Numerical analysis of dual-phase-lag heat transfer for a moving finite medium subjected to laser heat source

Haw-Long Lee, Wen-Lih Chen, Win-Jin Chang, Ming-I Char, Yu-Ching Yang

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A B S T R A C T
To accommodate the micro-structural effect, this work applies the dual-phase-lag (DPL) heat transfer model to explore the transient heat transfer for a moving finite medium under the effect of a time-dependent laser heat source. Laser heating is modeled as an internal heat source. A numerical scheme has been developed to overcome the mathematical difficulties in dealing with the hyperbolic heat conduction equation. Comparison between present numerical results and the analytic solutions for the non-Fourier case is made to verify the accuracy of the present numerical method. Additionally, the effects of different medium parameters, for example, moving velocity, phase lags values of the heat flux and temperature gradient, on the behavior of heat transfer have been examined. It is found that there exists clear phase shifts in the temperature distributions due to the medium moving velocity. The heat-flux phase lag tends to induce thermal waves with sharp wave-fronts in the medium, the inclusion of temperature-gradient phase lag smoothenes the sharp wave-fronts by promoting conduction into the medium, resulting in non-Fourier diffusion-like conduction.

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1. Introduction

Due to its numerous applications related to microelectronics and material processing, the use of laser and microwave heat sources has attracted much attention in recent years. These can be used in processes such as laser patterning, micromachining, and laser surface treatment. In terms of theoretical analysis on heat conduction problems, classical Fourier's law has been widely used; and its predictive accuracy is supported extensively and successfully by results which are in agreement with experimental data for most of the conventional heat transfer conditions. However, in situations involving high heat fluxes, very short times, or very low temperatures, the applicability of Fourier's law is questionable [1,2]. The problem is rooted in the fact that the Fourier’s law predicts an infinite speed of heat propagation, that is, a thermal disturbance in any part of a medium can result in an instantaneous perturbation to everywhere in the sample.

To account for the finite thermal wave speeds, Cattaneo [3] and Vernotte [4] proposed a thermal wave model with a single-phase time lag in which the temperature gradient established after a certain elapsed time:

\[ q(r, t + \tau_q) = -k \nabla T(r, t), \] (1)

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where $\tau_q$ denotes the relaxation time or the time delay of the heat flux relative to the temperature gradient in the transient energy transport process. From Eq. (1), it can be seen that when $\tau_q > 0$, the thermal wave propagates through the medium with a finite speed of $v = \sqrt{\alpha/\tau_q}$, where $\alpha$ is the thermal diffusivity. However, when $\tau_q$ approaches zero, the thermal wave has an infinite speed and thus the single-phase-lag (SPL) or non-Fourier model reduces to the traditional Fourier model.

Although the thermal wave model can capture the micro-scale response in time [5,6], the wave concept does not capture the micro-scale response in space [7]. Therefore, the validity of the thermal wave model becomes debatable in the aspect of fast-transient response with micro-structural interaction effects [8]. To remove the precedence assumption implied by the thermal wave model, the dual-phase-lag heat conduction model was developed and experimentally verified by Tzou [9]. The model accounts for spatial and temporal effects in both macro- and micro-scale heat transfer in a one-temperature formulation and takes the form:

$$q(r, t + \tau_q) = -k\nabla T(r, t + \tau_T),$$  

(2)

where $\tau_T$ is the phase lag of the temperature gradient. In the DPL model with $\tau_q > \tau_T$, temperature gradient within the medium induces heat flux; hence, the temperature gradient is the cause for energy transport and the heat flux is the effect; whereas, with
\( \tau_q < \tau_T \), heat flux is the cause for energy transport and temperature gradient is the effect. For \( \tau_q = \tau_T \) with homogeneous initial temperature, the DPL model reduces to the classical Fourier’s law [6].

