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Dual-phase-lag heat conduction in a furnace wall made of functionally graded materials☆

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ABSTRACT

In this article, the transient heat transfer in a furnace wall, which is made of functionally graded materials (FGMs), is investigated based on the hyperbolic-type dual-phase-lag (DPL) heat conduction model to consider the micro-structural interactions in the fast transient process of heat conduction. All material properties of the furnace wall are assumed to vary following a power-law form along the radial direction with arbitrary non-homogeneity indices. For simplicity, the values of the phase lags are taken constant. A semi-analytical solution for the temperature field is obtained in the Laplace domain. The transformed temperature solution is inverted to the physical quantity by using numerical inversion of the Laplace transform. A comparison between the hyperbolic-type DPL model and thermal wave model in the temperature responses of the furnace wall is made. Effects of different phase-lag values on the behavior of heat transfer are also investigated.

1. Introduction

In recent years, composition of several different materials is often used in structural components in order to optimize the thermal resistance and temperature distribution of structures subjected to thermal loading. In addition, for reducing the local stress concentration induced by abrupt transitions in material properties, the transition between different materials is made gradually. This idea, used originally by Japanese researchers [1], leads to the concept of functionally graded materials. These materials are expected to be used for thermal applications and high rate thermal loading. Although FGMs were first introduced to be utilized as a thermal shield, they are being used for many applications such as: heat exchanger tubes, heat-engine components, thermoelastic generators, and wear-resistant linings [2–4]. In recent years, FGMs have even been proposed as a solution for aerospace industry where temperature resistant, light-weight structures are required to meet the challenges faced by future high-speed space vehicles. Regarding the applications of FGM structures in extremely high temperature environments, therefore, the investigation of temperature field in FGMs is crucial in their design.

The conventional heat conduction theory, based on the classical Fourier's law, allows for the thermal disturbances to spread at an infinite speed because of the parabolic-type heat conduction equation. However, for situations involving very low temperatures near absolute zero, high-temperature gradient, very high frequencies of heat flux, and micro temporal and spatial scales, heat is found to propagate at a finite speed and Fourier's law becomes invalid. To remove these drawbacks, different non-Fourier heat conduction theories have been introduced to accommodate these extreme situations, with a majority including a hyperbolic-type heat conduction equation. One of the non-Fourier theories was proposed by Cattaneo [5] and Vernotte [6], to consider a finite speed for the thermal energy propagation by introducing the thermal relaxation time, which is defined as the time needed for acceleration of the heat flow. However, the thermal wave (C–V) model does not take into account the relaxation time between electrons and atomic lattice due to the macroscopic considerations.

In order to consider the relaxation time between electrons and atomic lattice in the fast transient process of heat transport, Tzou [7,8] introduced a phase lag for the temperature gradient absent in the thermal wave model. The corresponding model is called the dual-phase-lag (DPL) model. There have been various heat transfer problems described by the DPL model. For example, Dai et al. [9] presented a higher-order accurate and unconditionally stable compact finite difference scheme for solving the DPL equation in nano heat conduction with the temperature jump boundary condition. Lee et al. [10] applied the DPL heat transfer model to investigate the transient heat transfer in a thin metal film exposed to short-pulse laser heating. An efficient numerical scheme involving the hybrid application of the Laplace transform and control volume methods in conjunction with hyperbolic shape functions is used to solve the hyperbolic heat conduction equation. Liu and Wang [11] analyzed the thermal response for estimating thermal damage in laser-irradiated biological tissue. The effects of blood perfusion and metabolic heat generation on thermal response and thermal damage are explored. Recently, Wu et al. [12] solved the inverse hyperbolic

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Nomenclature

Bi	Biot number (hr_o/k_o)
c	specific heat ($\mathrm{J\,kg^{-1}K^{-1}}$)
c_o	specific heat at the outer surface of the furnace ($\mathrm{J\,kg^{-1}K^{-1}}$)
h	convection heat transfer coefficient ($\mathrm{W\,m^{-2}K^{-1}}$)
k	thermal conductivity ($\mathrm{W\,m^{-1}K^{-1}}$)
k_o	thermal conductivity at the outer surface of the furnace ($\mathrm{W\,m^{-1}K^{-1}}$)
n_i	non-homogeneity index, $i = 1, 2, 3$
q	heat flux ($\mathrm{W\,m^{-2}}$)
r	spatial coordinate (m)
r_i	inner radius of the furnace (m)
r_o	outer radius of the furnace (m)
s	Laplace transform parameter
T	temperature (K)
T_i	temperature at the inner surface of the furnace (K)
T_∞	surrounding temperature (K)
t	time (s)
v	dimensionless thermal wave speed

