# The Effects of Temperature-Dependent Specific Heats of the Working Fluid on the Performance of a Dual Cycle with Heat Loss and Friction

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***Abstract*—The purpose of this study is to analyze the effects of variable specific heats of working fluid on the performance of a Dual engine with considerations of heat transfer and friction by using finite-time thermodynamics. The results show that the power output as well as the efficiency where maximum power output occurs will increase with increasing the amount of heat added to the engine due to combustion or with decreasing the quantity of heat loss through the cylinder wall. The temperature-dependent specific heats of working fluid have a significant influence on the performance. The power output and the working range of the cycle increase with the increase of specific heats of working fluid, while, the efficiency decreases with the increase of specific heats of working fluid. Furthermore, the power output and efficiency of the cycle decrease with increasing the parameter b related to the friction loss. It is noteworthy that the effects of heat transfer with considerations of variable specific heats of working fluid and friction loss on the performance of a Dual engine are significant and should be considered in practice cycle analysis.***

***Keywords: Finite-time thermodynamics; Dual cycle; Heat transfer; Friction; Irreversible***

## I. INTRODUCTION

For an ideal engine cycle, the heat loss and friction loss do not occur, however, for a real engine cycle, these losses indeed exist and should be not negligible. Some attention has been paid to analyze the effects of heat transfer losses and/or friction losses on the performance of internal combustion engines [1-4]. The above studies were done without considering the variable specific heats of working fluid [2-4]. However, in the real engine cycle, the specific heat of working fluid is not a constant and should be considered in practice cycle analysis [5-7].

Few studies have investigated the effects of variable specific heats of working fluid and heat losses on the performance of a reciprocating heat engine. In particular, no performance analysis with emphasis on the Dual cycle with simultaneous considerations of variable specific heats of working fluid, heat transfer, and friction is available in the literature. In this study, we aim at analyzing the effects of heat loss, friction loss, and variable specific heats of working fluid on the net work output and the indicated thermal efficiency of an air standard Dual cycle. Accordingly, we relax the assumptions that there are not heat losses during combustion, that there are not friction losses of the piston for the cycle, and that specific heats of working fluid are constant. In other words, heat transfer between the working fluid and the environment through the cylinder wall is considered; friction loss of the piston on the power stroke is taken into account. Furthermore, we consider the variable specific heats of working fluid that is significant in practice cycle analysis. The results obtained in the study may offer good guidance for the design and operation of the Dual engine.

## II. THERMODYNAMIC ANALYSIS

Figure 1 shows the (P-V) diagram of an air standard Dual cycle model. The compression process (1→2) is an isentropic process; the heat addition takes place in two steps: process (2→3) is isochoric and process (3→4) is isobaric; the expansion process (4→5) is an isentropic process; and the heat rejection process (5→1) is an isochoric process. As is common in finite-time thermodynamic heat engine cycle models, we assume that the adiabatic processes (the compression process 1→2 and the expansion process 4→5) are taken as approximately instantaneous, and that the heat addition process (2→4) , and the heat rejection process (5→1) proceed heat transfer according to constant temperature rates. That is,

$$\frac{dT}{dt}=\frac{1}{K_1} \quad \text{(for } 2\rightarrow 4\text{)}, \tag{1}$$

and

$$\frac{dT}{dt}=\frac{1}{K_2} \quad \text{(for } 5\rightarrow 1\text{)}, \tag{2}$$

where $T$ is the absolute temperature and $t$ is time, $K_1$ and $K_2$ are constants linked to the mean variation rate of the temperatures. Integrating Eq. (1) and Eq. (2), respectively, yield

$$t_1 = K_1(T_3 - T_2) \tag{3}$$

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and

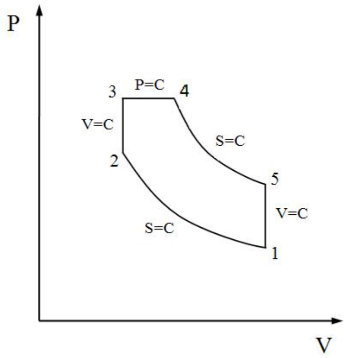


Fig. 1. P-V diagram of an air standard Dual cycle model.