The physical meanings of the DPL model have been shown by the experimental results in [9,10], and there have been various heat transfer problems described by the DPL model. For example, Ordoñez-Miranda and Alvarado-Gil [11] analytically studied the one-dimensional thermal wave transport in a semi-infinite medium under the framework of the DPL model of heat conduction considering a modulated thermal excitation for Dirichlet and Neumann boundary conditions. The behaviors of characteristic propagation properties of the thermal waves, such as the wavelength, penetration depth, and phases and group velocities were discussed. Rukolaine [12] considered the model of heat conduction based on the first-order approximation to the dual-phase-lag constitutive relation and concluded that the DPL model in the form of the Jeffreys-type equation is not in general an appropriate model of heat conduction.

In open literature, there have been various heat transfer problems described by hyperbolic heat conduction models in a finite medium heated with laser source [13]. But, to the best knowledge of the authors, there have been only few studies on the problems involving moving medium. For example, Shen and Zhang [14] studied the DPL thermal behavior of a moving finite medium subjected to constant boundary conditions. A number of notable physical anomalies concerning non-Fourier heat conduction under the DPL model were observed and investigated. Al-Khairy and Al-Ofey [15] applied the SPL model to investigate the thermal behavior of a moving semi-infinite medium subjected to time-dependent laser heat source. The solution was obtained by the Laplace transforms method, and the discussion of solutions for different time characteristics of heat source capacity was also presented. Recently, Al-khairy [16] analytically solved the hyperbolic heat conduction equation based on the SPL model for a moving finite medium under the effect of time-dependent laser heat source. Laser heating was modeled as an internal heat source, and the effect of the dimensionless medium velocity on the temperature profiles was also examined.

In the present work, the dual-phase-lag heat conduction model is used to study the thermal behavior of a moving finite medium subjected to time-dependent laser heat source. The problem is solved numerically. To the best of the authors’ knowledge, this is the first time that such a problem based on DPL heat conduction has been modeled.

2. Physical model and mathematical formulation

In this paper, the heat conduction in a thin slab of thickness \( l \), shown in Fig. 1, with constant thermophysical properties and insulated walls, is investigated using the DPL model. The thin slab is subjected to laser heating which is fixed in space, and the slab starts to move with a constant velocity \( w \) at time \( t > 0 \) when the laser heat generation starts at the front surface \( x = 0 \) of the slab. The one-dimensional energy conservation equation for the present problem can be given as [16,17]:

\[
-\frac{\partial q(x,t)}{\partial x} + g(x,t) = \rho c \left[ \frac{\partial T(x,t)}{\partial t} + w \frac{\partial T(x,t)}{\partial x} \right].
\]  

(3)

The source term \( g(x,t) \) which depicts the absorption of laser radiation is modeled as:

\[
g(x,t) = I(t)(1 - R)\mu \exp(-\mu x),
\]  

(4)

where \( I(t) \) is the laser incident intensity, \( R \) is the surface reflectance of the body, and \( \mu \) is the absorption coefficient.

Considering the constant motion \( w \) of the slab, applying Taylor’s series expansion formula on both sides of Eq. (2), and neglecting the second- and higher-order terms results in the linear DPL constitutive model with the following form [16,17]:

\[
q(x,t) + \tau_q \left[ \frac{\partial q(x,t)}{\partial t} + w \frac{\partial q(x,t)}{\partial x} \right] = -k \frac{\partial T(x,t)}{\partial x} - k\tau_T \frac{\partial^2 T(x,t)}{\partial t \partial x}.
\]  

(5)

![Fig. 1. Geometry and coordinate system.](image-url)
Elimination of the heat flux $q(x, t)$ between Eqs. (3) and (5) leads to the DPL heat conduction equation of this problem as:

$$
\tau_q \frac{\partial^2 T}{\partial t^2} + \frac{\partial T}{\partial t} + w \frac{\partial T}{\partial x} + 2 \tau_q w \frac{\partial^2 T}{\partial x \partial t} + \tau_q (w^2 - \nu^2) \frac{\partial^2 T}{\partial x^2} - \alpha \tau_q \frac{\partial^2 T}{\partial x^2} \left( \frac{\partial T}{\partial t} \right)
$$

$$
= \frac{1}{\rho c} \left[ g + \tau_q \frac{\partial g}{\partial t} + \tau_q w \frac{\partial g}{\partial x} \right].
$$

(6)