Greek symbols

α	thermal diffusivity ($\mathrm{m^2s^{-1}}$)
α_o	dimensionless thermal diffusivity of the outer surface of the furnace
δ_o	dimensionless phase lag of the temperature gradient
ε_o	dimensionless phase lag of the heat flux
η	dimensionless radius of the furnace
θ	dimensionless temperature
ξ	dimensionless time
ρ	density ($\mathrm{kg\,m^{-3}}$)
ρ_o	density at the outer surface of the furnace ($\mathrm{kg\,m^{-3}}$)
τ_q	phase lag of the heat flux (s)
τ_T	phase lag of the temperature gradient (s)

heat conduction problem with the DPL heat transfer model to estimate the unknown boundary pulse heat flux in an infinitely long solid cylinder from the temperature measurements taken within the medium.

In this study, the DPL heat transfer model is applied to investigate the non-Fourier heat transfer in a furnace wall which is made of functionally graded materials with power-varying material properties. First, the problem is formulated using the hyperbolic-type DPL model in general form. Then Laplace transform method is adopted to remove the time-dependent terms from the governing equations. A semi-analytical solution for the temperature field is obtained in the Laplace domain. Finally, the transformed temperature solution is inverted to the physical quantity by using numerical inversion of the Laplace transform. A comparison between the hyperbolic-type DPL model and thermal wave model in the temperature responses of furnace wall is made. Effects of different phase-lag values on the behavior of heat transfer are also investigated.

2. Physical model and mathematical formulation

In this study a furnace wall system, as shown in Fig. 1, is considered. The inner and outer radii of the furnace wall are r_i and r_o, respectively. The one-dimensional DPL heat conduction for the furnace wall can be expressed as [8]:

$$q(r, t+\tau_q) = -k\frac{\partial T(r, t+\tau_T)}{\partial r}, \tag{1}$$

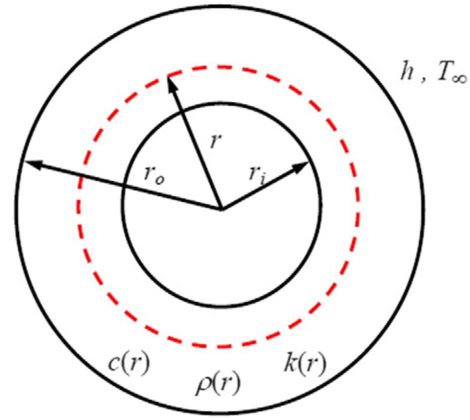

Fig. 1. Diagram for the furnace wall system.

where r is the spatial coordinate, t is the time, q is the heat flux, T is the absolute temperature, k is the thermal conductivity, τ_q is the phase lag of the heat flux, and τ_T is the phase lag of the temperature gradient. The time delay τ_q has been interpreted as the relaxation time due to the fast-transient effects of thermal inertia, while the time delay τ_T has been interpreted as the time required for the thermal activation in microscale, caused by the microstructural interactions such as electron–phonon interaction or phonon scattering [8,13]. Taylor's series expansion of Eq. (1) up to the second order in τ_q and up to the first order in τ_T leads to [14]:

$$q(r,t) + \tau_q\frac{\partial q(r,t)}{\partial t} + \frac{\tau_q^2}{2}\frac{\partial^2 q(r,t)}{\partial t^2} = -k\left[\frac{\partial T(r,t)}{\partial r} + \tau_T\frac{\partial^2 T(r,t)}{\partial t\partial r}\right]. \tag{2}$$

