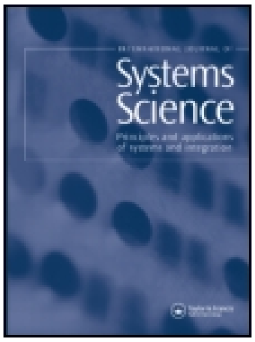

International Journal of Systems Science

ISSN: 0020-7721 (Print) 1464-5319 (Online) Journal homepage: http://www.tandfonline.com/loi/tsys20

Robust tracking control for a class of flexible-joint time-delay robots using only position measurements

Yeong-Chan Chang & Ming-Fang Wu

INTERNATIONAL JOURNAL OF SYSTEMS SCIENCE, 2016
VOL. 47, NO. 14, 3336–3349
http://dx.doi.org/10.1080/00207721.2015.1129677

Taylor & Francis
Taylor & Francis Group

Robust tracking control for a class of flexible-joint time-delay robots using only position measurements

Yeong-Chan Chang and Ming-Fang Wu

Department of Electrical Engineering, Kun-Shan University, Tainan, Taiwan, R.O.C.

ABSTRACT

In this paper, robust tracking control is investigated for a class of uncertain flexible-joint robots with time delays and time-varying perturbations. By employing the Lyapunov–Krasovskii functional technique and backstepping design technique, a novel robust tracking control scheme using only position measurements is developed such that all the states and signals of the closed-loop flexible-joint time-delay robot system remain bounded and the tracking error can asymptotically converge to a small neighbourhood around the origin. By appropriately choosing the weighting gains in the Lyapunov–Krasovskii functionals, the circular phenomenon in the controller design is overcome. Due to suitably designing the velocity observer and the virtual control input, the link-side dynamics does not need to be incorporated into the actuator-side tracking error dynamics, and so the complexity in the backstepping design is avoided. Consequently, we can easily construct the Lyapunov–Krasovskii functionals, and, in turn, the robust tracking control scheme developed here is a linear time-varying controller and can be simply implemented. Simulation examples are provided to verify the effectiveness of the proposed control algorithm.

ARTICLE HISTORY
Received 23 April 2015
Accepted 6 December 2015

KEYWORDS
Robot; flexible joint; time delay; robust tracking control; Lyapunov–Krasovskii functional

1. Introduction

The control of robot systems incorporating with the effects of joint flexibilities, time-delay uncertainties, and time-varying perturbations is of importance for numerous practical applications. The phenomenon of joint flexibility may be induced by various causes in industrial applications of robot manipulators, e.g. shaft windup, harmonic drives, and bearing deformation (Luca & Book, 2008; Spong & Vidyasagar, 1989). Both trajectory tracking control and regulation problems of flexible-joint robots have attracted tremendous attention of the control community, and many control strategies have been proposed in the literature, e.g. the feedback linearisation control (Luca & Book, 2008; Spong & Vidyasagar, 1989), passivity control (Forbes & Damaren, 2011; Son, Shim, & Seo, 2004), singular perturbation control (Ozgoli & Taghirad, 2009), adaptive sliding mode control (Huang & Chen, 2004), proportional-derivative control (Luca, Siciliano, & Zollo, 2005), adaptive backstepping control (Bang, Shim, Park, & Seo, 2010; Liu, Cheah, & Slotine, 2008), and observer-based control (Chang & Yen, 2011; Lightcap & Banks, 2010; Melhem & Wang, 2009). More recently, Yun and Su (2014) developed a robust optimal design method for a two-link manipulator with flexible joints which could achieve optimal disturbance suppression performance. Liu and Wu (2014) investigated modelling and adaptive tracking control problems for flexible-joint robots subjected to random disturbances. Korkmaz and Kemal Ider (2014) developed an inverse dynamics control algorithm for hybrid motion and force tracking control of flexible-joint parallel manipulators. Yoo (2012a) presented a fault detection and adaptive accommodation control scheme for flexible-joint robots with model uncertainties and multiple actuator faults. Chang and Yen (2012) designed a robust position feedback tracking controller for a large class of electrically driven flexible-joint robots.

Time delays often occur in many real engineering systems, e.g. robot systems, electrical-circuit models, and interconnected systems (Niculescu, 2001). In the flexible-joint robot system, the interconnection between the link-side subsystem and the actuator-side subsystem is often accompanied by energy transfer, and such a transfer process may lead to the time-delay phenomenon (Jiang, Xu, & Yan, 2014; Niculescu, 2001). Since the time-delay phenomenon, which may destroy the robustness stability of the interconnected mechanical systems and also limit the tracking performance, is unavoidable, how to override the effect of time delays becomes a very serious topic. Therefore, from the viewpoints of theoretic and practical applications, the stability analysis and controller synthesis for flexible-joint robot systems incorporating with the effect of time delays are quite important. In recent years, a large number of robust control schemes for general nonlinear time-delay systems have been developed (Li, Guan, & Luo, 2011; Philip Chen, Wen, Liu, & Wang, 2014; Wang, Ge, & Hong, 2010; Yoo, 2012b; Zhang, Xie, & Huang, 2015; Zhou, Shi, Xu, & Li, 2013). However, only a few of those control techniques for uncertain time-delay mechanical systems have been implemented (Chang, 2014; Kao & Pasumarthy, 2012; Li, Xia, & Sun, 2014; Sharma, Bhasin, Wang, & Dixon, 2011; Zhu, Wang, & Cai, 2010). The effect of joint flexibilities has not been compensated in these previous control designs of time-delay mechanical systems.

In this paper, robust tracking control is investigated for a class of uncertain flexible-joint robots with time delays and

CONTACT Yeong-Chan Chang ycchang@mail.ksu.edu.tw

time-varying perturbations using only position measurements. The time-delay functions are assumed to be bounded by functions of all the state variables, i.e. link position, link velocity, actuator position, and actuator velocity. By employing the Lyapunov–Krasovskii functional technique and backstepping design technique, a novel robust position feedback tracking control scheme is developed such that the boundedness of all the states and signals of the closed-loop system is guaranteed and the tracking error can asymptotically converge to a small neighbourhood around the origin. By appropriately choosing the weighting gains in the Lyapunov–Krasovskii functionals, the circular phenomenon in the controller design is overcome. Due to suitably designing the velocity observer and the virtual control input, the link-side dynamics does not need to be incorporated into the derivation of actuator-side tracking error dynamics, and so the complexity in the backstepping design is avoided. This also implies the time-delay function in the link-side dynamics does not appear in the actuator-side tracking error dynamics. Consequently, we can easily construct the Lyapunov–Krasovskii functionals, and, in turn, the robust tracking control scheme developed here is a linear time-varying controller and can be simply implemented. Compared with the existing literature of controlling flexible-joint robot systems, the control scheme developed here can be extended to handle a broader class of uncertain flexible-joint time-delay robot systems.

The rest of this paper is organised as follows. In Section 2, the model description is presented. The tracking control scheme is developed in Section 3. In Section 4, simulation examples are given. The conclusions are drawn in Section 5.

2. Model description and problem statement

The motion equations of an n-link flexible-joint manipulator incorporating with a class of time-delay uncertainties can be derived as (Luca & Book, 2008; Spong & Vidyasagar, 1989)

$$M(q_1)\ddot{q}_1 + C(q_1,\dot{q}_1)\dot{q}_1 + G(q_1) + H_1(q_1(t-\tau_1), \dot{q}_1(t-\tau_1)) = K(q_2 - q_1) + d_1 \quad (1)$$

$$J\ddot{q}_2 + B\dot{q}_2 + K(q_2 - q_1) + H_2(q_1(t-\tau_2), \dot{q}_1(t-\tau_2), q_2(t-\tau_2), \dot{q}_2(t-\tau_2)) = u + d_2 \quad (2)$$

where $q_1, \dot{q}_1, \ddot{q}_1 \in R^n$ are the link position, velocity, and acceleration, respectively, $M(q_1) \in R^{n\times n}$ is the inertia moment, $C(q_1,\dot{q}_1)\dot{q}_1 \in R^n$ is the centripetal and Coriolis forces, $G(q_1) \in R^n$ is the gravitational force, $q_2, \dot{q}_2, \ddot{q}_2 \in R^n$ are the actuator position, velocity, and acceleration, respectively, $K = \mathrm{diag}\{K_i\} \in R^{n\times n}$ is the joint-stiffness matrix, $J \in R^{n\times n}$ is the actuator inertia matrix, $B \in R^{n\times n}$ is the actuator damping matrix, $u \in R^n$ is the applied torques, and d_1 and d_2 are the external disturbances. $H_1(\cdot)$ and $H_2(\cdot)$ stand for the unknown time-delay functions, and τ_1 and τ_2 are the unknown time delays. Define the state variable $x(t) = [x_1^T(t), x_2^T(t), x_3^T(t), x_4^T(t)]^T = [q_1^T(t), \dot{q}_1^T(t), q_2^T(t), \dot{q}_2^T(t)]^T$ and let $x(t) = w(t)$, for $t \in [-\tau_{\max}, 0]$, be a continuous initial state, where $\tau_{\max} = \max\{\tau_1, \tau_2\}$ is the upper bound of time delays.

The following two fundamental properties for the link-side dynamics (1) can facilitate the controller design (Luca & Book, 2008; Spong & Vidyasagar, 1989):

P1: The matrix $M(q_1)$ is a positive-definite symmetric matrix bounded by $\lambda_m I \le M(x_1) \le \lambda_M I$, where λ_m and λ_M are the minimum and maximum eigenvalues.

P2: The matrix $\dot{M}(q_1) - 2C(q_1,\dot{q}_1)$ is skew-symmetric.

In the practical electromechanical system, the nominal values of elements in both mechanical and electrical dynamics are always known, but they are easily perturbed by time-varying uncertainties. Without loss of generality, we make the following assumptions in which $(\cdot)_0$ is the known nominal term and $\Delta(\cdot)$ is the time-varying perturbation.

A1: Let the parameter matrices in the link-side dynamics (1) be $M(\cdot) = M_0(q_1) + \Delta M(t, q_1)$, $C(\cdot) = C_0(q_1,\dot{q}_1) + \Delta C(t, q_1, \dot{q}_1)$, and $G(\cdot) = G_0(q_1) + \Delta G(t, q_1)$. □

A2: Let the parameter matrices in the actuator-side dynamics (2) be $J(\cdot) = J_0 + \Delta J(t)$, $B(\cdot) = B_0 + \Delta B(t)$, and $K(\cdot) = K_0 + \Delta K(t)$. □

Remark 2.1: In the flexible-joint robot system, the interconnection between the link-side dynamics (1) and the actuator-side dynamics (2) is accompanied by energy transfer, which is accomplished by the elastic coupling term $K(q_2 - q_1)$. Such a transfer process may cause the time-delay phenomenon (Jiang et al., 2014; Li et al., 2014; Niculescu, 2001, p. 48). Here, we concentrate all the effects of time-delay phenomena into both the lumped time-delay functions $H_1(\cdot)$ and $H_2(\cdot)$, and the unknown time delays τ_1 and τ_2 are assumed to be bounded. These assumptions are reasonable and have been commonly used in the procedure of control design to override the effects of time-delay functions (Chang, 2014; Philip Chen et al., 2014; Zhou et al., 2013). Therefore, the system dynamics (1) and (2) incorporated with assumptions A1 and A2 and time-delay functions $H_1(\cdot)$ and $H_2(\cdot)$ can be employed to express a large class of uncertain flexible-joint time-delay robots..

The objective of this paper is to construct a robust position feedback tracking controller without velocity measurements for the flexible-joint time-delay robots (1) and (2) such that the link position $q_1(t)$ and velocity $\dot{q}_1(t)$ can track the desired trajectories $q_{r1}(t)$ and $\dot{q}_{r1}(t)$, while all the states and signals of the closed-loop system remain bounded. Here, the desired signals $q_{r1}(t)$, $\dot{q}_{r1}(t)$, and $\ddot{q}_{r1}(t)$ are assumed to be available and bounded.

From the definitions of $x_1 = q_1$, $x_2 = \dot{q}_1$, $x_3 = q_2$, and $x_4 = \dot{q}_2$, the state-space expressions of the dynamic Equations (1) and (2) can be obtained as

$$\dot{x}_1 = x_2 \quad (3)$$

$$M(x_1)\dot{x}_2 = -C(x_1, x_2)x_2 - G(x_1) - H_1(x_{e12}(t-\tau_1)) - Kx_1 + Kx_3 + d_1 \quad (4)$$

$$\dot{x}_3 = x_4 \quad (5)$$

$$\dot{x}_4 = -J^{-1}Bx_4 + J^{-1}K(x_1 - x_3) - J^{-1}H_2(x_{e14}(t-\tau_2)) + J^{-1}u + J^{-1}d_2 \quad (6)$$

in which the augmented state is defined as $x_{e1i}(t) = [x_1^T(t)\cdots x_i^T(t)]^T$, $i = 2, 3, 4$. In this study, both the link and actuator positions, x_1 and x_3, are assumed to be available on the design of a controller, while both the link and actuator velocities, x_2 andx_4, are assumed to be unavailable. A $2n$-dimensional observer should be constructed to estimate these two lacking velocity signals.

The integrator backstepping technique is applied to the controller design and stability analysis. The state variables x_2, x_3, and x_4 are viewed as virtual control inputs in (3), (4), and (5), respectively. Moreover, we make the following assumptions in which $\|A\|$ denotes the induced 2-norm of the matrix A.

A3: Assume $\|\Delta K K_0^{-1}\| \le \varepsilon_K$ for some positive constant $0 \le \varepsilon_K < 1$.
A4: Assume $\|\Delta J J^{-1}\| \le \varepsilon_J$ for some positive constant $0 \le \varepsilon_J < 1$.
A5: Assume the time-delay functions $H_1(\cdot)$ and $H_2(\cdot)$ are bounded as $\|H_1(x_{e12})\| \le B_{H1}(x_{e12})$ and $\|H_2(x_{e14})\| \le B_{H2}(x_{e14})$ for some continuous functions $B_{H1}(x_{e12}) \ge 0$ and $B_{H2}(x_{e14}) \ge 0$.