The temperature field within the medium is assumed initially with a uniform value $T_0$, and there is no heat flow in the slab at the initial moment, i.e., $q(x, 0) = 0$. Then, according to the balance law in Eq. (3) for the internal energy, the initial conditions for the problem are:

$$
T(x, t) = T_0, \quad \text{for} \quad t = 0.
$$

(7)

$$
\frac{\partial T(x, t)}{\partial t} = \frac{g}{\rho c}, \quad \text{for} \quad t = 0.
$$

(8)

The boundary conditions assume insulated walls at the boundaries:

$$
\frac{\partial T(x, t)}{\partial x} = 0, \quad \text{at} \quad x = 0.
$$

(9)

$$
\frac{\partial T(x, t)}{\partial x} = 0, \quad \text{at} \quad x = l.
$$

(10)

For analytical convenience, the following dimensionless parameters and ratios of thermal properties are introduced:

$$
\eta = \frac{x}{2c \tau_q}, \quad \xi = \frac{t}{2 \tau_q}, \quad \theta = \frac{T - T_0}{T_0}, \quad W = \frac{w}{\nu}, \quad G = \frac{\tau_q g}{\rho c T_0}.
$$

(11)

Utilizing these dimensionless variables, Eq. (6) is expressed in dimensionless form as:

$$
\frac{\partial^2 \theta}{\partial \xi^2} + 2 \frac{\partial \theta}{\partial \eta} + 2W \frac{\partial \theta}{\partial \eta} + 2W \frac{\partial^2 \theta}{\partial \eta \partial \xi} - (1 - W^2) \frac{\partial^2 \theta}{\partial \eta^2} - \tau_{\tau q} \frac{\partial^2 \theta}{\partial \eta^2} \left( \frac{\partial \theta}{\partial \xi} \right)
$$

$$
= \left[ 4G + 2G \frac{\partial \theta}{\partial \xi} + 2W \frac{\partial G}{\partial \eta} \right].
$$

(12)

where $\tau_{\tau q} = \tau_r / 2 \tau_q$ and the dimensionless heat source $G$ according to Eq. (4) is:

$$
G = \psi_0 \phi(\xi) \exp(-\beta \eta).
$$

(13)

where

$$
\psi_0 = \frac{\tau_q L (1 - R)}{\rho c T_0}, \quad \phi(\xi) = \frac{\lambda^2 \pi \xi}{L}, \quad \beta = 2c \tau_q \mu.
$$

(14)

The dimensionless initial and boundary conditions become:

$$
\theta(\eta, \xi) = 0, \quad \text{for} \quad \xi = 0.
$$

(15)

$$
\frac{\partial \theta(\eta, \xi)}{\partial \xi} = 2 \psi_0 \phi(0) \exp(-\beta \eta), \quad \text{for} \quad \xi = 0.
$$

(16)

$$
\frac{\partial \theta(\eta, \xi)}{\partial \eta} = 0, \quad \text{at} \quad \eta = 0.
$$

(17)

$$
\frac{\partial \theta(\eta, \xi)}{\partial \eta} = 0, \quad \text{at} \quad \eta = L.
$$

(18)

where $L = l / 2c \tau_q$.

3. Numerical analysis

To reduce the complexity of the present problem, the Laplace transform method is employed to remove the time-dependent terms in Eqs. (12), (17), and (18). The unsteady problem can be mapped onto a steady one in the transform domain. Using the initial conditions given by Eqs. (15) and (16), the Laplace transform of Eq. (12) gives:

$$
\frac{\partial^2 \tilde{\theta}}{\partial \eta^2} - R_1 \tilde{\theta} \frac{\partial \tilde{\theta}}{\partial \eta} - R_2 \tilde{\theta} = R_3 \tilde{\phi} \exp(-\beta \eta),
$$