The energy conservation equation is written as:

$$-\frac{\partial q(r,t)}{\partial r} = \rho c\frac{\partial T(r,t)}{\partial t}, \tag{3}$$

where ρ is the density and c is the specific heat. Combining Eqs. (2) and (3), the DPL heat conduction equation for the furnace wall is obtained [8,15,16]:

$$\left[1 + \tau_q\frac{\partial}{\partial t} + \frac{\tau_q^2}{2}\frac{\partial^2}{\partial t^2}\right]\rho c\frac{\partial T(r,t)}{\partial t} = \frac{\partial}{\partial r}\left[k\left(1 + \tau_T\frac{\partial}{\partial t}\right)\right]\frac{\partial T(r,t)}{\partial r}. \tag{4}$$

Eq. (4) has been written based on the hyperbolic-type DPL heat conduction theory; however, discarding the second-order time derivative from the left-hand side of Eq. (4) leads to parabolic-type DPL model. Moreover, setting $\tau_T = 0$ and omitting the second-order Taylor series term in Eq. (4) results in the C–V or hyperbolic heat conduction theory. Finally, removing the second-order term of Taylor series expansion and letting $\tau_q = \tau_T$ gives the classical Fourier heat conduction theory.

The associated boundary and initial conditions are given as:

$$T(r,t) = T_i, \ \text{at} \ r = r_i, \tag{5}$$

$$-k\frac{\partial T(r,t)}{\partial r} = h[T(r,t) - T_\infty], \ \ at \ r = r_o, \tag{6}$$

$$T(r,t) = T_\infty, \ \text{for} \ t = 0, \tag{7}$$

$$\frac{\partial T(r,t)}{\partial t} = 0, \ \text{for} \ t = 0, \tag{8}$$

where T_i is the temperature at the inner surface of furnace wall, h is the convective heat transfer coefficient at the outer surface, and T_∞ is the surrounding temperature.

Unlike the standard heat conduction analysis, which assumes the material of furnace wall to be homogeneous with uniform material properties, the present analysis models the nonhomogeneity of the furnace wall by allowing the material properties of furnace wall to vary as a power function of radial coordinate r, that is [4]:

$$k(r) = k_o(r/r_o)^{n_1},\ \ \rho(r) = \rho_o(r/r_o)^{n_2},\ \ c(r) = c_o(r/r_o)^{n_3}, \tag{9}$$

where k_o, ρ_o, and c_o are material constants; $n_i(i=1,2,3)$are nonhomogeneity indices of the material properties. Such a power dependence of the material properties occurs in some aerospace and automotive structures [17].

For the convenience of comparison and discussion, the following dimensionless parameters are introduced as:

$$\eta = \frac{r}{r_o},\ \ \xi = \frac{\alpha_o t}{r_o^2},\ \ \theta = \frac{T-T_\infty}{T_i-T_\infty},\ \ \varepsilon_o = \frac{\alpha_o\tau_q}{r_o^2},\ \ \delta_o = \frac{\alpha_o\tau_T}{r_o^2},\ \ \alpha_o = \frac{k_o}{\rho_o c_o}. \tag{10}$$

Using material properties along with non-dimensional parameters defined in Eqs. (9) and (10), the heat conduction equation, boundary and initial conditions in Eqs. (4)–(8) are rewritten as:

$$\eta^{n_2+n_3}\left(1+\varepsilon_o\frac{\partial}{\partial\xi}+\frac{\varepsilon_o^2}{2}\frac{\partial^2}{\partial\xi^2}\right)\frac{\partial\theta}{\partial\xi} = \left(1+\delta_o\frac{\partial}{\partial\xi}\right)\times\left[(n_1+1)\eta^{n_1-1}\frac{\partial\theta}{\partial\eta}+\eta^{n_1}\frac{\partial^2\theta}{\partial\eta^2}\right], \tag{11}$$

$$\theta(\eta,\xi) = 1,\ \ \text{at}\ \ \eta = \eta_i, \tag{12}$$

$$-\frac{\partial\theta(\eta,\xi)}{\partial\eta} = Bi\cdot\theta(\eta,\xi),\ \ at\ \ \eta = 1, \tag{13}$$

$$\theta(\eta,\xi) = 0,\ \ \text{for}\ \ \xi = 0, \tag{14}$$

$$\frac{\partial\theta(\eta,\xi)}{\partial\xi} = 0,\ \ \text{for}\ \ \xi = 0, \tag{15}$$

where Bi $(=h\cdot r_o/k_o)$ is the Biot number and $\eta_i = r_i/r_o$.

