Optimization of Mixed White Light Color Rendering

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Abstract

A method is developed for optimization of color rendering for a light mixture. No derivatives of the rendering function are required. Constraints of correlated color temperature and approximate white can be incorporated. The method is developed based on the Complex Method with modification. It is first applicable for 3-colors light mixtures and then extended to a hierarchical and iterative structure for higher-order light mixtures. Applicability of the method for a mixture of four colors is demonstrated by simulation. The results show that global and unique convergence to the optimal within required tolerances for CRI and spatial dispersivity is always achieved.

Introduction

Color rendering ability of mixed white light is an important index to evaluate its illumination quality in applications. However, due to the complexity in measurement of the rendering ability under designated constraints, a method for general mathematical formulation and global optimization of the rendering ability is difficult to develop. It would be very useful for the light emitted diode (LED) die and luminary manufacturers if such a method is available.

Zukauskas *et al.* [1] considered the problem of optimal light mixing as maximization of an objective function. The objective function is formed by linearly interpolating between the luminous efficacy (LE) and the color rendering index (CRI). A boundary finder was developed for the maximization by locating a segment of an outer boundary of the phase distribution (LE, CRI). Zukauskas' method is full of inspiration. But, as mentioned in Zukauskas' paper, the method may require large depth and large number of perturbations to access the global maximum. It could be very time-consuming if the dimension of the parameter space is increased. Ries *et al.* [2] applied a Mathematica routine to search for maximization of the CRI, the LE or the luminous flux (LF) of a light mixture. The maximization is solved for a given mixture color by iteration. During the iteration, three of the relative intensities were computed to meet the requirement of the given color while the remaining intensities are updated by the routine. Ries' method is computationally efficient. However, the difficulty to access the global optimum increases rapidly with the dimension of the mixture space. Without solving the difficulty, the routine may yield only local maximization. Hsu [3] considered an objective function as a weighting sum of square differences of color properties such as CRI, LE, LF, CCT and color gamut from their set-points. The weighting-sum method is promising to include all the color properties of interest for optimization. However, the approach may achieve only sub-optimal results if precise values for the set-points and the weightings are not available.

The CIE CRI is scored in the scale of 0 ~100. In this scale, a CRI of 100 represents 8 sample objects illuminated by a test light may emit zero color differences in the CIE 1964 $W^*U^*V^*$ color space as illuminated by a reference. However, the color space is not of good uniformity at the red region. Currently, the CIE recommends use of the CIE 1976 $L^*a^*b^*$ and 1976 $L^*u^*v^*$ color spaces for calculating the color differences instead. More problems with the CRI may also include inadequate chromatic adaptation correction, limited applicability at extreme CCTs, few sample objects, flawed color fidelity, etc. [4]. Despite the applicability of the CIE 1964 $W^*U^*V^*$ color space for some case may be questionable [5, 6], it is still the one most widely used currently. Therefore, it is used in this study and methods for the CRI optimization developed are also applicable if a substitute of the CRI is given.

This study is devoted to working out mathematical formulation and a numerical method for the optimization of CRI of light mixture. Unlike the weighting-sum method, the proposed one requires no preknowledge of the set-points and gives clear-cut restraining border to a mixture space and global optimization. The Complex Method [7] is implemented with modification to determine the relative intensities of light mixture for the optimization. The Complex Method with modification in a hierarchical iterative structure is proposed so that the method becomes generally applicable to this study. Examples for simulation studies are given.

Problem Definition

In this study, a test illuminator is a mixture of multi-colors. Without changing the chromaticity and CRI of the mixture, the relative intensities of its compound illuminators can be normalized. Moreover, as it will be explained later for a light mixture of higher order (n > 3) for optimization of the CRI, it may be necessary to consider the constraints of the relative intensities as below:

$$\sum_{i=1}^{n} w_i = 1 \tag{1a}$$

$$w_{il} \leq w_i \leq w_{iu}, \ i = 1, 2, \ldots, n \tag{1b}$$

where w_{il} and w_{iu} denote the lower and upper bounds for a relative intensity. For a light mixture of lower order, the nominal lower and upper bounds are 0 and 1, respectively.