(19)
where the parameters \( R_1 \sim R_3 \) are defined as:
\[
R_1 = \frac{2W(1 + s)}{(1 - W^2 + s\tau q)}, \quad R_2 = \frac{s(2 + s)}{(1 - W^2 + s\tau q)}, \quad R_3 = -2\psi_0(2 + s - W\beta)/(1 - W^2 + s\tau q),
\]
where \( s \) is the Laplace transform parameter and the bar “\( \bar{\ } \)” denotes the Laplace transform of the function. The Laplace transforms of the boundary conditions Eqs. (17) and (18) are:
\[
\frac{d\tilde{\theta}_j(\eta, s)}{d\eta} = 0, \quad \text{at} \quad \eta = 0, \quad (21)
\]
\[
\frac{d\tilde{\theta}_j(\eta, s)}{d\eta} = 0, \quad \text{at} \quad \eta = L. \quad (22)
\]

In this work, Eq. (19) with boundary conditions Eqs. (21) and (22) is solved by using a numerical scheme [18–20]. Eq. (19) can be discretized within the \( \eta \) domain as follows. First, Eq. (19) is discretized by the following equation in the interval \([\eta_j, \eta_{j+1}]\) (i.e., within the \( j \)th element) as:
\[
\frac{d^2\tilde{\theta}_j}{d\eta^2} - R_1 \frac{d\tilde{\theta}_j}{d\eta} - R_2 \tilde{\theta}_j = R_3 \tilde{\phi} \exp(-\beta \eta), \quad \text{for} \quad \eta_j \leq \eta \leq \eta_{j+1}, \quad j = 1, 2, ..., N, \quad (23)
\]
where the subscript symbol \( j \) represents the \( j \)th element.

The general solution of Eq. (23) in the interval \([\eta_j, \eta_{j+1}]\) is:
\[
\tilde{\theta}_j(\eta, s) = A_1 e^{d_1 \eta} + A_2 e^{d_2 \eta} + A_3 e^{-\beta \eta}, \quad (24)
\]
where
\[
d_1 = \frac{R_1 + \sqrt{R_1^2 + 4R_2}}{2}, \quad d_2 = \frac{R_1 - \sqrt{R_1^2 + 4R_2}}{2}, \quad A_3 = \frac{R_3 \tilde{\phi}}{\beta^2 + R_1 \beta - R_2}. \quad (25)
\]
The constants \( A_1 \) and \( A_2 \) in Eq. (24) can be determined by the following boundary conditions:
\[
\tilde{\theta}_j(\eta_j) = \tilde{\theta}_j \quad \text{and} \quad \tilde{\theta}_j(\eta_{j+1}) = \tilde{\theta}_{j+1}, \quad (26)
\]
thus
\[
A_1 = \frac{e^{d_1 \eta_j} (A_1 e^{-\beta \eta_j} - \tilde{\theta}_j) - e^{d_1 \eta_j} (A_3 e^{-\beta \eta_{j+1}} - \tilde{\theta}_{j+1})}{e^{d_1 \eta_j} + d_1 \eta_j - e^{d_1 (\eta_j + d_1 \eta_j)}}, \quad (27)
\]
\[
A_2 = -e^{-d_1 \eta_j} (A_3 e^{-\beta \eta_j} - \tilde{\theta}_j) + \frac{e^{(d_1 - d_2) \eta_j} [e^{d_1 \eta_j} (A_3 e^{-\beta \eta_{j+1}} - \tilde{\theta}_{j+1}) - e^{d_1 \eta_j} (A_3 e^{-\beta \eta_j} - \tilde{\theta}_j)]}{e^{d_1 \eta_j} + d_1 \eta_j - e^{d_1 (\eta_j + d_1 \eta_j)}}. \quad (28)
\]