The dimensionless thermal wave speed $v_{DPL}(\eta)$ in the furnace wall based on the hyperbolic-type DPL model is [3]:

$$v_{DPL}(\eta) = \frac{\sqrt{2\delta_o}}{\varepsilon_o}\eta^{(n_1-n_2-n_3)/2}, \tag{16}$$

while the dimensionless thermal wave speed $v_{C-V}(\eta)$ in the furnace wall based on the C–V model is [3]:

$$v_{C-V}(\eta) = \frac{1}{\sqrt{\varepsilon_o}}\eta^{(n_1-n_2-n_3)/2}. \tag{17}$$

Eqs. (16) and (17) show that the dimensionless thermal wave speed depends on the position η, the non-homogeneity indices n_i, and the dimensionless phase lags ε_o and δ_o. Moreover, it is seen that:

$$\frac{v_{DPL}(\eta)}{v_{C-V}(\eta)} = \sqrt{\frac{2\delta_o}{\varepsilon_o}} = \sqrt{\frac{2\tau_T}{\tau_q}}. \tag{18}$$

3. Numerical method

The Laplace transform technique is used to map the transient problem into the steady one. The Laplace transform of a function $f(\xi)$ with respect to ξ is defined as follows:

$$\overline{f}(s) = \int_0^\infty e^{-s\xi}f(\xi)\mathrm{d}\xi, \tag{19}$$

where $\overline{f}(s)$ denotes the Laplace transform of function $f(\xi)$ and s is the Laplace transform parameter. Applying the Laplace transform to Eqs. (11)–(13) with respect to the initial conditions of Eqs. (14) and (15) results in:

$$\eta^2\frac{\mathrm{d}^2\overline{\theta}}{\mathrm{d}\eta^2}+(n_1+1)\eta\frac{\mathrm{d}\overline{\theta}}{\mathrm{d}\eta}-E\eta^{n_2+n_3-n_1+2}\overline{\theta} = 0, \tag{20}$$

$$\overline{\theta} = 1/s,\ \ \text{at}\ \ \eta = \eta_i, \tag{21}$$

$$-\frac{\mathrm{d}\overline{\theta}}{\mathrm{d}\eta} = Bi\cdot\overline{\theta},\ \ \text{at}\ \ \eta = 1, \tag{22}$$

where $E = s(1+\varepsilon_0 s+\varepsilon_0^2 s^2/2)/(1+\delta_0 s)$.

The ordinary differential Eq. (20) can be solved as [14]:

$$\overline{\theta}(\eta,s) = \eta^{-n_1/2}\left[A_1J_G(I\eta^H)+A_2Y_G(I\eta^H)\right],\ \ for\ \ n_1-n_2-n_3\neq 2, \tag{23}$$

or

$$\overline{\theta}(\eta,s) = A_1\eta^{\lambda_1})+A_2\eta^{\lambda_2},\ \ \text{for}\ \ n_1-n_2-n_3 = 2, \tag{24}$$

where

$$G = n_1/(n_2+n_3-n_1+2), \tag{25}$$

$$H = 1+(n_2+n_3-n_1)/2, \tag{26}$$

$$I = 2\sqrt{-E}/(n_2+n_3-n_1+2), \tag{27}$$

$$\lambda_{1,2} = \left[-n_1\pm\sqrt{n_1^2+4E}\right]/2, \tag{28}$$

where J_G and Y_G are the Gth-order Bessel function of the first and second kinds, respectively; A_1 and A_2 are integration constants to be found by satisfying the thermal boundary conditions of Eqs. (21) and (22).

To restore the time effect, the solutions in the Laplace domain, $\overline{\theta}$, must be inverted. As this inversion may not be simply done analytically, a numerical treatment is employed. The inverse Laplace transform of the temperatures $\overline{\theta}$ is completed by the application of the following numerical inversion formula, known as the Fourier series technique [18]:

$$\theta(t)\cong\frac{e^{bt}}{t}\left[\frac{1}{2}\overline{\theta}(b)+\mathrm{Re}\sum_{n=1}^{N}\overline{\theta}(b+in\pi/t)(-1)^n\right], \tag{29}$$

where b is a selected, positive arbitrary constant, which should be greater than the real parts of all singularities of $\overline{\theta}$. From the above equation, the Laplace inverse $\theta(t)$ of the function $\overline{\theta}(s)$at time t can be found.