3. Design of a robust position feedback tracking controller

Consider the link-side dynamics (1). Define the tracking error $\bar{x}_1(t) = q_1(t) - q_{r1}(t)$ and the filter link tracking error $\bar{x}_2(t) = \dot{q}_1(t) - \dot{q}_{r1}(t) + k_1(q_1(t) - q_{r1}(t))$, where $k_1 > 0$ is a controller gain. Taking into account (3) and (4), we get

$$\dot{\bar{x}}_1 = -k_1\bar{x}_1 + \bar{x}_2 \quad (7)$$

$$M(x_1)\dot{\bar{x}}_2 = -F(x_e) - C(x_1, x_2)\bar{x}_2 - H_1(x_{e12}(t-\tau_1)) - Kx_1 + Kx_3 + d_1 \quad (8)$$

where $x_e = [x_1{}^T, x_2{}^T, q_{r1}^T, \dot{q}_{r1}^T, \ddot{q}_{r1}^T]^T$ and $F(x_e) = M(x_1)(\ddot{q}_{r1} - k_1\dot{\bar{x}}_1) + C(x_1, x_2)(\dot{q}_{r1} - k_1\bar{x}_1) + G(x_1)$. Define $F_d(q_{re}) = M(q_{r1})\ddot{q}_{r1} + C(q_{r1}, \dot{q}_{r1})\dot{q}_{r1} + G(q_{r1})$, where $q_{re} = [q_{r1}^T, \dot{q}_{r1}^T, \ddot{q}_{r1}^T]^T$. After simple computations, $F(x_e)$ can be decomposed into

$$F(x_e) = F_1(x_e)\bar{x}_1 + F_2(x_e)\bar{x}_2 + F_d(q_{re}) \quad (9)$$

where both $F_1(\cdot)$ and $F_2(\cdot)$ can be found in Chang and Yen (2011).

Since the system matrices $M(\cdot)$, $C(\cdot)$, and $G(\cdot)$ are perturbed by time-varying uncertainties in A1, we express $F_d(q_{re}) = F_{d0}(q_{re}) + \Delta F_d(q_{re})$, where $F_{d0}(q_{re}) = M_0(q_{r1})\ddot{q}_{r1} + C_0(q_{r1}, \dot{q}_{r1})\dot{q}_{r1} + G_0(q_{r1})$ and $\Delta F_d(q_{re}) = \Delta M(q_{r1})\ddot{q}_{r1} + \Delta C(q_{r1}, \dot{q}_{r1})\dot{q}_{r1} + \Delta G(q_{r1})$. Define

$$F_0(q_{re}) = F_{d0}(q_{re}) + K_0 q_{r1} \quad (10)$$

Substituting (9), (10) and assumption A2 into (8) leads to

$$M(x_1)\dot{\bar{x}}_2 = -C(x_1, x_2)\bar{x}_2 - F_0(q_{re}) - F_1(x_e)\bar{x}_1 - F_2(x_e)\bar{x}_2 - \Delta F_d(q_{re}) - \Delta K q_{r1} - H_1(x_{e12}(t-\tau_1)) - K\bar{x}_1 + Kx_3 + d_1 \quad (11)$$

Now, construct a linear observer to estimate the behaviour of x_2:

$$\dot{\eta}_1 = (k_1k_{01} - k_{01}^2)\bar{x}_1 - k_{01}\eta_1 \quad (12)$$

$$\hat{\bar{x}}_2 = \eta_1 + k_{01}\bar{x}_1 \quad (13)$$

where η_1 is the observer state, $\hat{\bar{x}}_2$ is the observer output, and $k_{01} > 0$ is the observer gain. Define the observation error $\bar{e}_2(t) = \bar{x}_2(t) - \hat{\bar{x}}_2(t)$. Taking into account (7), (12), and (13), we can compute $\dot{\hat{\bar{x}}}_2 = k_{01}\bar{e}_2$, and, in turn, the observation error dynamics is

$$M(x_1)\dot{\bar{e}}_2 = -C(x_1, x_2)\bar{x}_2 - F_0(q_{re}) - F_1(x_e)\bar{x}_1 - F_2(x_e)\bar{x}_2 - \Delta F_d(q_{re}) - \Delta K q_{r1} - H_1(x_{e12}(t-\tau_1)) - K\bar{x}_1 + Kx_3 + d_1 - k_{01}M(x_1)\bar{e}_2 \quad (14)$$

Consider the virtual control input x_3 in (11) and (14), and choose

$$x_3^* = K_0^{-1}(F_0(q_{re}) - k_2\hat{\bar{x}}_2) \quad (15)$$

where $k_2 > 0$ is a controller gain. Define $\bar{x}_3(t) = x_3(t) - x_3^*(t)$. Taking into account A2 leads to

$$Kx_3 = F_0(q_{re}) - k_2(I + \Delta K K_0^{-1})\hat{\bar{x}}_2 + \Delta K K_0^{-1}F_0(q_{re}) + K\bar{x}_3 \quad (16)$$

Therefore, the error dynamics (11) and (14) are modified to

$$M(x_1)\dot{\bar{x}}_2 = -C(x_1, x_2)\bar{x}_2 - F_1\bar{x}_1 - F_2\bar{x}_2 - \Delta F_d - \Delta K q_{r1} - K\bar{x}_1 - H_1(x_{e12}(t-\tau_1)) - k_2(I + \Delta K K_0^{-1})\hat{\bar{x}}_2 + \Delta K K_0^{-1}F_0 + K\bar{x}_3 + d_1 \quad (17)$$

$$M(x_1)\dot{\bar{e}}_2 = -C(x_1, x_2)\bar{x}_2 - F_1\bar{x}_1 - F_2\bar{x}_2 - \Delta F_d - \Delta K q_{r1} - K\bar{x}_1 - H_1(x_{e12}(t-\tau_1)) - k_2(I + \Delta K K_0^{-1})\hat{\bar{x}}_2 + \Delta K K_0^{-1}F_0 + K\bar{x}_3 - k_{01}M\bar{e}_2 + d_1 \quad (18)$$

Now, consider the actuator-side dynamics (2). From (5) and (6), the error dynamics of $\bar{x}_3(t)$ is computed as

$$\dot{\bar{x}}_3 = x_4 - K_0^{-1}\dot{F}_0(q_{re}) + k_{01}k_2K_0^{-1}\bar{e}_2 \quad (19)$$

Recall that the velocity signal x_4 is assumed to be unavailable in the control design, and a linear observer will be constructed to estimate its value. Let η_2 be the observer state and $\hat{x}_4$ denotes its output. Let the observation error be $\bar{e}_4(t) = x_4(t) - \hat{x}_4(t)$. Then,

$$\dot{\bar{x}}_3 = \hat{x}_4 + \bar{e}_4 - K_0^{-1}\dot{F}_0(q_{re}) + k_{01}k_2K_0^{-1}\bar{e}_2 \quad (20)$$

Consider the virtual control input $\hat{x}_4$ in (20) and the corresponding desired trajectory x_4^*. Define $\bar{x}_4(t) = \hat{x}_4(t) - x_4^*(t)$. Choose

$$x_4^* = K_0^{-1}\dot{F}_0(q_{re}) - k_3\bar{x}_3 \tag{21}$$

where $k_3 > 0$ is a controller gain. Then, (20) is modified to

$$\dot{\bar{x}}_3 = \bar{e}_4 + \bar{x}_4 - k_3\bar{x}_3 + k_{01}k_2K_0^{-1}\bar{e}_2 \tag{22}$$

Design a linear observer to estimate the behaviour of x_4:

$$\dot{\eta}_2 = -k_{02}(\eta_2 + k_{02}x_3) + K_0^{-1}\ddot{F}_0 - k_4\bar{x}_4 \tag{23}$$

$$\hat{x}_4 = \eta_2 + k_{02}x_3 \tag{24}$$

where $k_4 > 0$ and $k_{02} > 0$ are control gains. From (21)–(24), the error dynamics of $\bar{x}_4(t)$ is obtained as

$$\dot{\bar{x}}_4 = k_{02}\bar{e}_4 + k_3(\bar{e}_4 + \bar{x}_4 - k_3\bar{x}_3 + k_{01}k_2K_0^{-1}\bar{e}_2) - k_4\bar{x}_4 \tag{25}$$

Since the matrices J, B, and K are perturbed by time-varying uncertainties in A2, we can decompose $J^{-1}B = J_0^{-1}B_0 + \Delta J_B$ and $J^{-1}K = J_0^{-1}K_0 + \Delta J_K$ for some ΔJ_B and ΔJ_K. Taking into account (6), (23), and (24), and the aforementioned two equalities, the error dynamics of $\bar{e}_4(t)$ can be computed as

$$\begin{aligned}\dot{\bar{e}}_4 = &-(J_0^{-1}B_0 + \Delta J_B)x_4 + (J_0^{-1}K_0 + \Delta J_K)(x_1 - x_3)\\ &- J^{-1}H_2(x_{e14}(t-\tau_2)) + J^{-1}u + J^{-1}d_2\\ &- (-k_{02}\hat{x}_4 + K_0^{-1}\ddot{F}_0 - k_4\bar{x}_4) - k_{02}x_4\end{aligned} \tag{26}$$

Finally, design the applied input as

$$u = B_0\hat{x}_4 - K_0(x_1 - x_3) + J_0(K_0^{-1}\ddot{F}_0 - k_4\bar{x}_4) \tag{27}$$

Substituting the equalities $J^{-1} = J_0^{-1}(I - \Delta JJ^{-1})$, $x_4(t) = \bar{e}_4(t) + \hat{x}_4(t)$, and (27) into (26) leads to

$$\begin{aligned}\dot{\bar{e}}_4 = &-\Delta J_B\bar{e}_4 - J^{-1}H_2(x_{e14}(t-\tau_2)) - k_4\Delta JJ^{-1}\bar{x}_4\\ &+ \Delta F_4(\cdot) - J_0^{-1}B_0\bar{e}_4 - k_{02}\bar{e}_4 + J^{-1}d_2\end{aligned} \tag{28}$$

where $\Delta F_4(\cdot) = -\Delta J_B\hat{x}_4 + \Delta J_K(x_1 - x_3) - \Delta JJ^{-1}(J_0^{-1}B_0\hat{x}_4 - J_0^{-1}K_0(x_1 - x_3)) - \Delta JJ^{-1}K_0^{-1}\ddot{F}_0$.

For the convenience of representation, the bounded functions $B_{H1}(\cdot)$and $B_{H2}(\cdot)$ for the time-delay functions $H_1(\cdot)$ and $H_2(\cdot)$defined in A5 are assumed to be expressed as $B_{H1}(x_{e12}) \le b_{11}\|x_1\| + b_{12}\|x_2\|$ and $B_{H2}(x_{e14}) \le b_{21}\|x_1\| + b_{22}\|x_2\| + b_{23}\|x_3\| + b_{24}\|x_4\|$, where $b_{11} > 0$, $b_{12} > 0$, $b_{21} > 0$, $b_{22} > 0$, $b_{23} > 0$, and $b_{24} > 0$ are known constants. Later, these two bounded functions will be relaxed in assumption A6.

Theorem 3.1: Consider the flexible-joint time-delay robot system (1)–(2). Let an observer-based position feedback tracking controller without velocity measurements be given by (12), (13), (23), (24), and (27) with $\eta_1(0) = -k_{01}(x_1 - q_{r1}) + \eta_{10}$and $\eta_2(0) = -k_{02}x_3 + \eta_{20}$ for some η_{10} and η_{20}. Then, for all bounded initial conditions, there exists a choice of gains $k_1, k_2, k_{01}, k_3, k_4,$and k_{02} such that the tracking error can be shown to be uniformly ultimately bounded (UUB) and all the states and signals remain bounded.

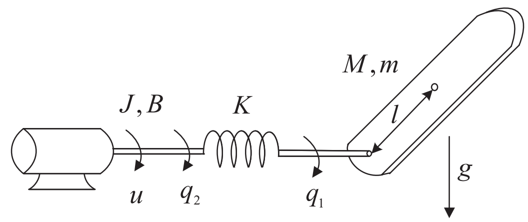

Figure 1. A single-link robot arm with a flexible joint.

Proof: Refer to the proof in Appendix 1. □

In the proof of Theorem 3.1, the weighting values b_{ij} in the bounded conditions $B_{H1}(\cdot)$ and $B_{H2}(\cdot)$ are assumed to be constant. Now, these two bounded conditions will be relaxed as follows:

A6: There exist continuous functions $B_{11}(x_1)>0$, $B_{12}(x_{e12}) > 0$, $B_{21}(x_1) > 0$, $B_{22}(x_{e12}) > 0$, $B_{23}(x_{e13})>0$, and $B_{24}(x_{e14}) > 0$ such that $B_{H1}(x_{e12}) \le B_{11}(x_1)\|x_1\| + B_{12}(x_{e12})\|x_2\|$ and $B_{H2}(x_{e14}) \le B_{21}(x_1)\|x_1\| + B_{22}(x_{e12})\|x_2\| + B_{23}(x_{e13})\|x_3\| + B_{24}(x_{e14})\|x_4\|$.

Choose the Lyapunov function candidate as

$$\begin{aligned}V(t) = &\frac{\alpha_1}{2}\bar{x}_1^T\bar{x}_1 + \frac{1}{2}\bar{x}_2^TM(x_1)\bar{x}_2 + \frac{1}{2}\bar{e}_2^TM(x_1)\bar{e}_2 + \frac{\alpha_3}{2}\bar{x}_3^T\bar{x}_3\\ &+ \frac{\alpha_4}{2}\bar{x}_4^T\bar{x}_4 + \frac{\alpha_5}{2}\bar{e}_4^T\bar{e}_4 + V_p(t)\end{aligned} \tag{29}$$

where the Lyapunov–Krasovskii functional term is modified as

$$V_P(t) = \frac{\alpha_2}{2}\sum_{j=1}^{2}\int_{t-\tau_j}^{t} P_{j1}(\cdot)\,d\tau + \frac{\alpha_6}{2}\int_{t-\tau_2}^{t} P_2(\cdot)\,d\tau \tag{30}$$

with $P_{j1}(\cdot) = B_{j1}^2(x_1)\|x_1(t)\|^2 + B_{j2}^2(x_{e12})\|x_2(t)\|^2$and $P_2(\cdot) = B_{23}^2(x_{e13})\|x_3(t)\|^2 + B_{24}^2(x_{e14})\|x_4(t)\|^2$.