Similarly, the general solution of Eq. (23) in the interval \([\eta_{j-1}, \eta_j]\) is:
\[
\tilde{\theta}_{j-1}(\eta, s) = B_1 e^{d_1 \eta} + B_2 e^{d_2 \eta} + B_3 e^{-\beta \eta}, \quad (29)
\]
where \( B_3 = A_3 \) and
\[
B_1 = \frac{e^{d_1 \eta_j} (B_3 e^{-\beta \eta_j} - \tilde{\theta}_j) - e^{d_1 \eta_j} (B_3 e^{-\beta \eta_{j-1}} - \tilde{\theta}_{j-1})}{e^{d_1 \eta_j} + d_1 \eta_j - e^{d_1 (\eta_j + d_1 \eta_j)}}, \quad (30)
\]
\[
B_2 = -e^{-d_1 \eta_j} (B_3 e^{-\beta \eta_j} - \tilde{\theta}_j) + \frac{e^{(d_1 - d_2) \eta_j} [e^{d_1 \eta_j} (B_3 e^{-\beta \eta_{j-1}} - \tilde{\theta}_{j-1}) - e^{d_1 \eta_j} (B_3 e^{-\beta \eta_j} - \tilde{\theta}_j)]}{e^{d_1 \eta_j} + d_1 \eta_j - e^{d_1 (\eta_j + d_1 \eta_j)}}. \quad (31)
\]
The continuity of the temperature and heat flux between \( \tilde{\theta}_{j-1}(\eta, s) \) and \( \tilde{\theta}_j(\eta, s) \) at \( \eta = \eta_j \) yields the following conditions:
\[
\tilde{\theta}_{j-1}(\eta_j, s) = \tilde{\theta}_j(\eta_j, s), \quad (32)
\]
\[
\frac{d\tilde{\theta}_{j-1}(\eta_j, s)}{d\eta} = \frac{d\tilde{\theta}_j(\eta_j, s)}{d\eta}. \quad (33)
\]
Substituting Eq. (32) and the approximation functions of \( \tilde{\theta} \) Eqs. (24) and (29) into Eq. (33) produces the following discretized forms for the interior nodes as:
\[
E_{j-1} \tilde{\theta}_{j-1} + E_j \tilde{\theta}_j + E_{j+1} \tilde{\theta}_{j+1} = F_j, \quad \text{for} \quad j = 2, 3, ..., N - 1, \quad (34)
\]
where
\[
E_{j-1} = \frac{e^{d_1 \eta_j} (d_1 - d_2)}{e^{d_1 \eta_{j-1}} - e^{d_1 (\eta_{j-1} - \eta_j) + d_1 \eta_j}}. \quad (35)
\]
\[ E_j = \frac{e^{d_1 + d_2} \eta_j \left( e^{d_2 \eta_{j-1} + d_1 \eta_j} - e^{d_1 \eta_{j-1} + d_2 \eta_j} \right) \left( d_1 - d_2 \right)}{\left( e^{d_2 \eta_{j-1} + d_1 \eta_j} - e^{d_1 \eta_{j-1} + d_2 \eta_j} \right) \left( e^{d_2 \eta_{j-1} + d_2 \eta_{j+1}} - e^{d_1 \eta_{j-1} + d_2 \eta_{j+1}} \right)}, \]  
\[ E_{j+1} = \frac{e^{d_1 + d_2} \eta_j \left( d_1 - d_2 \right)}{e^{d_1 \eta_j + d_2 \eta_{j+1}} - e^{d_1 \eta_j + d_2 \eta_{j+1}}}, \]  
\[ F_j = A_3 \left[ \frac{e^{d_1 \eta_j - \beta \left( \eta_{j-1} + \eta_j \right)} \left( e^{d_1 \eta_j + d_2 \eta_{j-1}} - e^{d_1 \eta_j + d_2 \eta_j} \right)}{e^{d_2 \eta_{j-1} + d_2 \eta_{j-1}} - e^{d_1 \eta_{j-1} + d_2 \eta_{j-1}}} + \frac{e^{d_1 \eta_j - \beta \left( \eta_{j+1} + \eta_j \right)} \left( e^{d_1 \eta_j + d_2 \eta_{j+1}} - e^{d_1 \eta_j + d_2 \eta_{j+1}} \right)}{e^{d_2 \eta_{j+1} + d_2 \eta_{j+1}} - e^{d_1 \eta_{j+1} + d_2 \eta_{j+1}}}, \right] \]  
\[ + \frac{e^{-\beta \left( \eta_{j-1} + d_2 \eta_{j-1} \right)} \left( e^{\beta \eta_{j+1} + d_2 \eta_{j+1}} - e^{\beta \eta_{j+1} + d_2 \eta_{j+1}} \right)}{e^{d_2 \eta_{j+1} + d_2 \eta_{j+1}} - e^{d_1 \eta_{j+1} + d_2 \eta_{j+1}}} \]  
\[ + \frac{e^{-\beta \left( \eta_{j+1} + d_2 \eta_{j+1} \right)} \left( e^{\beta \eta_{j-1} + d_2 \eta_{j-1}} - e^{\beta \eta_{j-1} + d_2 \eta_{j-1}} \right)}{e^{d_2 \eta_{j-1} + d_2 \eta_{j-1}} - e^{d_1 \eta_{j-1} + d_2 \eta_{j-1}}}. \]  