4. Results and discussion

The aim of this study is to study the transient thermal behavior in a furnace wall which is made of functionally graded materials (FGMs) using the DPL model with constant phase lags. In the numerical calculations, the inner and outer radii are taken to be $r_i = 0.6$ and $r_o = 1$, respectively. The furnace is initially at the ambient temperature $T_\infty(\theta = 0)$. The outer surface of the furnace wall is subjected to convective boundary condition, while the temperature of the inner surface is suddenly raised to $T_i(\theta = 1)$, which causes the generation of a thermal wave traveling from the inner surface toward the outer surface. On the other hand, to protect the furnace wall from thermal damages in real applications, we usually expect the temperature of the wall as low as possible. To keep the temperature field in low values at the steady state for the furnace wall subjected to a specified temperature loading at the inner wall surface, we should choose a positive value for n_1 [4]. In this study, all the non-homogeneity indices are assumed to be the same, $n_1 = n_2 = n_3 = n$ [3]. The value of $n = 0$ corresponds to a furnace wall with uniform material properties.

Fig. 2 graphically demonstrates the effects of different heat conduction models on the predicted transient thermal responses of the furnace wall at various times. The dimensionless phase lags of heat flux and temperature gradient are taken to be $\varepsilon_o = 0.35$ and $\delta_o = 0.25$, respectively [3]. To focus on the comparison between the hyperbolic-type DPL and the C–V heat conduction models, all non-homogeneity indices are assumed to be the same, $n_i = n = 1$. For the hyperbolic-type DPL and C–V models, the temperature propagates through the furnace wall with a finite speed and, as a result, a finite time is required for the wall to adapt itself with the imposed thermal shock. Fig. 2 reveals that the thermal waves begin to travel from the inner surface to the outer surface of the furnace wall. According to Eq. (18) and the selected values for the phase lags, the thermal wave speed based on the hyperbolic-type DPL model is greater than that based on the C–V heat conduction theory. Therefore, the wave fronts for the hyperbolic-type DPL model are traveling ahead of those for the C–V model due to their greater thermal wave speed. In addition, the effects of hyperbolic-type DPL model become more distinct as the increase of time. On the other hand, Fig. 2 shows that the wave fronts for both the hyperbolic-type DPL and C–V models have reached and reflected from the outer surface at $\xi = 0.24$.

Fig. 3(a)–(c) depict the effects of dimensionless phase lag of heat flux, ε_o, on the temperature distributions at $\xi = 0.08$, 0.16, and 0.24, respectively, based on the hyperbolic-type DPL heat conduction in the furnace wall with $n_i = n = 1$, $\delta_o = 0.25$, and Bi = 2, respectively. These figures reveal that an increase in the phase lag, ε_o, decreases the thermal wave speed and makes the wavelike temperature distribution more pronounced, which agrees with the expression of the thermal wave speed in Eq. (16). For example, in Fig. 3(a), the wave front for $\varepsilon_o = 0.15$ has reflected from the outer surface at $\xi = 0.08$, while the wave front for $\varepsilon_o = 0.35$ just propagates to $\eta = 0.8$. Moreover, in Fig. 3(c), when the wave fronts for $\varepsilon_o = 0.25$ and 0.35 travel from the reflection of outer surface at $\xi = 0.24$, the wave front for $\varepsilon_o = 0.15$ has reflected again from the inner surface.