The derivative of $V(t)$ in (29) is computed in Appendix 2. Consequently, as in the proof of Theorem 3.1, the stability analysis described in Theorem 3.1 can be completed.

Remark 3.1: A novel robust position feedback tracking control scheme without velocity measurements is developed for a class of uncertain flexible-joint time-delay robots. From the viewpoint of practical applications, the control structure developed in Theorem 3.1 can be easily implemented. Indeed, all the weighting gains of the signals η_1, η_2, $\bar{x}_1$, x_1, $\hat{\bar{x}}_2$, x_3, $\hat{x}_4$, and $\bar{x}_4$ in the controller (12), (13), (23), (24), (27) are constant, and only one nonlinear term in the controller is the time-varying function $\ddot{F}_0(\cdot)$, which depends only on q_{r1}, $\dot{q}_{r1}$, $\ddot{q}_{r1}$. Consequently, the robust position feedback tracking control scheme developed here is a linear time-varying controller. Especially, a linear time-invariant controller can be obtained when the regulation problem is discussed. □

Remark 3.2: The elastic torque $K(q_2 - q_1)$ is only the coupling force between the link-side dynamics (1) and the actuator-side dynamics (2). Based on the design philosophy of the backstepping technique, the actuator position $x_3(t) = q_2(t)$ is viewed as a virtual control input in (4). Since the desired trajectory x_3^* in (15) depends only on the signals $\ddot{q}_{r1}$, $\dot{q}_{r1}$, q_{r1}, and $\hat{\bar{x}}_2$

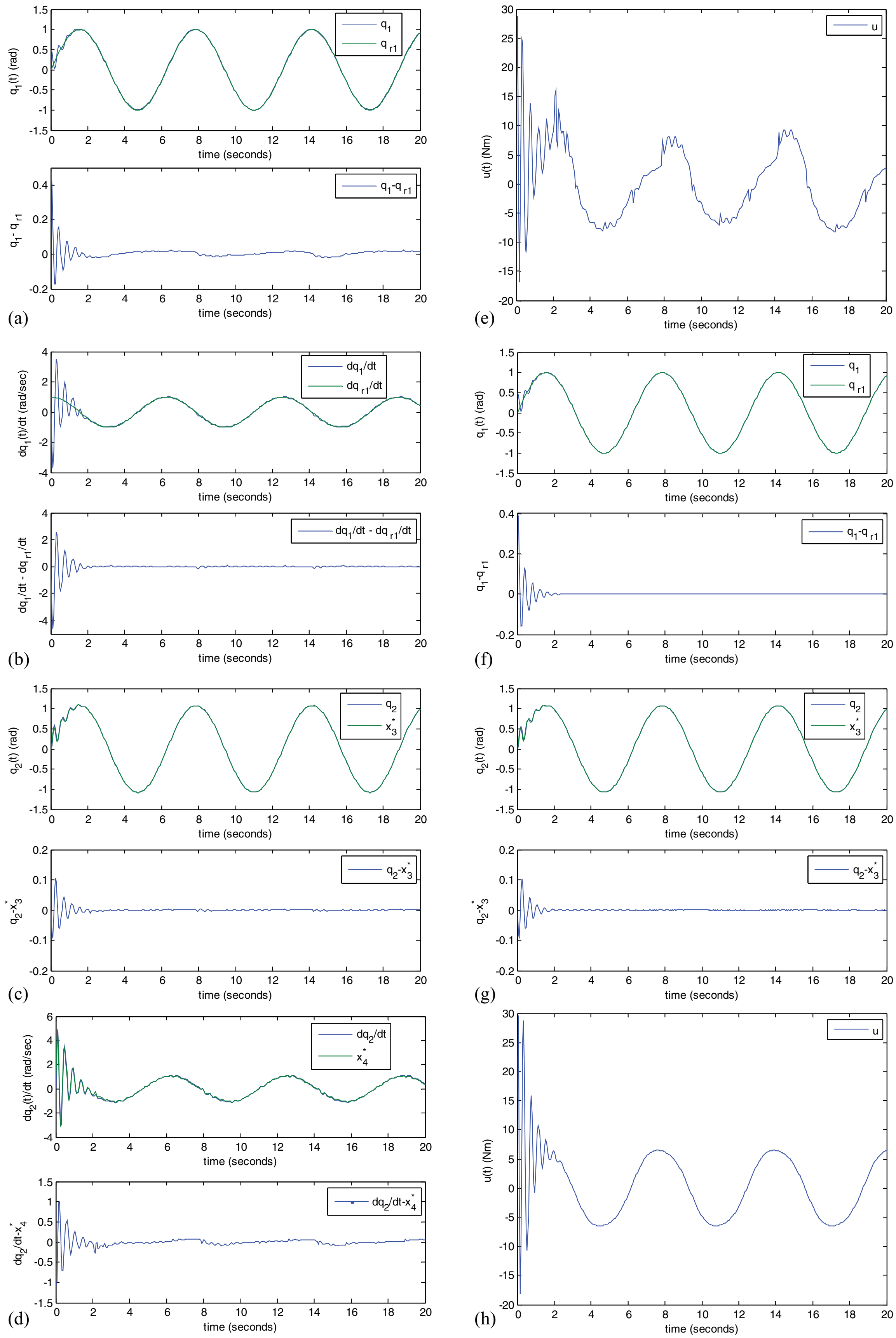

Figure 2. Responses of tracking $q_{r1}(t) = \sin(t)$ for the single-link flexible-joint time-delay robot arm using the developed controller in Theorem 3.1. (a) Angular position $q_1(t)$. (b) Angular velocity $\dot{q}_1(t)$. (c) Angular position $q_2(t)$. (d) Angular velocity $\dot{q}_2(t)$. (e) Applied torque $u(t)$. (f) Angular position $q_1(t)$. (g) Angular position $q_2(t)$. (h) Applied torque $u(t)$.

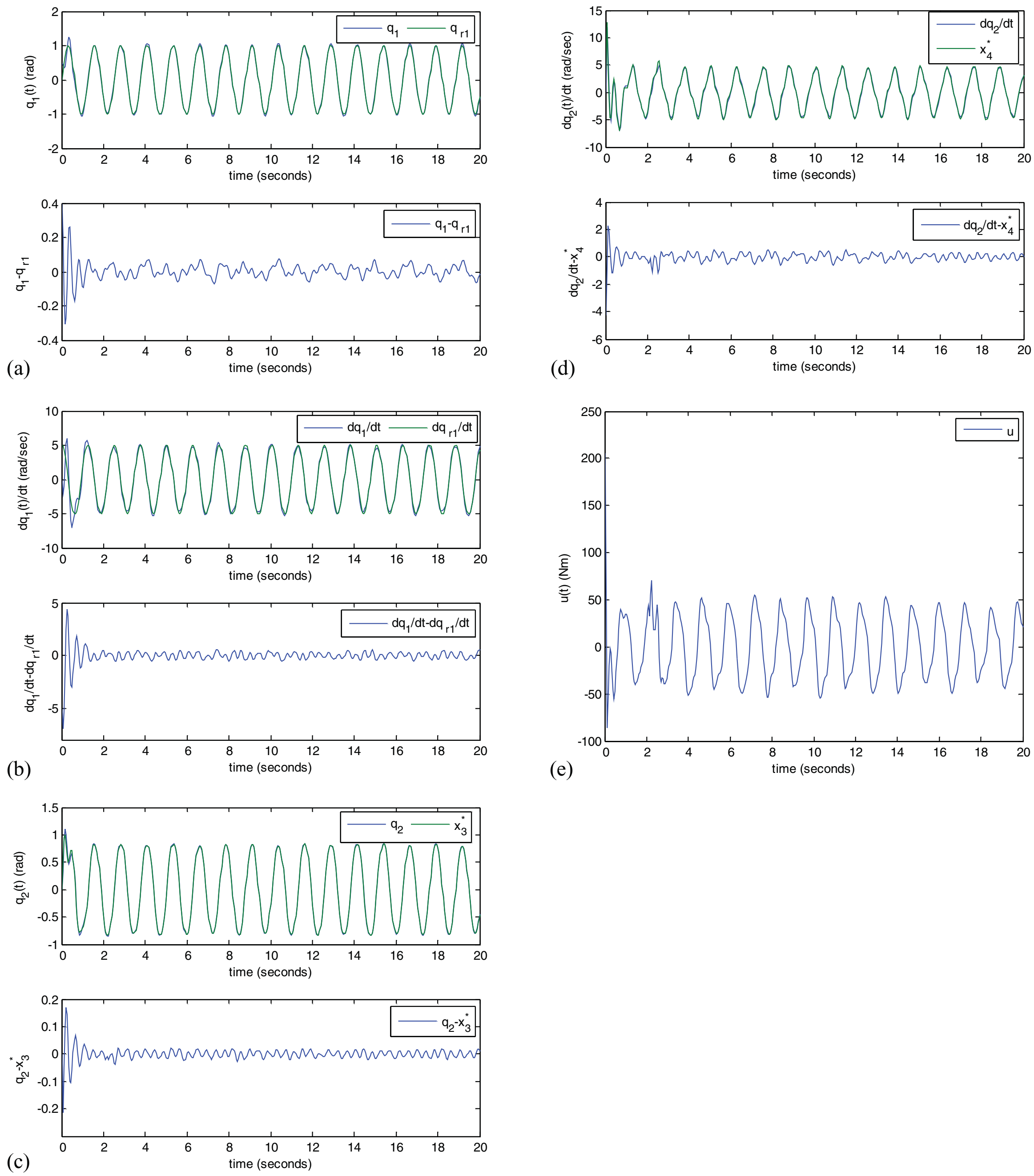

Figure 3. Responses of tracking $q_{r1}(t) = \sin(5t)$ for the single-link flexible-joint time-delay robot arm using the developed controller in Theorem 3.1. (a) Angular position $q_1(t)$. (b) Angular velocity $\dot{q}_1(t)$. (c) Angular position $q_2(t)$. (d) Angular velocity $\dot{q}_2(t)$. (e) Applied torque $u(t)$.

(but not $\dot{q}_1$), the link-side dynamics $\ddot{q}_1 = M^{-1}(-C\dot{q}_1 - G - H_1(x_{e12}(t-\tau_1)) + Kq_2 - Kq_1 + d_1)$ from (1) does not need to be incorporated into the actuator position tracking error dynamics, and, in turn, the time-delay function $H_1(x_{e12}(t-\tau_1))$ does not appear in the derivation of the error dynamics (19). Therefore, the Lyapunov–Krasovskii function candidate can be easily constructed. Indeed, from (29) the Lyapunov–Krasovskii functional term is

$$\frac{\alpha_2}{2}\sum_{j=1}^{2}\int_{t-\tau_j}^{t} P_{j1}(x_1(\tau), x_2(\tau))\, d\tau + \frac{\alpha_6}{2}\int_{t-\tau_2}^{t} P_2(x_3(\tau), x_4(\tau))\, d\tau$$

$$= \frac{\alpha_2}{2}\int_{t-\tau_1}^{t} (b_{11}^2\|x_1(\tau)\|^2 + b_{12}^2\|x_2(\tau)\|^2)\, d\tau + \frac{\alpha_2}{2}\int_{t-\tau_2}^{t} (b_{21}^2\|x_1(\tau)\|^2 + b_{22}^2\|x_2(\tau)\|^2)\, d\tau + \frac{\alpha_6}{2}\int_{t-\tau_2}^{t} (b_{23}^2\|x_3(\tau)\|^2 + b_{24}^2\|x_4(\tau)\|^2)\, d\tau$$

It is clear that the last two terms in the above equality are designed to override the effect of the time-delay function $H_2(\cdot)$, but do not need to simultaneously override the effect of time-delay function $H_1(x_{e12}(t-\tau_1))$. In contrast, the common feature existing in the previous adaptive backstepping control algorithms for perturbed nonlinear time-delay systems is that all the effects of the time-delay functions $H_i(\cdot)$, for $i = 1, \ldots, i-1$, are incorporated into the error dynamics in step i. The choice of the Lyapunov–Krasovskii function in step i must simultaneously compensate the effects of $H_1(\cdot), \ldots, H_{i-1}(\cdot)$, and so such a choice is quite complicated.

Remark 3.3: (1) Compared to the previous control schemes for flexible-joint robots without compensating for the effect of time delays (Chang & Yen, 2011, 2012; Korkmaz & Kemal Ider, 2014; Liu & Wu, 2014; Yoo, 2012a; Yun & Su, 2014), the tracking control scheme developed in Theorem 3.1 can be extended to handle a broader class of flexible-joint robots in the presence of time-delay uncertainties. (2) Compared with the published literature (Chang, 2014; Kao & Pasumarthy, 2012; Sharma et al., 2011; Zhu et al., 2010) in which the control design of uncertain mechanical systems with time delays is addressed, the tracking control scheme developed in Theorem 3.1 can be employed to solve the robust position feedback control problem without velocity measurements for a class of uncertain flexible-joint robots perturbed simultaneously by time-varying perturbations, time-delay uncertainties, and external disturbances.