Rearranging Eq. (34) in combination with the transformed boundary conditions of Eqs. (21) and (22) gives the following matrix equation:

\[ [E][\vec{\theta}] = [F], \]  

where \([E]\) is a matrix with complex numbers, \(\vec{\theta}\) is a column vector representing the unknown dimensionless nodal temperatures in the Laplace transform domain, and \([F]\) is a column vector representing the forcing term. Thereafter, the following numerical inversion formula, known as the Fourier series technique [21], is applied to obtain the nodal temperatures \(\{\theta\}\) in the physical domain:

\[ \theta(\xi) \approx \frac{e^{\beta \xi}}{\xi} \left[ \frac{1}{2} \tilde{\phi} (b) + \text{Re} \sum_{m=1}^{M} \tilde{\phi} (b + im\pi / \xi) (-1)^m \right]. \]

4. Results and discussion

The present work numerically studies the thermal behavior of a moving finite medium subjected to time-dependent laser heat source based on the DPL heat conduction model. In order to demonstrate the capability of the presented numerical scheme, two special cases of heat source capacity with \(\psi_0 = 1\) are examined. In these cases, the same parameters used in Ref. [16] are adopted. The velocity \(v\) of the thin slab (dimensionless thickness \(L = 1\)) is assumed not to exceed the speed of heat propagation \(v\), i.e., \(|W| < 1\).

Figs. 2–6 depict the results of calculation for the instantaneous heat source \(\phi(\xi) = \delta(\xi)\) with different parametric effects. In order to verify the accuracy of the presented numerical method, comparison between the present numerical results and

![Fig. 2. Temperature distributions in the slab at \(\xi = 0.1\) with \(\beta = 5\) for the instantaneous heat source \(\phi(\xi) = \delta(\xi)\).](image_url)
Fig. 3. Temperature distributions in the slab at $\xi = 0.3$ with $\beta = 5$ for the instantaneous heat source $\phi(\xi) = \delta(\xi)$.

Fig. 4. Temperature distributions in the slab at various times with $\beta = 5$ and $W = 0.1$ for the instantaneous heat source $\phi(\xi) = \delta(\xi)$. 
Fig. 5. Temperature distributions in the slab at $\xi = 0.2$ with $\beta = 1.5$ and $W = 0.4$ for the instantaneous heat source $\phi(\xi) = \delta(\xi)$.

Fig. 6. Temperature distributions in the slab at $\xi = 0.3$ with $\beta = 5$, $W = 0.1$, and $\tau_r/\tau_q = 0.1, 0.3, 0.6$ for the instantaneous heat source $\phi(\xi) = \delta(\xi)$.
the analytical results in Ref. [16] for the special case of \( \tau_T = 0 \) is illustrated in Figs. 2–6. Figs. 2 and 3 show the effect of dimensionless velocity \( W \) on temperature distributions in the slab along the \( \eta \) direction with \( \beta = 5, \tau_T/\tau_q = 0 \) and 0.1, and time \( \xi = 0.1 \) and \( \xi = 0.3 \), respectively. The results for \( \tau_T = 0 \) are corresponding to the hyperbolic or non-Fourier case. It can be found in Figs. 2 and 3 that the numerical solutions in this study are in very good agreement with those of the analytical values in Ref. [16] for the non-Fourier case of \( \tau_T = 0 \). Therefore, the accuracy of the current numerical scheme can be verified. For the hyperbolic case (\( \tau_T = 0 \)), the instantaneous heat source gives rise to a thermal pulse which travels along the medium and decays exponentially with time through the dissipation of its energy. During a period of \( \xi \), the maximum of the pulse moves over a distance \( \eta = \xi (1 + W) \). In addition, it is found that there exists clear phase shifts in conjunction with velocity \( W \) in the spatial temperature distributions. Moreover, the phase differences are the sole product of dimensionless velocity \( W \), and an increase in \( W \) results in larger phase difference. On the other hand, the thermal pulse in the slab is smoothed by the existence of \( \tau_T \) in the DPL case (\( \tau_T \neq 0 \)) in Figs. 2 and 3. The smoothing of temperature profiles for the DPL case of \( \tau_T \neq 0 \) is expected, since the effect of \( \tau_T \) is to promote conduction into the slab and result in non-Fourier diffusion-like conduction. Hence the temperature profiles are smoother in the DPL case.

