency domain analysis. The following steps are offered as a guideline:

1. the magnitude frequency response analysis

First, input a sinusoidal wave to the system. Then the frequency response of the system can be gained by measuring the output steady state sinusoidal amplitude. Repeating this at several frequencies yields data for a magnitude response plot. The magnitude value M can be gained by calculating the ratio of the output sinusoid's magnitude to the input sinusoid's magnitude. Converting the magnitude value M into dB, i.e. $20\log M$, and then plot these dB values as y-axis against the log$w$ as x-axis. The log-magnitude frequency response curve as function of log$w$ is called Bode plots or Bode diagram. When the magnitude response curve is 3dB down from its value at zero frequency or the value of M is equal to 0.707，the frequency at this condition is defined as the magnitude bandwidth. If this bandwidth value is not an integer value, and it cannot be found at log-magnitude plot. In this case, the frequency values ω of input sinusoidal wave are repeated each 0.1Hz, and repeating the above process until finding the exact value of magnitude bandwidth.

2. the phase frequency response analysis

Similarly, the phase response of the system can be obtained by measuring the delay time between the output and the input sinusoids. This delay time value is the difference in phase angle between the output and the input sinusoids. Then plot these phase values as y-axis against the log$w$ as x-axis. The phase frequency response curve as function of log$w$ is also called Bode plots or Bode diagram. When the phase angle is equal to -90°, the frequency at this condition is defined as the phase bandwidth.

VI. SIMULATION RESULTS AND DISCUSSION

The reference model is represented by a type-1 and second order closed loop control system with a PID compensator, its damping ratio ζ=0.707 and natural frequency ω_n=20 Hz. According to the simplified reference model, a suitable PID controller is designed to make the output of reference model track the controller's input well. The boundaries of the PID controller are K_P=1, K_i=0, K_d=1, respectively. Then, the PID controller along the plant uses the same control boundaries and plant's damping ratio ζ=0.1 and natural frequency ω_n=10 Hz. In this case, these four boundaries f_{11}, f_{12}, f_{21}, f_{22} are all null. Figure 7 shows the tracking performance at the unit step function input. The results show the plant cannot track the reference model response.

In the next condition, the boundaries of K_P, K_i and K_d are not changed and the boundaries in the adaptation mechanism are tuned as f_{11}=150, f_{12}=10, f_{21}=150, f_{22}=10, the combined control signal from the PID controller and adaptation mechanism make the plant track the reference model well. The simulated results are shown in Figure 8.

Figure 9 show that a sinusoidal wave input to the AMFC system and it's frequency value ω=1.45 Hz. The

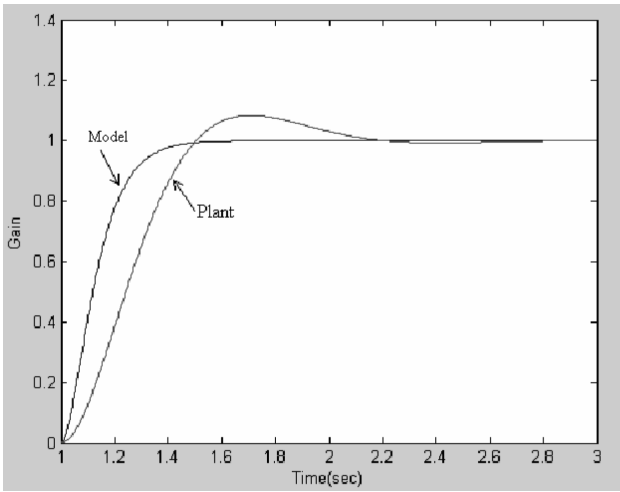

Fig. 7 Tracking performance of plant and model at PID control

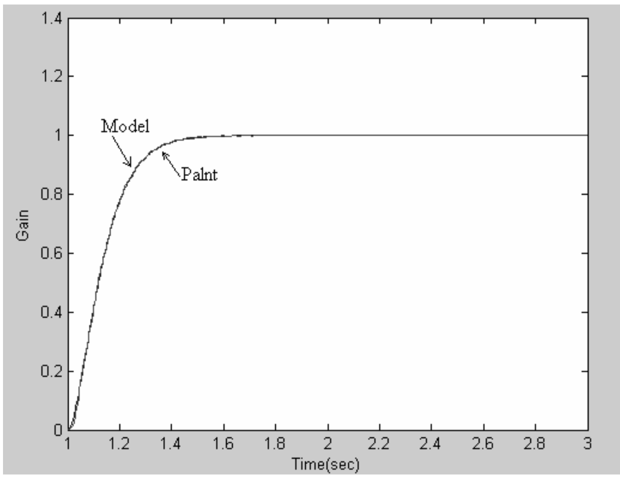

Fig. 8 Tracking performance of plant and model at AMFC control

magnitude value can be obtained by calculating the ratio of the output sinusoid's magnitude to the input sinusoid's magnitude and the ratio equal to 0.707, thus the magnitude bandwidth=1.45 Hz.

Figure 10 show that a sinusoidal wave input to the AMFC system and it's frequency value ω=2.5 Hz. The phase response of the system can be gained by measuring the delay time between the output and the input sinusoids, this delay time value can be calculated as 0.1 sec. The delay time value can be changed to phase angle by multiplying 360° and input frequency value (2.5Hz), the resulting phase angle is -90°. Therefore, the phase bandwidth is 2.5 Hz.