4. Simulation examples

Example 4.1: The motion equations of a single-link flexible-joint robot arm with time delays shown in Figure 1 are given by (Luca & Book, 2008; Spong & Vidyasagar, 1989)

$$M\ddot{q}_1 + mgl\sin(q_1) + H_1(x_{e12}(t-\tau_1)) = K(q_2 - q_1) + d_1$$

$$J\ddot{q}_2 + B\dot{q}_2 - K(q_1 - q_2) + H_2(x_{e14}(t-\tau_2)) = u + d_2$$

For the purpose of simulation, let $M_0 = 1\,\text{kg}$, $\Delta M = 0.2\sin(2t)\,\text{kg}$, $mgl_0 = 10\,\text{Nm}$, $\Delta mgl = \sin(t/2)\,\text{Nm}$, $K_0 = 100\,\text{Nm / rad}$, $\Delta K = 5\cos(t/2)\,\text{Nm / rad}$, $J_0 = 1\,\text{kg m}^2$, $\Delta J = 0.2\sin(t/5)\,\text{kg m}^2$, $B_0 = 0.9\,\text{Nm s / rad}$, and $\Delta B = 0.1\cos(t/10)\,\text{Nm s/rad}$. The time-delay uncertainties $H_1(\cdot) = q_1(t-\tau_1)\cos(\dot{q}_1(t-\tau_1))$ and $H_2(\cdot) = \sin(q_1(t-\tau_2))\dot{q}_2(t-\tau_2) + 1.2\dot{q}_1^2(t-\tau_2)q_2(t-\tau_2)$ with $\tau_1 = 1\,\text{s}$ and $\tau_1 = 2\,\text{s}$. The external disturbances $d_1(t)$ and $d_2(t)$ are incorporated with a viscous friction force, dynamic friction force, and an exogenous periodic perturbation:

$$d_1 = 0.5\dot{q}_1 + 0.2\text{sgn}(\dot{q}_1) + 0.5(1 + \text{sgn}(\sin 2t)) \quad \text{and}$$
$$d_2 = 0.5\dot{q}_1 + 0.5\dot{q}_2 + 0.25\text{sgn}(\dot{q}_2) + 0.5\text{sgn}(\sin 2t)$$

where $\text{sgn}(\cdot)$ denotes the signum function and $\text{sgn}(\sin 2t)$ is a square wave with period π, i.e. $\text{sgn}(\sin 2t) = 1$, for $0 \le t < \pi/2$, and $\text{sgn}(\sin 2t) = -1$, for $\pi/2 \le t < \pi$.

Let the desired trajectory $q_{r1}(t) = \sin(t)$. Construct the observer-based position feedback tracking controller (12), (13), (23), (24), (27). Choose $k_1 = 2$, $k_2 = 10$, $k_{01} = 20$, $k_3 = 40$, $k_4 = 60$, and $k_{02} = 80$. Set the initial positions and velocities as $q_1(0) = 0.5$, $y_2(0) = 0$, $\dot{q}_1(0) = 0$, and $\dot{q}_2(0) = 0$, and the initial observer states as $\eta_1(0) = -10$ and $\eta_2(0) = 0$. The angular position $q_1(t)$ and velocity $\dot{q}_1(t)$ on the link side are depicted in Figure 2(a) and 2(b). The angular position $q_2(t)$ and velocity $\dot{q}_2(t)$ on the actuator side are shown in Figure 2(c) and 2(d).

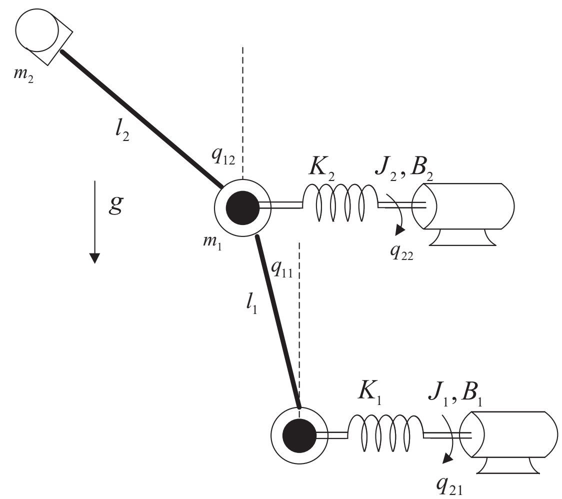

Figure 4. A two-link robot arm with flexible joints.

The applied torque $u(t)$ is plotted in Figure 2(e). These simulation results indicate that a satisfactory tracking and convergent performance for the single-link flexible-joint robot involving time-varying perturbations, time-delay uncertainties, and external disturbances is achieved. Moreover, we make an additional comparative simulation in which all the time-varying perturbations, ΔM, Δmgl, ΔK, ΔJ, ΔB, time-delay uncertainties, $H_1(\cdot)$, and $H_2(\cdot)$, and external disturbances, $d_1(t)$ and $d_2(t)$, are set to zero, and simulation results are shown in Figure 2(f)–(h). It can be seen that the tracking performance in Figure 2(f) and 2(g) is better than that in Figure 2(a) and 2(c) in which the effects of time-varying perturbations, time-delay uncertainties, and external disturbances are involved, and the curve of applied torque in Figure 2(h) is smoother than that in Figure 2(e).

Finally, in order to demonstrate the capability of tracking faster trajectories, we also make a comparative simulation in which the desired trajectory is chosen to be $q_{r1}(t) = \sin(5t)$. The simulation results are shown in Figure 3. These results indicate that the tracking control performance is achieved, so the developed controller can also be employed to track faster trajectories. However, for previously given performance specifications (e.g. bandwidth, maximum error, and damping ratio), how to analytically choose the design parameters in the developed control scheme to satisfy these specifications will be further investigated in the future research.

Example 4.2: Consider the two-link flexible-joint manipulator with time delays shown in Figure 4 (Luca & Book, 2008; Spong & Vidyasagar, 1989). The system parameters in the form of (1) and (2) with $q_1 = [q_{11}, q_{12}]^T$, $q_2 = [q_{21}, q_{22}]^T$, and $u = [u_1, u_2]^T$ are

$$M = \begin{bmatrix} (m_1 + m_2)l_1^2,\ m_1 l_1 l_2(s_1 s_2 + c_1 c_2) \\ m_2 l_1 l_2(s_1 s_2 + c_1 c_2),\ m_2 l_2^2 \end{bmatrix},$$

$$C = m_2 l_1 l_2(c_1 s_2 - s_1 c_2)\begin{bmatrix} 0,\ -\dot{q}_{12} \\ -\dot{q}_{11},\ 0 \end{bmatrix},$$

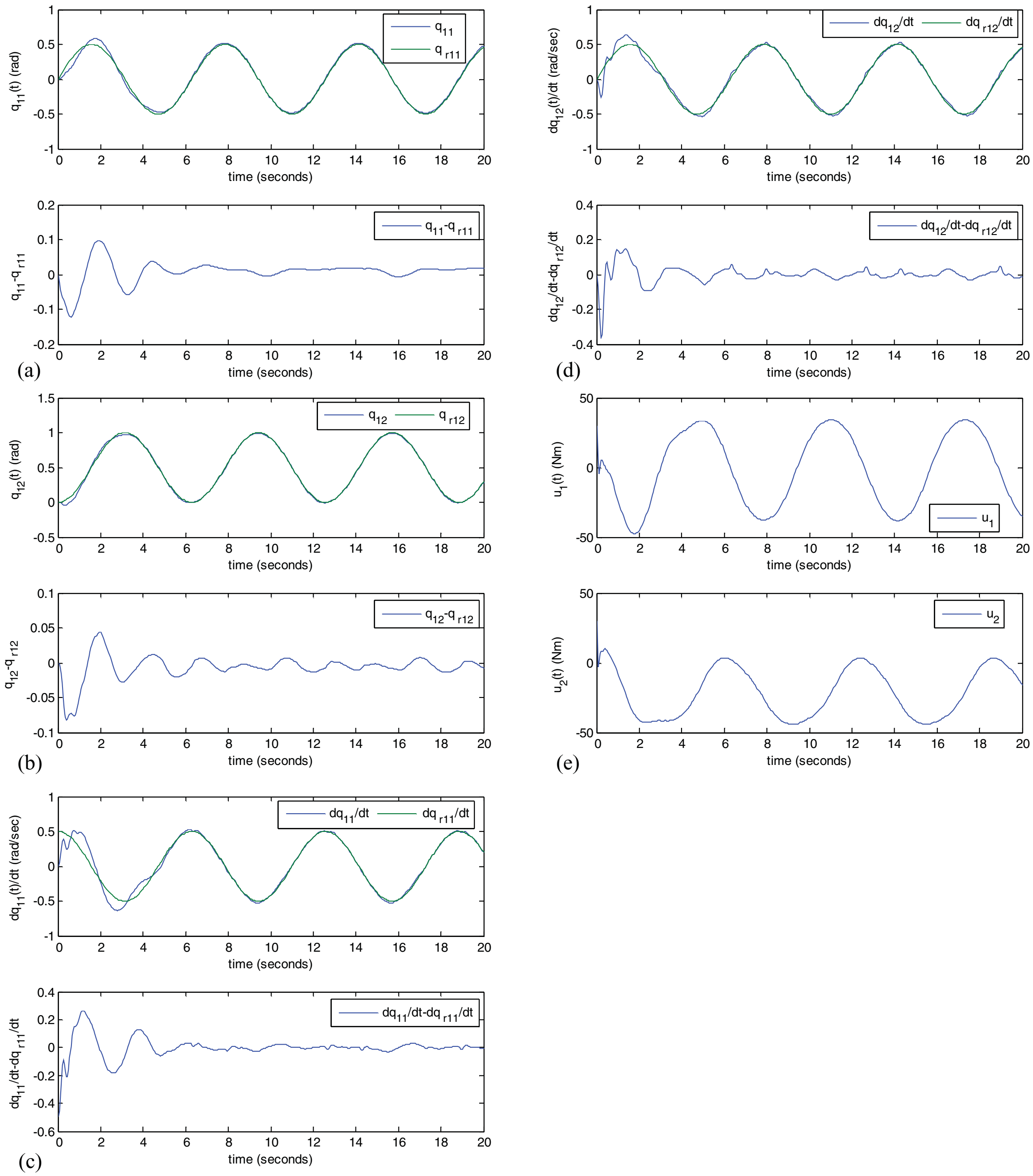

Figure 5. Responses for the two-link flexible-joint time-delay robot arm using the developed controller in Theorem 3.1. (a) Angular position $q_{11}(t)$. (b) Angular position $q_{12}(t)$. (c) Angular velocity $\dot{q}_{11}(t)$. (d) Angular velocity $\dot{q}_{12}(t)$. (e) Applied torques $u_1(t)$ and $u_2(t)$.

$$G = \begin{bmatrix} -(m_1+m_2)l_1 g s_1 \\ -m_2 l_2 g s_2 \end{bmatrix}$$

$$K = \begin{bmatrix} K_1, 0 \\ 0, K_2 \end{bmatrix},\ J = \begin{bmatrix} J_1, 0 \\ 0, J_2 \end{bmatrix},\ B = \begin{bmatrix} B_1, 0 \\ 0, B_2 \end{bmatrix},$$

$$H_1 = \begin{bmatrix} H_{11} \\ H_{12} \end{bmatrix},\ H_2 = \begin{bmatrix} H_{21} \\ H_{22} \end{bmatrix},$$

where $c_1 = \cos(q_{11})$, $s_1 = \sin(q_{11})$, $c_2 = \cos(q_{12})$, and $s_2 = \cos(q_{12})$.

For the purpose of simulation, let $l_1 = 1\,\text{m}$, $l_2 = 1\,\text{m}$, and$g = 9.8\,\text{m/s}^2$. Suppose the system parameters are perturbed by time-varying perturbations, and let $m_{10} = 2\,\text{kg}$ and $\Delta m_{10}(t) = 0.1\sin t\,\text{kg}$, $m_{20} = 5\,\text{kg}$ and $\Delta m_{20}(t) = 0.1\sin t\,\text{kg}$, $K_{10} = 100\,\text{Nm/rad}$ and $\Delta K_{10}(t) = 2\cos t\,\text{Nm/rad}$, $K_{20} = 100\,\text{Nm/rad}$ and $\Delta K_{20}(t) = 2\cos t\,\text{Nm/rad}$, $J_{10} = 1\,\text{kg m}^2$ and $\Delta J_{10}(t) = 0.1\sin 0.5t\,\text{kg m}^2$, $J_{20} = 1\,\text{kg m}^2$ and $\Delta J_{20}(t) = 0.1\sin 0.5t\,\text{kg m}^2$, $B_{10} = 0.9\,\text{Nm s/rad}$ and $\Delta B_{10}(t) = 0.1\cos 0.5t\,\text{Nm s/rad}$, and $B_{20} = 0.9\,\text{Nm s/rad}$ and $\Delta B_{20}(t) = 0.1\cos 0.5t\,\text{Nm s/rad}$. The time-delay uncertainties $H_{11}(\cdot) = q_{11}(t-\tau_1)\cos(\dot{q}_{12}(t-\tau_1))$, and $H_{12}(\cdot) = q_{12}(t-\tau_1)\sin(\dot{q}_{11}(t-\tau_1))$ with $\tau_1 = 1\,\text{s}$, and $H_{21}(\cdot) = \sin(q_{11}(t-\tau_2))\dot{q}_{21}(t-\tau_2) + \dot{q}_{12}(t-\tau_2)q_{22}(t-\tau_2)$, and