Fig. 4 illustrates the temperature distributions for instantaneous heat source \( \phi(\xi) = \delta(\xi) \) in the slab that moves with velocity \( W = 0.1 \) for \( \beta = 5 \) at time \( \xi = 0.3, 0.6, \) and 1.2, respectively. It is noticeable in Fig. 4 that the present numerical results agree excellently with the analytical solutions in Ref. [16] for the special case of \( \tau_T = 0 \). It is seen that, for the hyperbolic case (\( \tau_T = 0 \)), the thermal pulse travels from the left boundary to the right boundary as time elapses, and the pulse decays exponentially with time along the \( \eta \)-axis. The pulse has reached the right boundary and reflected at time \( \xi = 1.2 \). It can also be found in Fig. 4 that the existence of \( \tau_T \) enlarges the penetration depth of thermal signal and smoothens the temperature profiles. Therefore, the non-Fourier case predicts a temperature higher than that of the DPL case within the time interval in Fig. 4. In order to investigate the effect of \( \beta \) on the temperature profiles, Fig. 5 demonstrates the temperature distributions in the slab at time \( \xi = 0.2 \) with moving velocity \( W = 0.4 \) for \( \beta = 1 \) and 5. Fig. 5 illustrates that the numerical solutions are in very good agreement with those of the analytical values for the case of \( \tau_T = 0 \). As shown in Fig. 5, for the non-Fourier case (\( \tau_T = 0 \)), the smaller the \( \beta \), the more blunt the thermal pulse and the shorter the time of its decay. That is, when the slope of the space characteristics of the heat source capacity increases, a blunt wave-front can be observed. Once again, the existence of \( \tau_T \) promotes diffusion of heat ahead of wave-fronts that would be induced by \( \tau_q \), making the temperature profiles smoother in the DPL case in Fig. 5.

Fig. 6 depicts the temperature profiles in the slab at time \( \xi = 0.3 \) with \( \beta = 5, W = 0.1 \), and \( \tau_T/\tau_q = 0.1, 0.3, \) and 0.6 for the instantaneous heat source \( \phi(\xi) = \delta(\xi) \). Once again, Fig. 6 shows that the present numerical results agree very well with the analytical solutions for the special case of \( \tau_T = 0 \). On the other hand, it can be found in Fig. 6 that, compared with the non-Fourier case of \( \tau_T = 0 \), the phase lag \( \tau_T \) of the temperature gradient promotes the diffusion of heat ahead of the wave-fronts, increases the penetration depth of thermal disturbance and flattens the temperature profile. Therefore in Fig. 6, the smoothing effect of \( \tau_T \) becomes more evident as the value of \( \tau_T/\tau_q \) increases.

Next, to prove the capability of the presented numerical scheme in solving different types of heat source, Figs. 7–11 depict the results of calculation for the heat source of step function \( \phi(\xi) = u(\xi) - u(\xi - 0.4) \) with different parametric effects. Figs. 7 and 8

![Image](image_url)

Fig. 7. Temperature distributions in the slab at \( \xi = 0.2 \) with \( \beta = 5 \) for the heat source of step function \( \phi(\xi) = u(\xi) - u(\xi - 0.4) \).
Fig. 8. Temperature distributions in the slab at $\xi = 0.5$ with $\beta = 5$ for the heat source of step function $\phi(\xi) = u(\xi) - u(\xi - 0.4)$.