Equations (1) denote many hyperplanes in R^n and the hyperplanes confine the bounds of the mixture space. A hyperplane is a generalization of the concept of a plane into R^n where n > 3 and it can be converted into a line segment in the two-dimensional chromaticity space C^2 of the CIE 1931. For example, the mixture space of 3 colors is a bounded polyhedron in R^3 but it is a bounded triangle in C^2 where the chromaticity of each illuminator denotes a vertex of the triangle.

Further constraints on the mixture space may include a designated CCT and an approximate white mixture as below:

$$T_1 \leq CCT \leq T_2 \tag{2a}$$

$$DC = \sqrt{(u - u_r)^2 + (v - v_r)^2} < 0.0054 \tag{2b}$$

where T_1 and T_2 denote the lower and upper bounds of the CCT of the light mixture; (u, v) the CIE 1960 UCS coordinate of the mixture; (u_r, v_r) the coordinate of a reference illuminator which has the same CCT as the mixture.

The goal of this study is devoted to seeking the global maximum of CRI of a light mixture under the constraints of Eqs (1) and (2),

$$\underset{\substack{\text{under the constraints} \\ \text{of Eqs (1) and (2)}}}{\text{Max}} \ R_a(f, \tilde{f}) \tag{3}$$

where R_a denotes the general CRI of the light mixture; f the spectral power distribution (SPD) of the light mixture; $\tilde{f}$ the SPD of a reference illuminator.

Consistency of the Constraints

Four real LED chips of RGB and white (RGBW) colors are to be used for simulation studies. Among the chips, the RGB LEDs are in one package. The white LED is a commercial product from a local dealer in Taiwan. The SPDs of the LED illuminators shown in Fig. 1 were measured using a spectrometer. The chromaticity coordinates of the red, green, blue, and white illuminators are denoted by p_1= (0.547, 0.318), p_2= (0.270, 0.608), p_3= (0.176, 0.142) and p_4= (0.346, 0.406), respectively. The mixture spaces of RGB and RGBW are of the same triangle $\Delta p_1 p_2 p_3$ as p_4 is enclosed within the triangle as shown in Fig. 2.

Shown also in Fig. 2 are iso-CCTs from 3000K to 9000K and an iso-DC of 0.0054 for illustration. An iso-CCT is a straight line while an iso-DC curve is of non-convexity. The iso-DC curve under the constraint of Eqs (2b) has a significant offset at the CCT of 5000K due to different reference illuminators used for computation of the DC. Moreover, the constraints of Eq. (1b) and Eqs (2) could conflict to each other and must be set in consistency. This is especially true for a higher-order light mixture as one of the constraints could conflict with the achievable space under another. For an RGBW light mixture, the relative intensity of white must be properly upper-bounded, otherwise the mixture could be neither approximately white nor in consistency with a designated CCT.

Numerical Schemes for CRI optimization

Since the mapping from R^2 and R^3 to C^2 is bijective, the Complex Method [7] has been proved by simulation to be applicable for 2- and 3-color light mixtures where the CRI distribution has a unique maximum. However, the method may not be applicable for higher-order light mixtures where the mapping is generally not bijective.

Among the above RGBW light mixtures, the CRI distributions of 3-color mixtures are plotted in Fig. 3. Figure 3(a) shows the CRI distribution of the RGB light mixture in the triangle $\Delta p_1 p_2 p_3$ and Fig. 3(b) shows the distributions of the RGW, GBW, and RBW light mixtures in the triangles $\Delta p_1 p_3 p_4$, $\Delta p_1 p_2 p_4$ and $\Delta p_2 p_3 p_4$, respectively. The CRI distribution for 4 different values of w_4 is plotted in Fig. 4. As w_4 is increased from 0 to 0.2, the CRI maximum is growing in strength and approaching to the mixture space under the constraint of Eq. (3b). As w_4 is further increased, the maximum will reside at the edge $\overline{p_{14}p_{34}}$ but diminish in strength until reach the point p_4. Based on the computed values, a unique maximum exists in the distribution for each w_4. However, the overlapping of all the distributions may result in multiple maxima.