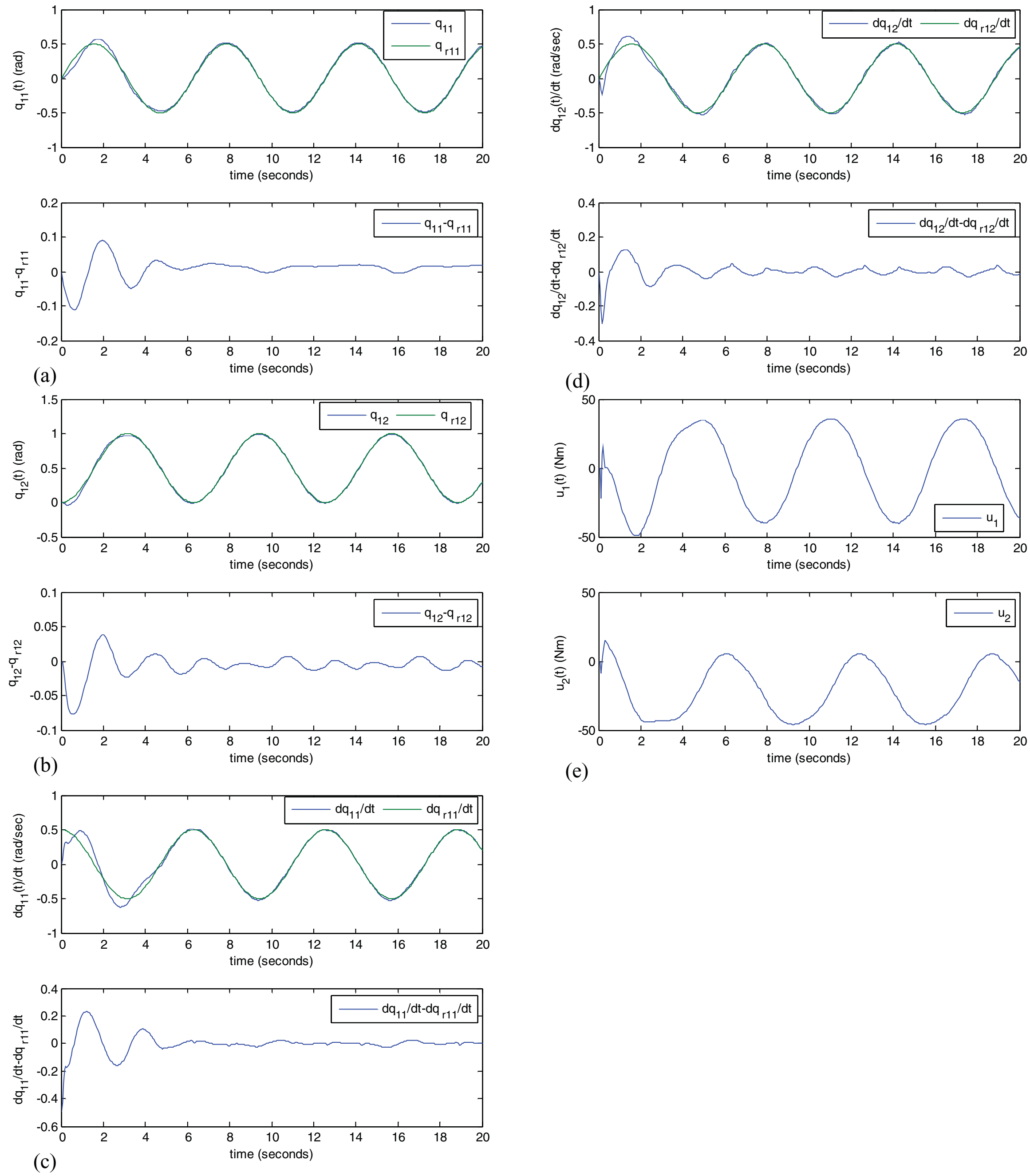

Figure 6. Responses for the two-link flexible-joint time-delay robot arm using the developed controller in (27) incorporating with the numerical position differentiation technique. (a) Angular position $q_{11}(t)$. (b) Angular position $q_{12}(t)$. (c) Angular velocity $\dot{q}_{11}(t)$. (d) Angular velocity $\dot{q}_{12}(t)$. (e) Applied torque $u_1(t)$ and $u_2(t)$.

$H_{22}(\cdot) = q_{12}(t-\tau_2)\dot{q}_{22}(t-\tau_2) + q_{21}(t-\tau_2)\cos(\dot{q}_{11}(t-\tau_2))$ with $\tau_2 = 2\,\text{s}$. The external disturbances $d_1(t)$ and $d_2(t)$ are incorporated with a viscous friction force, dynamic friction force, and an exogenous perturbation:

$$d_1 = \begin{bmatrix} 0.2\dot{q}_{11} + 0.25\text{sgn}(\dot{q}_{11}) + 0.5 \\ 0.25\dot{q}_{11} + 0.2\dot{q}_{12} + 0.25\text{sgn}(\dot{q}_{12}) - 0.5 \end{bmatrix},$$

$$d_2 = \begin{bmatrix} 0.25\dot{q}_{21} + 0.2\text{sgn}(\dot{q}_{21}) - 0.5 \\ 0.2\dot{q}_{21} + 0.25\dot{q}_{22} + 0.2\text{sgn}(\dot{q}_{22}) + 0.5 \end{bmatrix}$$

Let the desired trajectories $q_{r11}(t) = 0.5\sin(t)$ and $q_{r12}(t) = 0.5 - 0.5\cos(t)$.Construct the observer-based position feedback tracking controller (12), (13), (23), (24), and (27) with $k_1 = 2,\ k_2 = 10,\ k_{01} = 20,\ k_3 = 30,\ k_4 = 40,\ \text{and} k_{02} = 60$. Set the initial conditions $q_1(0) = [0, 0]^T$, $q_2(0) = [0, 0]^T$, $\dot{q}_1(0) = [0, 0]^T$, $\dot{q}_2(0) = [0, 0]^T$, $\eta_1(0) = [0, 0]^T$, and $\eta_2(0) = [0, 0]^T$. The angular positions $q_1(t)$ and velocities $\dot{q}_1(t)$ are depicted in Figure 5(a)–(d). The applied torques $u(t)$ are plotted in Figure 5(e) and 5(f). Consequently, the proposed approach can confirm a good tracking performance and effectively compensate the effects due to time-varying perturbations, time-delay uncertainties, and external disturbances.

Next, we perform the following comparative simulation in which the velocity is approximated by numerically differentiating position measurements (Jaritz & Spong, 1996). That is, replace the velocity signals in the applied input (27) by the estimated values from the difference algorithms. Let $\hat{\dot{q}}_{1n}(nT) = (q_1(nT) - q_1((n-1)T))/T$ and $\hat{x}_{4n}(nT) = (q_2(nT) - q_2((n-1)T))/T$, where $\hat{\dot{q}}_{1n}(\cdot)$ denotes the estimate of the velocity $\dot{q}_1(t)$, $\hat{x}_{4n}(\cdot)$ denotes the estimate of the velocity $\dot{q}_2(t)$, n represents the sampling instant, and T represents the sampling interval. Replacing $\hat{x}_4(t)$ by $\hat{x}_{4n}(nT)$ and $\hat{\bar{x}}_2(t)$ by $\hat{\bar{x}}_{2n}(t) = \hat{\dot{q}}_{1n}(nT) - \dot{q}_{r1}(t) + k_1(q_1(t) - q_{r1}(t))$, the applied input in (27) is modified as $u = B_0\hat{x}_{4n} - K_0(x_1 - x_3) + J_0(K_0^{-1}\ddot{F}_0 - k_4\bar{x}_{4n})$ with $\bar{x}_{4n}(t) = \hat{x}_{4n}(nT) - x_{4n}^*(t)$, $x_{4n}^* = K_0^{-1}\dot{F}_0(q_{re}) - k_3\bar{x}_{3n}$, and $\bar{x}_{3n}(t) = x_3(t) - K_0^{-1}(F_0(q_{re}) - k_2\hat{\bar{x}}_{2n})$. The simulation results are shown in Figure 6. Obviously, this comparative simulation demonstrates that the observer-based position feedback control algorithm proposed in this study enables the achievement of a good tracking performance, which is similar to that obtained from the frequently used numerical position differentiation technique. However, compared with this numerical position differentiation technique (in which no theoretical justification was developed to show the closed-loop stability property), a mathematical analysis and control design is employed here to theoretically show that the closed-loop robot system possesses stability property and tracking performance.

5. Conclusion

Robust tracking control of flexible-joint time-delay robots using only position measurements is proposed and solved. By employing the Lyapunov–Krasovskii functional technique and backstepping design technique, a novel robust position feedback tracking control scheme without velocity measurements is developed such that all the states and signals remain bounded and the tracking error is UUB. By appropriately choosing the Lyapunov–Krasovskii functionals and designing the velocity observer and the virtual control input, the circular phenomenon in the controller design is overcome and the complexity in the backstepping design is avoided. Consequently, a linear time-varying robust tracking control scheme is developed and that can be easily implemented. Compared with the existing literature of controlling flexible-joint robots, the robust tracking control scheme developed in this study can be extended to handle a broader class of uncertain flexible-joint time-delay robots.

Acknowledgments

The author would like to thank the associate editor and the anonymous reviewers for their valuable comments and suggestions that have improved the presentation of this paper. This work was supported by the National Science Council, Taiwan, R.O.C., under contract NSC 102-2221-E-168-011.

Disclosure statement

No potential conflict of interest was reported by the authors.

Funding

National Science Council, Taiwan, R.O.C. [contract NSC 102-2221-E-168-011].

Notes on contributors

Yeong-Chan Chang was born in Tainan, Taiwan, R.O.C. He received his PhD degree in electrical engineering from the National Tsing-Hua University in 1995. Since 2000 he has been working as a professor in the Department of Electrical Engineering at Kun-Shan University, Tainan, Taiwan, R.O.C. His current research interests are in nonlinear control, adaptive control, intelligent control, robotic control, and constrained mechanical systems.

Ming-Fang Wu was born in Taichung, Taiwan, R.O.C. He received his PhD degree in electrical engineering from the National Taiwan University of Science and Technology in 1995. Since 1996 he has been working as an associate professor in the Department of Electrical Engineering at Kun-Shan University, Tainan, Taiwan, R.O.C. His current research interests are in machine vision control, image processing, and signal processing.

References

Bang, J.S., Shim, H., Park, S.K., & Seo, J.H. (2010). Robust tracking and vibration suppression for a two-inertia system by combining backstepping approach with disturbance observer. *IEEE Transactions on Industrial Electronics, 57*, 3197–3206.

Chang, Y.C. (2014). Robust adaptive neural tracking control for a class of electrically driven robots with time delays. *International Journal of Systems Science, 45*, 2418–2434.

Chang, Y.C., & Yen, H.M. (2011). Design of a robust position feedback tracking controller for flexible-joint robots. *IET Control Theory and Applications, 5*, 351–363.

Chang, Y.C., & Yen, H.M. (2012). Robust tracking control for a class of electrically driven flexible-joint robots without velocity measurements. *International Journal of Control, 85*, 194–212.

Forbes, J.R., & Damaren, C.J. (2011). Design of optimal strictly positive real controllers using numerical optimization for the control of flexible robotic systems. *Journal of the Franklin Institute, 348*, 2191–2215.

Huang, A.C., & Chen, Y.C. (2004). Adaptive sliding control for single-link flexible-joint robot with mismatched uncertainties. *IEEE Transactions on Control Systems Technology, 12*, 770–775.

Jaritz, A., & Spong, M.W. (1996). An experimental comparison of robust control algorithms on a direct drive manipulator. *IEEE Transactions on Control Systems Technology, 4*, 627–640.

Jiang, S.Y., Xu, J., & Yan, Y. (2014). Stability and oscillations in a slow-fast flexible joint system with transformation delay. *Acta Mechanica Sinica, 30*, 727–738.

Kao, C.Y., & Pasumarthy, R. (2012). Stability analysis of interconnected Hamiltonian systems under time delays. *IET Control Theory and Applications, 6*, 570–577.

Khalil, H.K. (2002). *Nonlinear systems* (3rd ed.). Upper Saddle River, NJ: Prentice-Hall.

Korkmaz, O., & Kemal Ider, S. (2014). Hybrid force and motion control of flexible joint parallel manipulators using inverse dynamics approach. *Advanced Robotics, 28*, 1221–1230.

Li, Y., Guan, X., & Luo, X. (2011). Robust exponential stability criteria of uncertain stochastic systems with time-varying delays. *International Journal of Systems Science, 42*, 601–608.

Li, Z., Xia, Y., & Sun, F. (2014). Adaptive fuzzy control for multilateral cooperative teleoperation of multiple robotic manipulators under random network-induced delays. *IEEE Transactions on Fuzzy Systems, 22*, 437–450.

Lightcap, C.A., & Banks, S.A. (2010). An extended Kalman filter for real-time estimation and control of a rigid-link flexible-joint manipulator. *IEEE Transactions on Control Systems Technology, 18*, 91–103.

Liu, Z.G., & Wu, Y.Q. (2014). Modelling and adaptive tracking control for flexible joint robots with random noises. *International Journal of Control, 87*, 2499–2510.

Liu, C., Cheah, C.C., & Slotine, J.J.E. (2008). Adaptive task-space regulation of rigid-link flexible-joint robots with uncertain kinematics. *Automatica, 44*, 1806–1814.

Luca, A.D., & Book, W. (2008). Robots with flexible elements. In B. Siciliano & O. Khatib (Eds.), *Springer Handbook of Robotics* (pp. 287–319). Berlin/Heidelberg: Springer-Verlag.