Fig. 9. Temperature distributions in the slab at various times with $\beta = 5$ and $W = 0.1$ for the heat source of step function $\phi(\xi) = u(\xi) - u(\xi - 0.4)$.
Fig. 10. Temperature distributions in the slab at $\xi = 0.2$ with $\beta = 1.5$ and $W = 0.4$ for the heat source of step function $\phi(\xi) = u(\xi) - u(\xi - 0.4)$.

Fig. 11. Temperature distributions in the slab at $\xi = 0.3$ with $\beta = 5$, $W = 0.1$, and $\tau_T/\tau_q = 0.1, 0.3, 0.6$ for the heat source of step function $\phi(\xi) = u(\xi) - u(\xi - 0.4)$.
illustrate the effects of $W$ and $\tau_p$ on the temperature profiles in the slab along the $\eta$ direction with $\beta = 5$ at time $\xi = 0.2$ and $\xi = 0.5$, respectively. The case of $\tau_p = 0$ corresponds to the hyperbolic or non-Fourier case. In Figs. 7 and 8, it is found that an increase in dimensionless velocity $W$ results in larger phase difference. Since the effect of $\tau_p$ is to promote the non-Fourier diffusion-like conduction in the slab, the presence of $\tau_p$ in the DPL case of Figs. 7 and 8 flattens the temperature profiles.

Fig. 9 shows the temperature profiles in the slab with moving velocity $W = 0.1, \beta = 5$ at various moments of time, namely $\xi = 0.3, 0.6$, and 1.2. It is seen in Fig. 9 that the step-function heat source gives rise to a thermal pulse which travels along the medium and decays exponentially with time. On the other hand, due to the effect of $\tau_p$ on enlarging the penetration depth of thermal signal, the DPL case also displays smoother temperature distributions than the hyperbolic case. Therefore in Fig. 9, the non-Fourier case predicts a temperature higher than that of the DPL case within the time interval. Fig. 10 depicts the temperature profiles in the medium at time $\xi = 0.2$ for various values of $\beta = 1, 5$ with dimensionless moving velocity $W = 0.4$ for the heat source of step function $\phi(\xi) = u(\xi) - u(\xi - 0.4)$. Compared with the results in Fig. 5, the same tendency of temperature variation can be found in Fig. 10, and a blunter wave-front is observed as the $\beta$ decreases. That is, a blunter wave-front can be observed when the slope of the space characteristics of the heat source capacity increases. The existence of the temperature-gradient phase lag $\tau_p$ also promotes the diffusion-like conduction in the slab and makes the temperature profiles slightly smoother in the DPL case in Fig. 10.

Finally, in order to show the effect of $\tau_p/\tau_q$ ratio on the temperature variations in the slab, Fig. 11 demonstrates the temperature distributions in the medium at time $\xi = 0.3$ with $\beta = 5, W = 0.1$, and $\tau_p/\tau_q = 0.1, 0.3$, and 0.6, respectively. It can be found in Fig. 11 that, similar to the results in Fig. 6, the existence of $\tau_p$ dissipates the thermal disturbance and extends the penetration depth of thermal disturbance by inducing the diffusion-like conduction in the medium.

5. Conclusion

To accommodate the micro-structural effect, this work applies the dual-phase-lag model to explore the transient heat transfer in a moving finite medium which is subjected to a time-dependent laser heat source. This work presents a numerical scheme to overcome the mathematical difficulties in dealing with the hyperbolic heat conduction equation. A comparison between the present numerical results and the analytic solutions for the hyperbolic or non-Fourier case ($\tau_p = 0$) is made to prove the accuracy of the presented method. This work also examines the effects of different medium parameters on the behavior of heat transfer. It is found that the spatial temperature distributions show clear phase shifts due to the existence of the medium moving velocity, and increasing medium velocity predicts larger phase difference. In addition, the existence of temperature-gradient phase lag results in non-Fourier diffusion-like conduction in the medium and smoothens the sharp wave-fronts induced by the heat-flux phase lag.

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References