The trace of all the maxima as w_4 varies from 0 to 1 is plotted in chromaticity in Fig. 5. The trace of maxima forms a one-dimensional space in C^2 which is named a hierarchy space (HS) in this study. As w_4 is an independent variable for this case, the HS is denoted by H_4. If another relative intensity w_i is defined as an independent variable, the HS will be denoted by H_i.

The above hierarchy concept can be extended for higher-order light mixtures. An HS of higher hierarchy will be defined when an additional color is added into the mixture. It is postulated that the CRI distribution in an HS has a unique maximum and the highest HS contains the global maximum. The adequacy of this postulation will be demonstrated by a RGBW light mixture in the simulation studies later.

Without loss of generality, all the HS will be denoted by H_n, H_{n-1}, ..., H_4, from the highest hierarchy to the lowest. This hierarchical structure is proposed for optimization of the CRI of higher-order light mixtures in this study.

Proposed Hierarchical Complex Method

By hierarchy, optimization of the CRI of a higher-order light mixture over the mixture space with constraints $\tilde{R}^n$ is based on maximization over the space H_n and optimization over the reduced space $\tilde{R}^{n-1}$, the latter is in turn based on maximization over the space H_{n-1} and optimization over the reduced space $\tilde{R}^{n-2}$, and so on. This hierarchical structure of optimization is indeed performed from the lowest HS, say H_4, until the highest HS, say H_n.

The relative intensities in all HS form a set below:

$$W = \{w_{ij} \mid w_{il} \le w_{ij} \le w_{iu}, i = n, n-1, ..., 4 \text{ and } j = 1, 2, ..., M^{n-i+1}\} \tag{4}$$

where M denotes the number of points to be defined for maximization over an HS; w_{ij} denotes the jth point in H_i. In H_n are w_{n1}, w_{n2},..., w_{nM}, in H_{n-1} are $w_{(n-1)1}$, $w_{(n-1)2}$,..., $w_{(n-1)M}{}^2$, and in H_4 are w_{41}, w_{42},..., $w_{4M}{}^{n-3}$. The number of points expands in a power of M as the number of HS increases by one.

The proposed hierarchical Complex Method [8] is summarized below:
Step 1:
Generate the relative intensities in all HS in the following sequence:

$$w_{nj} = \gamma_{1j}, j = 1, 2, ..., M \tag{5a}$$

$$w_{ij} = (1 - \sum_{k=i+1}^{n} w_{kj})\gamma_{(n-i+1)j}, \quad i = n-1, n-2, ..., 4 \text{ and } j = 1, 2, ..., M^{n-i+1} \tag{5b}$$

where $\gamma_{(n-i+1)j}$ denotes a random number uniformly distributed over the interval $[w_{il}, w_{iu}]$.
Divide the above relative intensities into subsets,

$$W_s = \{w_{ij} \mid j = [s \, DIV \, (M^{i-4} + 1)] + 1; i = n, n-1, ..., 4\}, s = 1, 2, ..., M^{n-3} \tag{6}$$

Determine the complement subsets of W_s using the modified Complex Method,

$$\tilde{W}_s = \{w_{is} \mid i = 3, 2, 1\}, s = 1, 2, ..., M^{n-3} \tag{7}$$

such that all the elements in W_s and the corresponding ones in $\tilde{W}_s$ form a initial set of points as below for later manipulation:

$$P_s = W_s \cup \tilde{W}_s, s = 1, 2, ..., M^{n-3} \tag{8}$$

The following hierarchical iteration completes optimization of the CRI over the space $\tilde{R}^n$. However, the iteration first starts from the 1st set of points (j =1) in the space H_4 (i = 4).