Luca, A.D., Siciliano, B., & Zollo, L. (2005). PD control with on-line gravity compensation for robots with elastic joints: Theory and experiments. *Automatica, 41*, 1809–1819.
Melhem, K., & Wang, W. (2009). Global output tracking control of flexible joint robots via factorization of the manipulator mass matrix. *IEEE Transactions on Robotics, 25*, 428–437.
Niculescu, S.L. (2001). *Delay effects on stability: a robust control approach*. New York: Springer-Verlag.
Ozgoli, S., & Taghirad, H.D. (2009). Fuzzy error governor: a practical approach to counter actuator saturation on flexible joint robots. *Mechatronics, 19*, 993–1002.
Philip Chen, C.L., Wen, G.X., Liu, Y.J., & Wang, F.Y. (2014). Adaptive consensus control for a class of nonlinear multiagent time-delay systems using neural networks. *IEEE Transactions on Neural Networks and Learning Systems, 25*, 1217–1226.
Sharma, N., Bhasin, S., Wang, Q., & Dixon, W.E. (2011). Predictor-based control for an uncertain Euler–Lagrange system with input delay. *Automatica, 47*, 2332–2342.
Son, Y.I., Shim, H., & Seo, J.H. (2004). A dynamic output feedback control law for elastic joint robots via feedback-passivity approach. *Journal of the Franklin Institute, 341*, 477–490.
Spong, M.W., & Vidyasagar, M. (1989). *Robot dynamics and control*. New York, NY: Wiley.
Wang, M., Ge, S.S., & Hong, K.S. (2010). Approximation-based adaptive tracking control of pure-feedback nonlinear systems with multiple unknown time-varying delays. *IEEE Transactions on Neural Networks, 21*, 1804–1816.
Yoo, S.J. (2012a). Actuator fault detection and adaptive accommodation control of flexible-joint robots. *IET Control Theory and Applications, 6*, 1497–1507.
Yoo, S.J. (2012b). Adaptive neural output-feedback control for a class of nonlinear systems with unknown time-varying delays. *IET Control Theory and Applications, 6*, 130–140.
Yun, J.N., & Su, J.B. (2014). Design of a disturbance observer for a two-link manipulator with flexible joints. *IEEE Transactions on Control Systems Technology, 22*, 809–815.
Zhang, X.H., Xie, X.J., & Huang, Y.X. (2015). Global state feedback stabilization of high-order nonlinear systems with multiple time-varying delays. *Journal of the Franklin Institute, 352*, 271–290.
Zhou, Q., Shi, P., Xu, S., & Li, H. (2013). Adaptive output feedback control for nonlinear time-delay systems by fuzzy approximation approach. *IEEE Transactions on Fuzzy Systems, 21*, 301–313.
Zhu, B., Wang, X., & Cai, K.Y. (2010). Tracking control for angular-rate-sensorless vertical take-off and landing aircraft in the presence of angular-position measurement delay. *IET Control Theory and Applications, 4*, 957–969.

Appendices

Appendix 1: Proof of Theorem 3.1.

Choose the Lyapunov–Krasovskii function candidate $V(t)$ as

$$\begin{aligned} V(t) &= \frac{\alpha_1}{2}\bar{x}_1^T\bar{x}_1 + \frac{1}{2}\bar{x}_2^T M(x_1)\bar{x}_2 + \frac{1}{2}\bar{e}_2^T M(x_1)\bar{e}_2 \\ &\quad + \frac{\alpha_2}{2}\sum_{j=1}^{2}\int_{t-\tau_j}^{t} P_{j1}(x_1(\tau), x_2(\tau))\,d\tau + \frac{\alpha_3}{2}\bar{x}_3^T\bar{x}_3 \\ &\quad + \frac{\alpha_4}{2}\bar{x}_4^T\bar{x}_4 + \frac{\alpha_5}{2}\bar{e}_4^T\bar{e}_4 + \frac{\alpha_6}{2}\int_{t-\tau_2}^{t} P_2(x_3(\tau), x_4(\tau))\,d\tau \end{aligned} \tag{31}$$

for some weighting gains $\alpha_1 > 0, \ldots, \alpha_6 > 0$, where $P_{j1}(\cdot) = b_{j1}^2\,\|x_1(t)\|^2 + b_{j2}^2\,\|x_2(t)\|^2$ and $P_2(\cdot) = b_{23}^2\,\|x_3(t)\|^2 + b_{24}^2\|x_4(t)\|^2$. Taking into account (7), (17), (18), (22), (25), and (28), the derivative of $V(t)$ is computed as

$$\begin{aligned} \dot{V} &= \alpha_1\bar{x}_1^T(-k_1\bar{x}_1 + \bar{x}_2) + (1/2)\bar{x}_2^T\dot{M}\bar{x}_2 + (1/2)\bar{e}_2^T\dot{M}\bar{e}_2 \\ &\quad + (\bar{x}_2^T + \bar{e}_2^T)(-C\bar{x}_2 - F_1\bar{x}_1 - F_2\bar{x}_2 - \Delta F_d - \Delta K q_{r1} - K\bar{x}_1 \\ &\quad - H_1(x_{e12}(t-\tau_1)) - k_2(I + \Delta K K_0^{-1})\hat{\bar{x}}_2 + \Delta K K_0^{-1}F_0 \\ &\quad + K\bar{x}_3 + d_1) - k_{01}\bar{e}_2^T M\bar{e}_2 \\ &\quad + (\alpha_2/2)(b_{11}^2x_1^2(t) + b_{12}^2x_2^2(t)) + (\alpha_2/2)(b_{21}^2x_1^2(t) \\ &\quad + b_{22}^2x_2^2(t)) - (\alpha_2/2)(b_{11}^2x_1^2(t-\tau_1) + b_{12}^2x_2^2(t-\tau_1)) \\ &\quad - (\alpha_2/2)(b_{21}^2x_1^2(t-\tau_2) + b_{22}^2x_2^2(t-\tau_2)) \\ &\quad + \alpha_3\bar{x}_3^T(\bar{e}_4 + \bar{x}_4 - k_3\bar{x}_3 + k_{01}k_2K_0^{-1}\bar{e}_2) \\ &\quad + \alpha_4\bar{x}_4^T(k_{02}\bar{e}_4 + k_3(\bar{e}_4 + \bar{x}_4 - k_3\bar{x}_3 + k_{01}k_2K_0^{-1}\bar{e}_2) - k_4\bar{x}_4) \\ &\quad + \alpha_5\bar{e}_4^T(-\Delta J_B\bar{e}_4 - J^{-1}H_2(x_{e14}(t-\tau_2)) - k_4\Delta J J^{-1}\bar{x}_4 \\ &\quad + \Delta F_4 - J_0^{-1}B_0\bar{e}_4 - k_{02}\bar{e}_4 + J^{-1}d_2) \\ &\quad + (\alpha_6/2)(b_{23}^2x_3^2(t) + b_{24}^2x_4^2(t)) - (\alpha_6/2)(b_{23}^2x_3^2(t-\tau_2) \\ &\quad + b_{24}^2x_4^2(t-\tau_2)) \end{aligned} \tag{32}$$

By using Young's inequalities, the uncertain terms containing time-delay functions are bounded as

$$\begin{aligned} &-\bar{x}_2^T(t)H_1(x_{e12}(t-\tau_1)) \le (2/\alpha_2)\bar{x}_2^2(t) \\ &+ (\alpha_2/4)(b_{11}^2x_1^2(t-\tau_1) + b_{12}^2x_2^2(t-\tau_1)) \end{aligned} \tag{33}$$

$$\begin{aligned} &-\bar{e}_2^T(t)H_1(x_{e12}(t-\tau_1)) \le (2/\alpha_2)\bar{e}_2^2(t) \\ &+ (\alpha_2/4)(b_{11}^2x_1^2(t-\tau_1) + b_{12}^2x_2^2(t-\tau_1)) \end{aligned} \tag{34}$$

$$\begin{aligned} &-\alpha_5\bar{e}_4^TJ^{-1}H_2(x_{e14}(t-\tau_2)) \le (1/\alpha_2 + 1/\alpha_6)\alpha_5^2\lambda_{J^{-1}}^2\bar{e}_4^2(t) \\ &+ (\alpha_2/2)b_{21}^2x_1^2(t-\tau_2) + (\alpha_2/2)b_{22}^2x_2^2(t-\tau_2) \\ &+ (\alpha_6/2)b_{23}^2x_3^2(t-\tau_2) + (\alpha_6/2)b_{24}^2x_4^2(t-\tau_2) \end{aligned} \tag{35}$$

By substituting (33)–(35) into (32), we have

$$\begin{aligned} \dot{V} &\le \alpha_1\bar{x}_1^T(-k_1\bar{x}_1 + \bar{x}_2) + (1/2)\bar{x}_2^T\dot{M}\bar{x}_2 + (1/2)\bar{e}_2^T\dot{M}\bar{e}_2 \\ &\quad + (\bar{x}_2^T + \bar{e}_2^T)(-C\bar{x}_2 - F_1\bar{x}_1 - F_2\bar{x}_2 - \Delta F_d - \Delta K q_{r1} - K\bar{x}_1 \\ &\quad - k_2(I + \Delta K K_0^{-1})\hat{\bar{x}}_2 + \Delta K K_0^{-1}F_0 + K\bar{x}_3 + d_1) \\ &\quad + (2/\alpha_2)\bar{x}_2^2(t) + (\alpha_2/4)(b_{11}^2x_1^2(t-\tau_1) + b_{12}^2x_2^2(t-\tau_1)) \\ &\quad + (2/\alpha_2)\bar{e}_2^2(t) + (\alpha_2/4)(b_{11}^2x_1^2(t-\tau_1) + b_{12}^2x_2^2(t-\tau_1)) \\ &\quad - k_{01}\bar{e}_2^TM\bar{e}_2 + (\alpha_2/2)(b_{11}^2x_1^2(t) + b_{12}^2x_2^2(t)) \\ &\quad + (\alpha_2/2)(b_{21}^2x_1^2(t) + b_{22}^2x_2^2(t)) \\ &\quad - (\alpha_2/2)(b_{11}^2x_1^2(t-\tau_1) + b_{12}^2x_2^2(t-\tau_1)) \\ &\quad - (\alpha_2/2)(b_{21}^2x_1^2(t-\tau_2) + b_{22}^2x_2^2(t-\tau_2)) \\ &\quad + \alpha_3\bar{x}_3^T(\bar{e}_4 + \bar{x}_4 - k_3\bar{x}_3 + k_{01}k_2K_0^{-1}\bar{e}_2) \\ &\quad + \alpha_4\bar{x}_4^T(k_{02}\bar{e}_4 + k_3(\bar{e}_4 + \bar{x}_4 - k_3\bar{x}_3 + k_{01}k_2K_0^{-1}\bar{e}_2) - k_4\bar{x}_4) \\ &\quad + \alpha_5\bar{e}_4^T(-\Delta J_B\bar{e}_4 - k_4\Delta J J^{-1}\bar{x}_4 + \Delta F_4 - J_0^{-1}B_0\bar{e}_4 \\ &\quad - k_{02}\bar{e}_4 + J^{-1}d_2) \\ &\quad + (1/\alpha_2 + 1/\alpha_6)\alpha_5^2\lambda_{J^{-1}}^2\bar{e}_4^2(t) + (\alpha_2/2)b_{21}^2x_1^2(t-\tau_2) \\ &\quad + (\alpha_2/2)b_{22}^2x_2^2(t-\tau_2) \\ &\quad + (\alpha_6/2)b_{23}^2x_3^2(t-\tau_2) + (\alpha_6/2)b_{24}^2x_4^2(t-\tau_2) \\ &\quad + (\alpha_6/2)(b_{23}^2x_3^2(t) + b_{24}^2x_4^2(t)) \\ &\quad - (\alpha_6/2)(b_{23}^2x_3^2(t-\tau_2) + b_{24}^2x_4^2(t-\tau_2)) \end{aligned} \tag{36}$$

For the sake of clarity, we analyse the terms in the right-hand side of (36) in the following steps.

(1) Taking into account the skew-symmetric property P2, we obtain $\bar{x}_2^T(1/2\dot{M}-C)\bar{x}_2=0$ and $\bar{e}_2^T(1/2\dot{M}-C)\bar{e}_2=0$. Then,

$$(1/2)\bar{x}_2^T\dot{M}\bar{x}_2+(1/2)\bar{e}_2^T\dot{M}\bar{e}_2+(\bar{x}_2^T+\bar{e}_2^T)(-C\bar{x}_2) = -\bar{e}_2^TC\bar{x}_2+\bar{e}_2^TC\bar{e}_2 \tag{37}$$

(2) From the fact $\hat{\bar{x}}_2(t)=\bar{x}_2(t)-\bar{e}_2(t)$ and A3, we get

$$-k_2(\bar{x}_2^T+\bar{e}_2^T)(I+\Delta KK_0^{-1})\hat{\bar{x}}_2 \le -(1-\varepsilon_K)k_2\bar{x}_2^T\bar{x}_2 + (1+\varepsilon_K)k_2\bar{e}_2^T\bar{e}_2 \tag{38}$$

(3) The summation of all the nonlinear terms containing the time-delay states $x_i(t-\tau_1)$, $i=1,2$, and $x_i(t-\tau_2)$, $i=1,\ldots,4$ is equal to zero, that is,

$$\begin{aligned}&(\alpha_2/4)(b_{11}^2x_1^2(t-\tau_1)+b_{12}^2x_2^2(t-\tau_1))\\&+(\alpha_2/4)(b_{11}^2x_1^2(t-\tau_1)+b_{12}^2x_2^2(t-\tau_1))\\&-(\alpha_2/2)(b_{11}^2x_1^2(t-\tau_1)+b_{12}^2x_2^2(t-\tau_1))\\&-(\alpha_2/2)(b_{21}^2x_1^2(t-\tau_2)+b_{22}^2x_2^2(t-\tau_2))\\&+(\alpha_2/2)(b_{21}^2x_1^2(t-\tau_2)+b_{22}^2x_2^2(t-\tau_2))\\&+(\alpha_6/2)(b_{23}^2x_3^2(t-\tau_2)+b_{24}^2x_4^2(t-\tau_2))\\&-(\alpha_6/2)(b_{23}^2x_3^2(t-\tau_2)+b_{24}^2x_4^2(t-\tau_2))=0\end{aligned} \tag{39}$$

(4) Consider the nonlinear delay-free terms $(\alpha_2/2)(b_{11}^2x_1^2+b_{12}^2x_2^2)+(\alpha_2/2)(b_{21}^2x_1^2+b_{22}^2x_2^2)$ that are induced owing to the use of Lyapunov–Krasovskii functional terms to handle the effects of time delays. From the equality $x_1=\bar{x}_1+q_{r1}$, we have $x_1^2\le 2\bar{x}_1^2+2q_{r1}^2$. From the equality $x_2=\bar{x}_2-k_1\bar{x}_1+\dot{q}_{r1}$, we have $x_2^2\le 2(\bar{x}_2-k_1\bar{x}_1)^2+2\dot{q}_{r1}^2\le 4\bar{x}_2^2+4k_1^2\bar{x}_1^2+2\dot{q}_{r1}^2$.