The log-magnitude frequency response curve as a function of logω are shown in Figure 11. The magnitude bandwidth of reference model is calculated as 1.65Hz and the magnitude bandwidth of plant under PID control is calculated as 1.1Hz. When the control signal from adaptation mechanism is combined to the PID controller, the plant's magnitude frequency response can be induced to follow the reference model's magnitude frequency response.

The phase frequency response curve as a function of logω is shown in Figure 12. The phase bandwidth of reference model is calculated as 2.2Hz and the phase bandwidth of plant under PID control is calculated as 0.9Hz. When the control signal from the adaptation mechanism is combined to the PID controller, the plant's phase response can also be induced to follow the reference model's phase response.

From the results, the model following performances on the frequency domain showed the bandwidth of the adaptive model-following PID controller can be improved by a suitable adaptation signal.

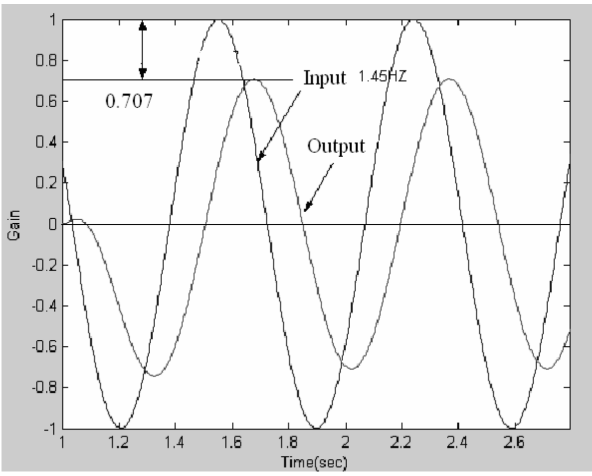

Fig. 9 The sinusoidal response at magnitude bandwidth case

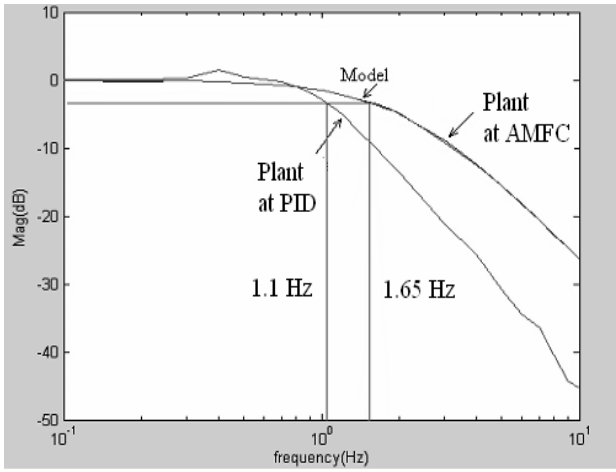

Fig. 11 The magnitude frequency response at PID and AMFC control

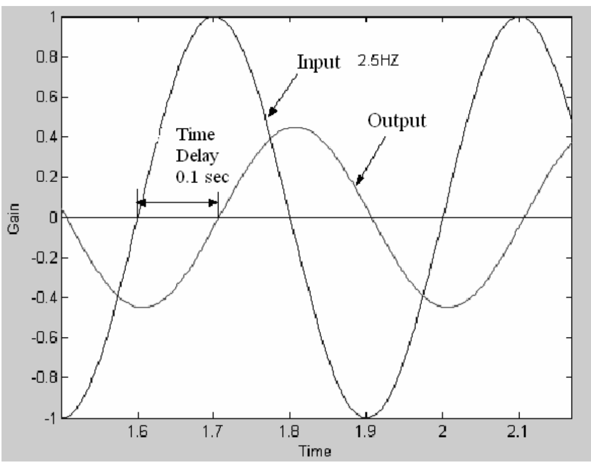

Fig. 10 The sinusoidal response at phase bandwidth Case

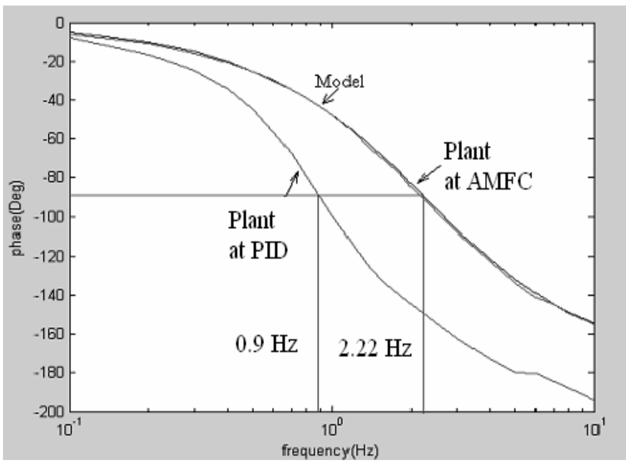

Fig. 12 The phase frequency response at PID and AMFC control

V. CONCLUSION

In this study, the hyperstability theory applied to the adaptive model-following PID controller was presented. A computer-aided method of frequency domain analysis with Matlab/Simulink software on the adaptive model- following PID controller was also presented. From the simulation results, the step responses of the PID controller with and without the adaptive model-following control effects were discussed. The model following performances on the frequency domain also showed the bandwidth of the adaptive model-following PID controller can be improved by a suitable adaptation signal.

When thesinusoidal transfer function of the adaptive control system cannot be gained by the classical method, the computer-aided method of frequency domain analysis shown in this paper is a suitable method for this case.

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