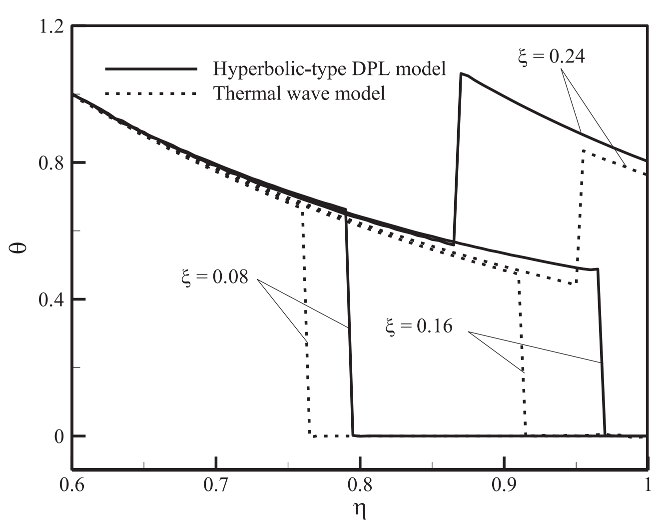

Fig. 2. Effects of different heat conduction models on the temperature distributions at various times with $n_i = n = 1, \varepsilon_o = 0.35, \delta_o = 0.25$, and Bi = 2, respectively.

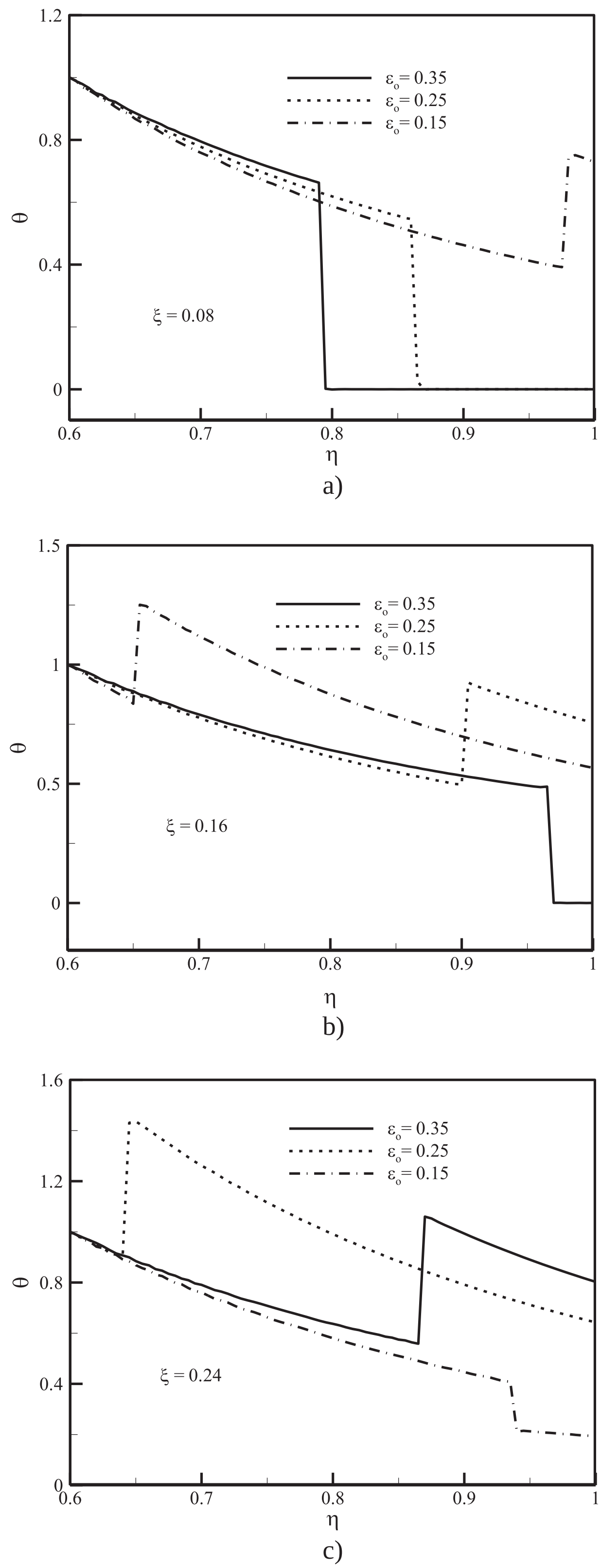

Fig. 3. Effects of the dimensionless phase lag of heat flux on the temperature distributions at various times with $n_i = n = 1, \delta_o = 0.25$, and Bi = 2, respectively. (a) $\xi = 0.08$, (b) $\xi = 0.16$, and (c) $\xi = 0.24$.