Choose the weighting scalar $\alpha_2=\beta_2/k_1^2$ for some $\beta_2>0$. Then, if $k_1>1$, we get

$$\begin{aligned}&(\alpha_2/2)(b_{11}^2x_1^2+b_{12}^2x_2^2+b_{21}^2x_1^2+b_{22}^2x_2^2)\\&\le(\beta_2/2k_1^2)((2b_{11}^2+4k_1^2b_{12}^2+2b_{21}^2+4k_1^2b_{22}^2)\bar{x}_1^2\\&\quad+(4b_{12}^2+4b_{22}^2)\bar{x}_2^2\\&+(2b_{11}^2q_{r1}^2+2b_{12}^2\dot{q}_{r1}^2+2b_{21}^2q_{r1}^2+2b_{22}^2\dot{q}_{r1}^2))\\&\le\delta_{11}\bar{x}_1^2+\delta_{12}\bar{x}_2^2+\delta_{10}\end{aligned} \tag{40}$$

where $\delta_{11}=\beta_2(b_{11}^2+2b_{12}^2+b_{21}^2+2b_{22}^2)$, $\delta_{12}=\beta_2(2b_{12}^2+2b_{22}^2)$, and $\delta_{10}=\beta_2(b_{11}^2q_{r1}^2+b_{12}^2\dot{q}_{r1}^2+b_{21}^2q_{r1}^2+b_{22}^2\dot{q}_{r1}^2)$. Here, owing to the suitable choice of α_2, the values of δ_{11}, δ_{12}, and δ_{10} do not depend upon k_1, k_2, k_{01}, k_3, k_4, and k_{02}.

(5) Consider the nonlinear delay-free term $(\alpha_6/2)b_{23}^2x_3^2$ that is also induced owing to the effects of time delays. From the equality $x_3=\bar{x}_3+K_0^{-1}F_0-k_2K_0^{-1}\hat{\bar{x}}_2$ and the inequality $\hat{\bar{x}}_2^2\le 2\bar{x}_2^2+2\bar{e}_2^2$, we have $x_3^2\le 4\bar{x}_3^2+8k_2^2(K_0^{-1}\bar{x}_2)^2+8k_2^2(K_0^{-1}\bar{e}_2)^2+2(K_0^{-1}F_0)^2$. Choose the weighting scalar $\alpha_6=\beta_6/k_2^2k_3^2$ for some $\beta_6>0$. Then, if $k_2k_3>1$, we get

$$\begin{aligned}(\alpha_6/2)b_{23}^2x_3^2&\le(\beta_6b_{23}^2/2k_2^2k_3^2)(4\bar{x}_3^2+8k_2^2(K_0^{-1}\bar{x}_2)^2\\&\quad+8k_2^2(K_0^{-1}\bar{e}_2)^2+2(K_0^{-1}F_0)^2)\\&\le\delta_{21}\bar{x}_3^2+\delta_{22}\bar{x}_2^2+\delta_{23}\bar{e}_2^2+\delta_{20}\end{aligned} \tag{41}$$

where $\delta_{21}=2\beta_6b_{23}^2$, $\delta_{22}=4\beta_6b_{23}^2\lambda_{K_0^{-1}}^2$, $\delta_{23}=4\beta_6b_{23}^2\lambda_{K_0^{-1}}^2$, and $\delta_{20}=\beta_6b_{23}^2(K_0^{-1}F_0)^2$.

(6) Consider the nonlinear delay-free term $(\alpha_6/2)b_{24}^2x_4^2$ that is induced owing to the effects of time delays. From the definitions of $\bar{x}_4,\bar{e}_4,x_4^*$, we get $x_4^2\le 8\bar{e}_4^2+8\bar{x}_4^2+4k_3^2\bar{x}_3^2+2(K_0^{-1}\dot{F}_0)^2$. Therefore, we can compute

$$(\alpha_6/2)b_{24}^2x_4^2\le\delta_{31}\bar{e}_4^2+\delta_{32}\bar{x}_4^2+\delta_{33}\bar{x}_3^2+\delta_{30} \tag{42}$$

where $\delta_{31}=4\beta_6b_{24}^2$, $\delta_{32}=4\beta_6b_{24}^2$, $\delta_{33}=2\beta_6b_{24}^2$, and $\delta_{30}=\beta_6b_{24}^2(K_0^{-1}\dot{F}_0)^2$. It is clear that owing to the suitable choice of α_6, all the values of δ_{21}, δ_{22}, δ_{23}, δ_{20}, δ_{31}, δ_{32}, δ_{33}, and δ_{30} do not depend upon k_1, k_2, k_{01}, k_3, k_4, and k_{02}.

Summarily, substituting (37)–(42) into (36), the derivative $\dot{V}$ is bounded as

$$\begin{aligned}\dot{V}\le&-\alpha_1k_1\bar{x}_1^T\bar{x}_1-(1-\varepsilon_K)k_2\bar{x}_2^T\bar{x}_2-k_{01}\bar{e}_2^TM\bar{e}_2\\&-\alpha_3k_3\bar{x}_3^T\bar{x}_3-\alpha_4k_4\bar{x}_4^T\bar{x}_4-k_{02}\alpha_5\bar{e}_4^T\bar{e}_4\\&+\alpha_1\bar{x}_1^T\bar{x}_2-\bar{e}_2^TC\bar{x}_2+\bar{e}_2^TC\bar{e}_2+(1+\varepsilon_K)k_2\bar{e}_2^T\bar{e}_2\\&+(\bar{x}_2^T+\bar{e}_2^T)(-F_1\bar{x}_1-F_2\bar{x}_2-\Delta F_d-\Delta Kq_{r1}-K\bar{x}_1\\&+\Delta KK_0^{-1}F_0+K\bar{x}_3+d_1)\\&+(2/\alpha_2)\bar{x}_2^2+(2/\alpha_2)\bar{e}_2^2+\delta_{11}\bar{x}_1^2+\delta_{12}\bar{x}_2^2+\delta_{10}\\&+\alpha_3\bar{x}_3^T(\bar{e}_4+\bar{x}_4+k_{01}k_2K_0^{-1}\bar{e}_2)\\&+\alpha_4\bar{x}_4^T(k_{02}\bar{e}_4+k_3(\bar{e}_4+\bar{x}_4-k_3\bar{x}_3+k_{01}k_2K_0^{-1}\bar{e}_2))\\&+\alpha_5\bar{e}_4^T(-\Delta J_B\bar{e}_4-k_4\Delta JJ^{-1}\bar{x}_4+\Delta F_4-J_0^{-1}B_0\bar{e}_4+J^{-1}d_2)\\&+(1/\alpha_2+1/\alpha_6)\alpha_5^2\lambda_{J^{-1}}^2\bar{e}_4^2+\delta_{21}\bar{x}_3^2+\delta_{22}\bar{x}_2^2\\&+\delta_{23}\bar{e}_2^2+\delta_{20}+\delta_{31}\bar{e}_4^2+\delta_{32}\bar{x}_4^2+\delta_{33}\bar{x}_3^2+\delta_{30}\end{aligned} \tag{43}$$

Let $R_0=\{x_0|\,\|q_1-q_{r1}\|\le c_1,\|\dot{q}_1-\dot{q}_{r1}\|\le c_2,\|\hat{\bar{x}}_2\|\le c_3,\|q_2\|\le c_4,\|\dot{q}_2\|\le c_5,\|\hat{x}_4\|\le c_6\}$ be an attraction region with $x_0=[q_1^T,\dot{q}_1^T,\hat{\bar{x}}_2^T,q_2^T,\dot{q}_2^T,\hat{x}_4^T]^T$ and constants $c_1>0,\ldots,c_6>0$. Recall that $\bar{x}_3=x_3-K_0^{-1}F_0(q_{re})+k_2K_0^{-1}\hat{\bar{x}}_2$. Define $c_7=\max_{x_0\in R_0}\|x_3-K_0^{-1}F_0(q_{re})\|$. Take the weighting scalar $\alpha_3=\beta_3/k_2^2$ for some $\beta_3>0$. Then, if $k_2>1$, we can obtain $(\alpha_3/2)\bar{x}_3^T\bar{x}_3\le(\beta_3/2k_2^2)(\|x_3-K_0^{-1}F_0\|+k_2\|K_0^{-1}\hat{\bar{x}}_2\|)^2\le(\beta_3/2)(c_7+c_3\lambda_{K_0^{-1}})^2$, where $\lambda_{K_0^{-1}}$ is the maximum eigenvalue of K_0^{-1}.Recall that $\bar{x}_4=\hat{x}_4-K_0^{-1}\dot{F}_0(q_{re})+k_3\bar{x}_3$. Define $c_8=\max_{x_0\in R_0}\|\hat{x}_4-K_0^{-1}\dot{F}_0(q_{re})\|$. Take the weighting scalar $\alpha_4=\beta_4/k_2^2k_3^2$ for some $\beta_4>0$. Then, if $k_2k_3>1$, we can compute $(\alpha_4/2)\bar{x}_4^T\bar{x}_4\le(\beta_4/2k_2^2k_3^2)(\|\hat{x}_4-K_0^{-1}\dot{F}_0\|+k_3\|\bar{x}_3\|)^2\le(\beta_4/2)(c_8+c_7+c_3\lambda_{K_0^{-1}})^2$. Define the augmented error vector $\bar{e}=[\bar{x}_1^T,\bar{x}_2^T,\bar{e}_2^T,\bar{x}_3^T,\bar{x}_4^T,\bar{e}_4^T]^T$. Let $R=\{\bar{e}|\,\|\bar{x}_1\|\le\sqrt{2V_{\max}/\alpha_1},\|\bar{x}_2\|\le\sqrt{2V_{\max}/\lambda_m},\|\bar{e}_2\|\le\sqrt{2V_{\max}/\lambda_m},\|\bar{x}_3\|\le\sqrt{2V_{\max}/\alpha_3},\|\bar{x}_4\|\le\sqrt{2V_{\max}/\alpha_4}$, and $\|\bar{e}_4\|\le\sqrt{2V_{\max}/\alpha_5}\}$, where $V_{\max}$ is a sufficiently large value such that $\max_{x_0\in R_0}V(t)\le V_{\max}$. In order to avoid the circular construction of the controller, the value of $V_{\max}$ should not depend upon the values of time delays τ_1, τ_2 and gains k_2, k_{01}, k_3, k_4, andk_{02}. According to the region R, define the bounded values as $\max_{\bar{e}\in R}\|F_1\| = M_1$, $\max_{\bar{e}\in R}\|F_2\| = M_2$, and $\max_{\bar{e}\in R}\|C\| = M_3$ for some constants $M_1 > 0$, $M_2 > 0$, and $M_3 > 0$. Suppose $\|K\| \le \lambda_K$, $\|J^{-1}\| \le \lambda_{J^{-1}}$, and $\|\Delta J_B\| \le \lambda_{\Delta J_B}$ for some constants $\lambda_K > 0$, $\lambda_{J^{-1}} > 0$, and $\lambda_{\Delta J_B} > 0$.

Furthermore, since the uncertain term $-\Delta F_d - \Delta K q_{r1} + \Delta K K_0^{-1} F_0$ depends only on the desired trajectories $q_{r1}, \dot{q}_{r1}, \ddot{q}_{r1}$, there exists an $\varepsilon_1 > 0$ such that $\| - \Delta F_d - \Delta K q_{r1} + \Delta K K_0^{-1} F_0\| \le \varepsilon_1$. For the uncertain term ΔF_4 defined in (28) suppose there are $\varepsilon_2 > 0, \ldots, \varepsilon_7 > 0$ such that $\|\Delta F_4\| \le \varepsilon_2 + \varepsilon_3\|\bar{x}_1\| + \varepsilon_4\|\bar{x}_2\| + \varepsilon_5\|\bar{e}_2\| + \varepsilon_6\|\bar{x}_3\| + \varepsilon_7\|\bar{x}_4\|$. It is clear that since both terms $-\Delta F_d - \Delta K q_{r1} + \Delta K K_0^{-1} F_0$ and ΔF_4 are induced owing to the small perturbations, these two terms can be omitted when all the small perturbations are neglected, and this also implies that the bounded values can be set to be $\varepsilon_1 = 0, \ldots, \varepsilon_7 = 0$.