Step 2:

Consider the jth set of points in the hierarchy space H_i that

$$P = \{P_s \mid s = k + M^{j-1}, k = 1,2,...,M\} \tag{9}$$

Update the point $P_L = (w_1, w_2, w_3, ..., w_{iL}, ..., w_{n-1}, w_n)$ corresponding to the lowest CRI in the above set by flipping it over the centroid of the points except the updated one and expanding the distance to the centroid as below:

$$w_{iL}^* = w_{iL} + \alpha(w_{iC} - w_{iL}) \tag{10}$$

where w_{iL}^* denotes the updated one of w_{iL}; w_{iC} the centroid.

Coerce the updated point $P_L^* = (w_1, w_2, w_3, ..., w_{iL}^*, ..., w_{n-1}, w_n)$ to fall within the space $\tilde{R}^n$ whenever necessary. If $i = 4$, apply the modified Complex Method to update w_1, w_2 and w_3. If $i > 4$, perform maximization from the space H_{i-1} down to H_4 to update the relative intensities of lower hierarchy before applying the modified Complex Method to achieve $P_L^* = (w_1^*, w_2^*, w_3^*, ..., w_{iL}^*, ..., w_{n-1}, w_n)$.

Then, proceed to the step 3.

Step 3:

In this step, one of the following conditions will be executed:

(a) If the updated point is infeasible, retract half its distance to the centroid as below until feasible.

$$w_{iL}^* = 0.5(w_{iC} + w_{iL}^*) \tag{11}$$

Then, proceed to the step 4.

(b) If the updated point is feasible but the corresponding CRI is not improved, retract half its distance to the centroid until improved and then proceed to the step 4.

(c) If the updated point is feasible and the corresponding CRI is improved, then proceed to the step 4.

Step 4:

Repeat the step 2 and step 3 until $i = n$, $j = 1$ and the criteria of convergence are satisfied; otherwise let $j = j + 1$ if $j < n-3$ or let $i = i + 1$, $j = 1$, and then go back to the step 2:

The hierarchical iteration and updating take much computer time when n is a large number. Fortunately, a light mixture of 4 colors is normally more than enough to have high CRI for applications.

Simulation and Experimental Studies

Optimization of the CRI of RGBW light mixtures is investigated. The SPDs of the RGBW LEDs given in Fig. 1 are used. For appropriate operations of point distance retraction to and flipping over the centroid during the optimization searching, the number of working points is set to be square of the dimension of the chromaticity space. Hence, $N = 4$ is used in this study. To shorten the simulation time, a small number of points for maximization over an HS is preferred and $M = 2$ is used in this study. Besides, the tolerances of initial spatial dispersivity, convergence for CRI and for spatial dispersivity set in this study are that $\varepsilon_D = 0.05$, $\varepsilon_R = 0.01$ and $\varepsilon_C = 0.001$. These tolerance values were determined by experiment.

The CRI distribution of the RGBW light mixture varies as w_4 varies. The CRI maxima over the space H_4 for w_4 varying from 0 to 1 with increment of 0.01 are shown in Fig. 6(a). The chromaticity of x and y of the maxima are plotted in Fig. 6(b) and 6(c), respectively. The CRI maxima in H_4 have a unique global maximum whose CRI is 96.5 when $w_4 = 0.29$. The chromaticity coordinate of the maximum is (0.3714, 0.3571) but it is outside of the mixture space if the constraint of Eq. (3b) is imposed. Meanwhile, a singularity among the CRI maxima occurs at 5000K when $w_4 = 0.24$. The corresponding chromaticity has an abrupt change at (0.3444, 0.3495) as shown in Fig. 3.