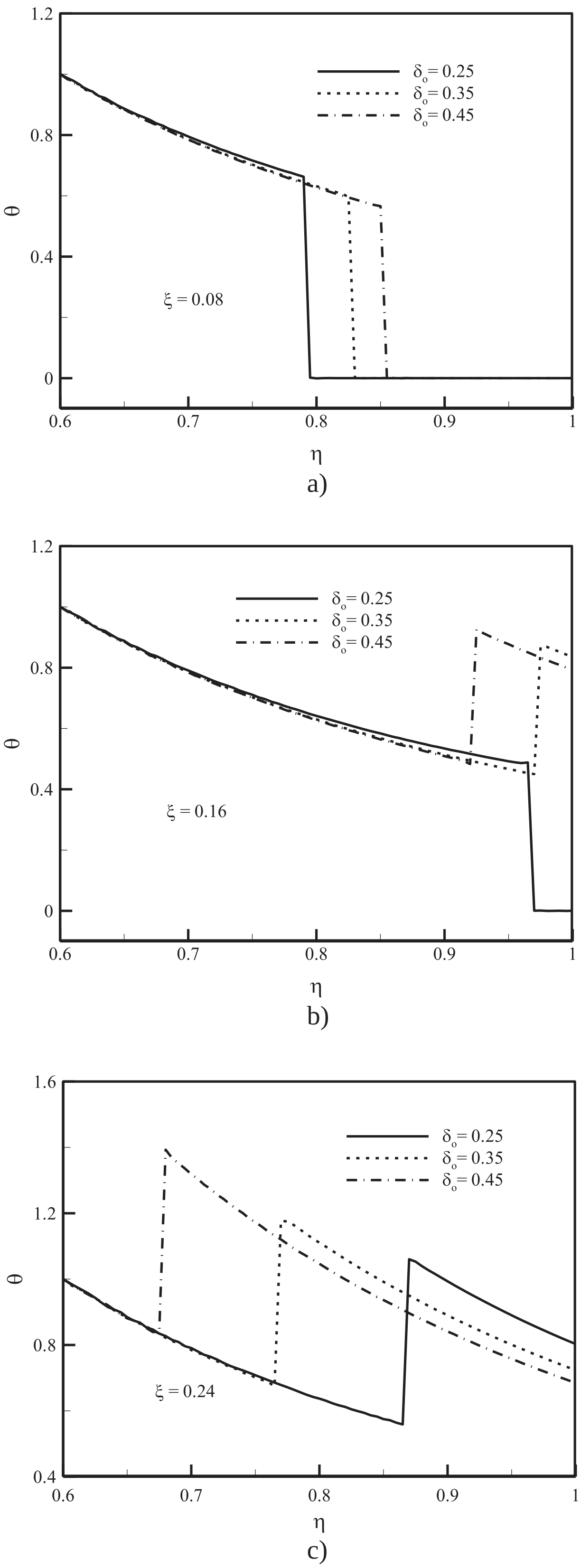

Fig. 4. Effects of the dimensionless phase lag of temperature gradient on the temperature distributions at various times with $n_i = n = 1, \varepsilon_o = 0.35$, and Bi = 2, respectively. (a) $\xi = 0.08$, (b) $\xi = 0.16$, and (c) $\xi = 0.24$.

The effects of the dimensionless phase lag of temperature gradient, δ_o, on the temperature distributions at $\xi = 0.08, 0.16$, and 0.24, respectively, for the hyperbolic-type DPL heat conduction in the furnace wall with $n_i = n = 1$, $\varepsilon_o = 0.35$, and Bi = 2, respectively, are illustrated in Fig. 4(a)–(c). These figures show that the thermal wave speed increases as the increase of temperature-gradient phase lag δ_o, which also agrees with the expression of the thermal wave speed in Eq. (16). In Fig. 4(a), we find that all the wave fronts for the three values of δ_o have not reached the outer surface of the furnace wall at $\xi = 0.08$. However, all the three wave fronts have reflected from the outer surface of the furnace at $\xi = 0.24$, as shown in Fig. 4(c).