Therefore, by using the aforementioned inequalities, the Cauchy–Schwartz inequality, and the procedure of completing the squares, $\dot{V}(t)$ can be further bounded as

$$\begin{aligned}
\dot{V} \le & -\alpha_1 k_1 \|\bar{x}_1\|^2 - (1-\varepsilon_K)k_2\|\bar{x}_2\|^2 - \lambda_m k_{01}\|\bar{e}_2\|^2 \\
& - \alpha_3 k_3 \|\bar{x}_3\|^2 - \alpha_4 k_4 \|\bar{x}_4\|^2 - \alpha_5 k_{02}\|\bar{e}_4\|^2 \\
& + M_2\|\bar{x}_2\|^2 + (1+\varepsilon_K)k_2\|\bar{e}_2\|^2 + M_3\|\bar{e}_2\|^2 \\
& + \alpha_4 k_3\|\bar{x}_4\|^2 + \alpha_5\lambda_{\Delta J_B}\|\bar{e}_4\|^2 \\
& + (\alpha_1 + M_1 + \lambda_K)\|\bar{x}_1\|\|\bar{x}_2\| + (M_1+\lambda_K)\|\bar{x}_1\|\|\bar{e}_2\| \\
& + \alpha_5\varepsilon_3\|\bar{x}_1\|\|\bar{e}_4\| \\
& + (M_2+M_3)\|\bar{x}_2\|\|\bar{e}_2\| + \lambda_K\|\bar{x}_2\|\|\bar{x}_3\| \\
& + \alpha_5\varepsilon_4\|\bar{x}_2\|\|\bar{e}_4\| \\
& + (\lambda_K + \alpha_3\lambda_{K_0^{-1}}k_{01}k_2)\|\bar{e}_2\|\|\bar{x}_3\| \\
& + \alpha_4\lambda_{K_0^{-1}}k_{01}k_2k_3\|\bar{e}_2\|\|\bar{x}_4\| \\
& + \alpha_5\varepsilon_5\|\bar{e}_2\|\|\bar{e}_4\| \\
& + (\alpha_3+\alpha_4k_3^2)\|\bar{x}_3\|\|\bar{x}_4\| + (\alpha_3+\alpha_5\varepsilon_6)\|\bar{x}_3\|\|\bar{e}_4\| \\
& + (\alpha_4k_{02} + \alpha_4k_3 + \alpha_5\varepsilon_J k_4 + \alpha_5\varepsilon_7)\|\bar{x}_4\|\|\bar{e}_4\| \\
& + \varepsilon_1\|\bar{x}_2\| + \varepsilon_1\|\bar{e}_2\| + \alpha_5\varepsilon_2\|\bar{e}_4\| + \|\bar{x}_2\|\|d_1\| \\
& + \|\bar{e}_2\|\|d_1\| + \alpha_5\lambda_{J^{-1}}\|\bar{e}_4\|\|d_2\| \\
& + (2/\alpha_2)\bar{x}_2^2 + (2/\alpha_2)\bar{e}_2^2 + \delta_{11}\bar{x}_1^2 + \delta_{12}\bar{x}_2^2 + \delta_{10} \\
& + (1/\alpha_2 + 1/\alpha_6)\alpha_5^2\lambda_{J^{-1}}^2\bar{e}_4^2 + \delta_{21}\bar{x}_3^2 + \delta_{22}\bar{x}_2^2 + \delta_{23}\bar{e}_2^2 \\
& + \delta_{20} + \delta_{31}\bar{e}_4^2 + \delta_{32}\bar{x}_4^2 + \delta_{33}\bar{x}_3^2 + \delta_{30} \\
\le & -\alpha_1 k_1\|\bar{x}_1\|^2 - (1-\varepsilon_K)k_2\|\bar{x}_2\|^2 - \lambda_m k_{01}\|\bar{e}_2\|^2 \\
& - \alpha_3k_3\|\bar{x}_3\|^2 - \alpha_4k_4\|\bar{x}_4\|^2 - \alpha_5k_{02}\|\bar{e}_4\|^2 \\
& + O_1\|\bar{x}_1\|^2 + O_2\|\bar{x}_2\|^2 + O_3\|\bar{e}_2\|^2 + O_4\|\bar{x}_3\|^2 \\
& + O_5\|\bar{x}_4\|^2 + O_6\|\bar{e}_4\|^2 + \rho^2\|d_1\|^2 + \rho^2\|d_2\|^2 + \varepsilon_0
\end{aligned} \tag{44}$$

where $O_1 = a_1 + a_2 + a_3 + \delta_{11}$,

$$\begin{aligned}
O_2 &= M_2 + (\alpha_1 + M_1 + \lambda_K)^2/4a_1 + a_4 + a_5 + a_6 \\
&\quad + a_{13} + 1/2\rho^2 + 2/\alpha_2 + \delta_{12} + \delta_{22} \\
O_3 &= (1+\varepsilon_K)k_2 + M_3 + (M_1+\lambda_K)^2/4a_2 + (M_2+M_3)^2/4a_4 \\
&\quad + a_7 + a_8 + a_9 + a_{14} + 1/2\rho^2 + 2/\alpha_2 + \delta_{23} \\
O_4 &= \lambda_K^2/4a_5 + (\lambda_K + \alpha_3\lambda_{K_0^{-1}}k_{01}k_2)^2/4a_7 \\
&\quad + a_{10} + a_{11} + \delta_{21} + \delta_{33} \\
O_5 &= (\alpha_4\lambda_{K_0^{-1}}k_{01}k_2k_3)^2/4a_8 + (\alpha_3+\alpha_4k_3^2)^2/4a_{10} \\
&\quad + \alpha_4k_3 + a_{12} + \delta_{32} \\
O_6 &= \alpha_5^2\varepsilon_3^2/4a_3 + \alpha_5^2\varepsilon_4^2/4a_6 + \alpha_5^2\varepsilon_5^2/4a_9 \\
&\quad + (\alpha_3 + \alpha_5\varepsilon_6)^2/4a_{11} + \alpha_5\lambda_{\Delta J_B} \\
&\quad + (\alpha_4k_{02} + \alpha_4k_3 + \alpha_5\varepsilon_Jk_4 + \alpha_5\varepsilon_7)^2/4a_{12} + \alpha_5^2a_{15} \\
&\quad + \alpha_5^2\lambda_{J^{-1}}^2/4\rho^2 + (1/\alpha_2 + 1/\alpha_6)\alpha_5^2\lambda_{J^{-1}}^2 + \delta_{31} \\
\varepsilon_0 &= \varepsilon_1^2/4a_{13} + \varepsilon_1^2/4a_{14} + \varepsilon_2^2/4a_{15} + \delta_{10} + \delta_{20} + \delta_{30}
\end{aligned}$$

for some constants $a_1 > 0, \ldots, a_{15} > 0$, $\rho > 0$ in which the fact that $\alpha\|x\|\|y\| \le a\|x\|^2 + (\alpha^2/4a)\|y\|^2$ for the vectors x, y, scalar α, and arbitrarily assigned constant $a > 0$ is used. Since all the values of δ_{ij} do not depend upon $k_1, k_2, k_{01}, k_3, k_4$, and k_{02}, and the values of M_1, M_2, and M_3 do not depend upon k_2, k_{01}, k_3, k_4, andk_{02}, the value of O_1 does not depend upon $k_1, k_2, k_{01}, k_3, k_4$,and $k_{02}k_{02}$, the value of O_2 does not depend upon k_2, k_{01}, k_3, k_4,and $k_{02}k_{02}$, the value of O_3 doe s not depend upon k_{01}, k_3, k_4,and $k_{02}k_{02}$, the value of O_4 does not depend upon k_3, k_4,and $k_{02}k_{02}$, the value of O_5 does not depend upon $k_4$and $k_{02}k_{02}$, and the value of O_6 does not depend upon k_{02}. Therefore, the circular phenomenon in the controller design is avoided.

Select the control gains as

$$\begin{aligned}
& k_1 > O_1/\alpha_1,\ k_2 > O_2/(1-\varepsilon_K),\ k_{01} > O_3/\lambda_m, \\
& k_3 > O_4/\alpha_3,\ k_4 > O_5/\alpha_4,\ k_{02} > O_6/\alpha_5
\end{aligned} \tag{45}$$

The derivative (44) can be rewritten as $\dot{V} \le -\bar{e}^T Q\bar{e} + \rho^2\|d_1\|^2 + \rho^2\|d_2\|^2 + \varepsilon_0$for some matrix$Q = Q^T > 0$. For the bounded disturbances d_1 and d_2, there is an $\varepsilon_d > 0$ such that $\|d_1\| \le \varepsilon_d$ and $\|d_2\| \le \varepsilon_d$. Let $\lambda_q$be the minimum eigenvalue of Q and $\mu = ((\varepsilon_0 + 2\rho^2\varepsilon_d^2)/\lambda_q)^{1/2}$. Therefore,

$$\dot{V} \le -\xi\|\bar{e}\|^2 < 0, \forall\,\|\bar{e}\| > \mu \tag{46}$$

for some $\xi > 0$.

Now, we proceed to prove that if $x_0 \in R_0$, then $\bar{e}(t) \in R$, $\forall\, t \ge 0$. Choose the initial states satisfying $\|q_1(0) - q_{r1}(0)\| \le c_1$, $\|\dot{q}_1(0) - \dot{q}_{r1}(0)\| \le c_2$, $\|q_2(0)\| \le c_4$, $\|\dot{q}_2(0)\| \le c_5$, $\eta_1(0) = -k_{01}(q_1(0) - q_{r1}(0)) + \eta_{10}$ with $\|\eta_{10}\| \le c_3$, and $\eta_2(0) = -k_{02}q_2(0) + \eta_{20}$ with $\|\eta_{20}\| \le c_6$. Since $\hat{\bar{x}}_2 = \eta_1 + k_{01}\bar{x}_1$, we have $\hat{\bar{x}}_2(0) = \eta_{10}$. Since $\hat{x}_4 = \eta_2 + k_{02}x_3$, we have $\hat{x}_4(0) = \eta_{20}$. Then, $x_0 \in R_0$. From the definition of $V_{\max}$, we have $V(0) \le V_{\max}$. Therefore, the inequality (46) implies $V(t) \le V(0) \le V_{\max}$. From (31) we get $\alpha_1\|\bar{x}_1\|^2 + \lambda_m\|\bar{x}_2\|^2 + \lambda_m\|\bar{e}_2\|^2 + \alpha_3\|\bar{x}_3\|^2 + \alpha_4\|\bar{x}_4\|^2 + \alpha_5\|\bar{e}_4\|^2 \le 2V_{\max}$, and so $\bar{e}(t) \in R$ for all $t \ge 0$. This concludes that all the states and signals remain bounded. Finally, from (46) it is also concluded that both the tracking errors $q_1(t) - q_{r1}(t)$ and $\dot{q}_1(t) - \dot{q}_{r1}(t)$ are UUB and the asymptotic bound μ can be made arbitrarily small by increasing the control gains sufficiently large (Khalil 2002). □

Appendix 2: The derivative of *V(t)* in (29).

Differentiate $V(t)$ defined in (29) along (7), (17), (18), (22), (25), and (28), and the uncertain terms containing the time-delay functions $H_1(\cdot)$ and $H_2(\cdot)$ are bounded as

$$-\bar{x}_2^T(t)H_1(x_{e12}(t-\tau_1)) \le (2/\alpha_2)\bar{x}_2^2(t) + (\alpha_2/4)(B_{11}^2(x_1(t-\tau_1))x_1^2(t-\tau_1) + B_{12}^2(x_{e12}(t-\tau_1))x_2^2(t-\tau_1)) \quad (47)$$

$$-\bar{e}_2^T(t)H_1(x_{e12}(t-\tau_1)) \le (2/\alpha_2)\bar{e}_2^2(t) + (\alpha_2/4)(B_{11}^2(x_1(t-\tau_1))x_1^2(t-\tau_1) + B_{12}^2(x_{e12}(t-\tau_1))x_2^2(t-\tau_1)) \quad (48)$$

$$-\alpha_5\bar{e}_4^T J^{-1}H_2(x_{e14}(t-\tau_2)) \le (1/\alpha_2 + 1/\alpha_6)\alpha_5^2\lambda_{J^{-1}}^2\bar{e}_4^2(t) + (\alpha_2/2)(B_{21}^2(x_1(t-\tau_1))x_1^2(t-\tau_2) + B_{22}^2(x_{e12}(t-\tau_1))x_2^2(t-\tau_2)) + (\alpha_6/2)(B_{23}^2(x_{e13}(t-\tau_2))x_3^2(t-\tau_2) + B_{24}^2(x_{e14}(t-\tau_2))x_4^2(t-\tau_2)) \quad (49)$$

Therefore, based on the choice of $V_P(t)$ defined in (30), we have

$$\dot{V}_P - \bar{x}_2^T(t)H_1(x_{e12}(t-\tau_1)) - \bar{e}_2^T(t)H_1(x_{e12}(t-\tau_1)) - \alpha_5\bar{e}_4^T J^{-1}H_2(x_{e14}(t-\tau_2)) \le (2/\alpha_2)\bar{x}_2^2(t) + (2/\alpha_2)\bar{e}_2^2(t) + (1/\alpha_2 + 1/\alpha_6)\alpha_5^2\lambda_{J^{-1}}^2\bar{e}_4^2(t) + (\alpha_2/2)(B_{11}^2(x_1(t))x_1^2(t) + B_{12}^2(x_{e12}(t))x_2^2(t)) + (\alpha_2/2)(B_{21}^2(x_1(t))x_1^2(t) + B_{22}^2(x_{e12}(t))x_2^2(t)) + (\alpha_6/2)(B_{23}^2(x_{e13}(t))x_3^2(t) + B_{24}^2(x_{e14}(t))x_4^2(t)). \quad (50)$$

Here, the fact that the summation of all the nonlinear terms containing the time-delay states is equal to zero has been used in the above analysis. According to the region R, define $\max_{\bar{e}\in R}\|B_{11}(x_1)\| = b_{11}$, $\max_{\bar{e}\in R}\|B_{12}(x_{e12})\| = b_{12}$, $\max_{\bar{e}\in R}\|B_{21}(x_1)\| = b_{21}$, $\max_{\bar{e}\in R}\|B_{22}(x_{e12})\| = b_{22}$, $\max_{\bar{e}\in R}\|B_{23}(x_{e13})\| = b_{23}$, and $\max_{\bar{e}\in R}\|B_{24}(x_{e14})\| = b_{24}$ for some constants $b_{11} > 0$, $b_{12} > 0$, $b_{21} > 0$, $b_{22} > 0$, $b_{23} > 0$, and $b_{24} > 0$. Similar to the derivation of inequalities (40)–(42), the inequality (50) is rewritten as

$$\dot{V}_P - \bar{x}_2^T(t)H_1(x_{e12}(t-\tau_1)) - \bar{e}_2^T(t)H_1(x_{e12}(t-\tau_1)) - \alpha_5\bar{e}_4^T J^{-1}H_2(x_{e14}(t-\tau_2)) \le (2/\alpha_2)\bar{x}_2^2(t) + (2/\alpha_2)\bar{e}_2^2(t) + (1/\alpha_2 + 1/\alpha_6)\alpha_5^2\lambda_{J^{-1}}^2\bar{e}_4^2(t) + \delta_{11}\bar{x}_1^2 + \delta_{12}\bar{x}_2^2 + \delta_{10} + \delta_{21}\bar{x}_3^2 + \delta_{22}\bar{x}_2^2 + \delta_{23}\bar{e}_2^2 + \delta_{20} + \delta_{31}\bar{e}_4^2 + \delta_{32}\bar{x}_4^2 + \delta_{33}\bar{x}_3^2 + \delta_{30} \quad (51)$$

Consequently, as in the proof of Theorem 3.1, the derivative $\dot{V}(t)$ can be shown to be bounded as in (44), and so the stability analysis described in Theorem 3.1 can be completed.