In this example, optimization of the CRI of the RGBW light mixture is first studied with the constraints of Eqs (2) where $T_1 = 4000K$ and $T_2 = 6000K$. Programming codes are written to determine an upper bound of w_4. The upper bound is determined by checking for w_4 starts from 1 down to 0 with the decrement of 0.1 and it terminates if there is a feasible point. For the CCT constraints above, the upper bound of w_4 determined by the codes is 0.4. Hence, w_4 were randomly generated with a uniform distribution over the interval [0, 0.4].

The adequacy of the hierarchical Complex Method has been proved by comprehensive tests. Since the mixture space encloses the global maximum and the singularity at 5000 K, iterations

could converge to both. The problem of non-uniqueness of convergence can be solved by setting the CCT bounds exclusive of the singularity. For example, consider the constraint that $3000K \leqq CCT \leqq 5000K$. The upper bound of w_4 can be the same without conflicting with the constraint. The simulation results show that the states of chromaticity for 50 tests always converge to the global maximum in Fig. 7 and the results are summarized below:

$$\overline{w}_1 = 0.529, \quad \sigma_{w_1} = 0.0008, \quad \overline{w}_2 = 0.023, \quad \sigma_{w_2} = 0.0009, \quad \overline{w}_3 = 0.171, \quad \sigma_{w_3} = 0.0009$$

$$\overline{w}_4 = 0.277, \quad \sigma_{w_4} = 0.0011, \quad \overline{x} = 0.368, \quad \sigma_x = 0.0002, \quad \overline{y} = 0.357, \quad \sigma_y = 0.0001,$$

$$\overline{R}_a(P_h) = 96.5, \quad \sigma_{CRI} = 0.001$$

Conclusion

This study is mainly devoted to developing a numerical method for the CRI optimization. The method is developed based on the Complex Method [7] with modification. Among the RGBW colors, the best option for a 2-color light mixture is the RW to achieve the highest CRI; the best option for a 3-color light mixture is the RBW which also has a large range of chromaticity for design.

The mapping from R^n ($n > 3$) to C^2 is not bijective in general. The CRI distribution is varying with one of the relative intensities of compound illuminators of mixture. An analytical approach for such applications is difficult to develop. So, the modified Complex Method is implemented in a hierarchical and iterative structure for the optimization. The method requires taking no derivatives of function and it is very adequate for the CRI optimization. This adequacy has been demonstrated by simulation for RGBW LED light mixtures and the results have shown the global and unique convergence to the optimal within required tolerances for CRI and spatial dispersivity.

The CRI optimization is studied under the constraints of bounded relative intensities, CCT and DC of a light mixture. Inconsistent constraints as indicated in Fig. 1 and the singularity at 5000 K are preventable. This requires the relative intensities and CCT need to be properly bounded to achieve convergence in iteration. An example for computation of bounded intensity to solve the problem of inconsistent constraints is illustrated while the singularity problem is solved by choosing bounded CCT exclusive of 5000 K.

In practice, an illuminator of high CRI may suffer from low LE. Accurate control of the relative intensities of light mixtures is also a challenging task due to LED junction temperature changes and device aging. Further inclusion of the LE and the effects of junction temperature as extra constraints are proposed for future studies.

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Figure 1: Measured SPDs of the RGBW LED illuminators used for this study.

Figure 2: Mixture spaces of RGB and RGBW; $p_1 \ldots p_4$: red, green, blue and white LED chromaticities, respectively.

Figure 3: CRI distributions of 3-color light mixtures (a) RGB (b) RGW, GBW and RBW.

Figure 4: CRI distributions of a RGBW light mixture at 4 values of w_4 (a) w_4 =0.1 (b) w_4 =0.2 (c) w_4 = 0.3 (d) w_4 =0.4.

Figure 5: CRI maxima in the H_4 space of an RGBW mixture. See text for additional details.

Figure 6: CRI maxima and chromaticities of RGBW light mixture vs. w_4.