5. Conclusion

In this study, to consider the microstructural interactions in the fast transient process of heat conduction, the hyperbolic-type DPL heat transfer model is applied to investigate the non-Fourier heat transfer in a furnace wall which is made of functionally graded materials with power-varying material properties. For simplicity, the values of the phase lags of heat flux and temperature gradient are taken constant. A semi-analytical solution for the temperature field is obtained in the Laplace domain. The transformed temperature solution is inverted to the physical quantity by using numerical inversion of the Laplace transform. A comparison between the hyperbolic-type DPL model and thermal wave model in the temperature responses of furnace wall is made. Effects of different phase-lag values on the behavior of heat transfer are also investigated. It is found that, with the phase lag of temperature gradient kept constant, an increase in the phase lag of heat flux decreases the thermal wave speed. However, the thermal wave speed increases with the increase of the phase lag of temperature gradient for a constant phase lag of heat flux.

Acknowledgments

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References

[1] M. Koizumi, The concept of FGM, Ceram. Trans. Funct. Graded Mater. 34 (1993) 3–10.

[2] A.H. Akbarzadeh, Z.T. Chen, Transient heat conduction in a functionally graded cylindrical panel based on the dual phase lag theory, Int. J. Thermophys. 33 (2012) 1100–1125.

[3] A.H. Akbarzadeh, Z.T. Chen, Dual phase lag heat conduction in functionally graded hollow spheres, Int. J. Appl. Mech. 6 (2014) 1450002-1–1450002-27.

[4] S.H. Wang, S.C. Lin, Y.C. Yang, Geometry estimation for the inner surface in a furnace wall made of functionally graded materials, Int. Commun. Heat Mass Transfer 67 (2015) 1–7.

[5] C. Cattaneo, A form of heat conduction which eliminates the paradox of instantaneous propagation, C. R. 247 (1958) 431–433.

[6] P. Vernotte, Paradoxes in the continuous theory of the heat equation, C. R. 246 (1958) 3154–3155.

[7] D.Y. Tzou, The generalized lagging response in small-scale and high-rate heating, Int. J. Heat Mass Transf. 38 (1995) 3231–3240.

[8] D.Y. Tzou, Macro- to Microscale Heat Transfer: The Lagging Behavior, Taylor and Francis, Washington, DC, 1996.

[9] W. Dai, F. Han, Z.Z. Sun, Accurate numerical method for solving dual-phase-lagging equation with temperature jump boundary condition in nano heat conduction, Int. J. Heat Mass Transf. 64 (2013) 966–975.

[10] H.L. Lee, W.L. Chen, W.J. Chang, E.J. Wei, Y.C. Yang, Analysis of dual-phase-lag heat conduction in short-pulse laser heating of metals with a hybrid method, Appl. Therm. Eng. 52 (2013) 275–283.

[11] K.C. Liu, J.C. Wang, Analysis of thermal damage to laser irradiated tissue based on the dual-phase-lag model, Int. J. Heat Mass Transf. 70 (2014) 621–628.

[12] T.S. Wu, H.L. Lee, W.J. Chang, Y.C. Yang, An inverse hyperbolic heat conduction problem in estimating pulse heat flux with a dual-phase-lag model, Int. Commun. Heat Mass Transfer 60 (2015) 1–8.

[13] D.Y. Tzou, On the thermal shock wave induced by a moving heat source, ASME J. Heat Transf. 111 (1989) 232–238.

[14] A.H. Akbarzadeh, Z.T. Chen, Heat conduction in one-dimensional functionally graded media based on the dual-phase-lag theory, Proc. IMechE C J. Mech. Eng. Sci. 227 (2013) 744–759.

[15] H.L. Lee, W.J. Chang, S.C. Wu, Y.C. Yang, An inverse problem in estimating the base heat flux of an annular fin based on the hyperbolic model of heat conduction, Int. Commun. Heat Mass Transfer 44 (2013) 31–37.
[16] W.L. Chen, H.M. Chou, H.L. Lee, Y.C. Yang, An inverse hyperbolic heat conduction problem in estimating base heat flux of two-dimensional cylindrical pin fins, Int. Commun. Heat Mass Transfer 52 (2014) 90–96.
[17] A.Z. Sahin, Optimal insulation of structures with varying thermal conductivity, J. Thermophys. Heat Transf. 11 (1997) 153–157.
[18] Y.C. Yang, H.L. Lee, W.L. Chen, J.L. Salazar, Estimation of thermal contact resistance and temperature distributions in the pad/disc tribosystem, Int. Commun. Heat Mass Transfer 38 (2011) 298–303.