$$t_2 = K_2(T_4 - T_1) \tag{4}$$

where $t_1$ and $t_2$ denote the heating time and cooling time, respectively. In this way, the cycle's period ( $\tau$ ) is

$$\tau = t_1 + t_2 = K_1(T_4 - T_2) + K_2(T_5 - T_1). \tag{5}$$

In practical internal combustion engine cycle, constant-pressure and constant-volume specific heats of the working fluid are variable and will greatly affect the performance of the cycle. According to Ref. [5], it can be assumed that the specific heats of the working fluid are functions of temperature alone and have the following forms:

$$C_{pm} = a_p + k_1 T \tag{6}$$

and

$$C_{vm} = b_v + k_1 T \tag{7}$$

where $C_{pm}$ and $C_{vm}$ are, respectively, the molar specific heats with respect to constant pressure and volume. $a_p$, $b_v$ and $k_1$ are constants. Accordingly, the molar gas constant ($R$) of the working fluid can be expressed as

$$R = C_{pm} - C_{vm} = a_p - b_v. \tag{8}$$

The heat added to the working fluid during process (2→3) and (3→4) are respectively as following

$$Q_{23} = n\left[\int_{T_2}^{T_3} C_{vm} dT\right] = n\left[\int_{T_2}^{T_3} (b_v + k_1 T) dT\right] = n\left[b_v(T_3 - T_2) + 0.5k_1(T_3{}^2 - T_2{}^2)\right]. \tag{9}$$

and

$$Q_{34} = n\left[\int_{T_3}^{T_4} C_{pm} dT\right] = n\left[\int_{T_3}^{T_4} (a_p + k_1 T) dT\right] = n\left[a_p(T_4 - T_3) + 0.5k_1(T_4{}^2 - T_3{}^2)\right] \tag{10}$$

Thus, the total heat addition ($Q_{in}$) during combustion is

$$Q_{in} = Q_{23} + Q_{34} = n\left[b_v(T_3 - T_2) + a_p(T_4 - T_3) + 0.5k_1(T_4{}^2 - T_2{}^2)\right]. \tag{11}$$

The heat rejected by working fluid during process 4→1 is

$$Q_{out} = n\left[\int_{T_1}^{T_5} C_{vm} dT\right] = n\left[\int_{T_1}^{T_5} (b_v + k_1 T) dT\right] = n\left[b_v(T_5 - T_1) + 0.5k_1(T_5^2 - T_1^2)\right] \tag{12}$$

where $n$ is the molar number of the working fluid.

The adiabatic exponent $k = C_{pm}/C_{vm}$ will vary with temperature since both $C_{pm}$ and $C_{vm}$ are dependent on temperature. Accordingly, the equation often used in reversible adiabatic process with constant $k$ cannot be used in reversible adiabatic process with variable $k$. According to Ref. [5], however, a suitable engineering approximation for reversible adiabatic process with variable $k$ can be made, i.e., this process can be divided into infinitesimally small processes, for each of these processes, adiabatic exponent $k$ can be regarded as constant. For instance, for any reversible adiabatic process between states i and j, we can regard the process as that it consists of numerous infinitesimally small processes with constant $k$. For any of these processes, when small changes in temperature $dT$, and in volume $dV$ of the working fluid take place, the equation for reversible adiabatic process with variable $k$ can be written as follows:

$$TV^{k-1} = (T + dT)(V + dV)^{k-1} \tag{13}$$

Form Eq.(15), we get the following equation

$$k_1(T_j - T_i) + b_v \ln(T_j / T_i) = -R \ln(V_j / V_i) \tag{14}$$

The compression ratio ( $\gamma_c$ ) is defined as $\gamma_c = V_1 / V_2$ . Therefore, the equations for processes 1→2 and 4→5 are shown, respectively, by the following equations

$$k_1(T_2 - T_1) + b_v \ln(T_2 / T_1) = R \ln \gamma_c \tag{15}$$

and

$$k_1(T_5 - T_4) + b_v \ln(T_5 / T_4) = R \ln\left[T_4 / (\gamma_c T_3)\right] \tag{16}$$

For an ideal Dual cycle, all the processes are reversible and thus no irreversible losses occur. However, there are heat losses in the cycle of a real engine that are neglected in air-standard analysis. Therefore, irreversibility due to heat transfer between the working fluid and the cylinder wall occurs, which is considered in this paper. The heat loss through the cylinder wall is assumed to be proportional to the average temperature of both working fluid and cylinder wall. Accordingly, the heats added to the working fluid during the constant-volume and the constant-pressure combustion processes can be given, respectively, in the following linear expressions [1-4]:

$$Q_{23} = n\left[A - B(T_2 + T_3)\right] \tag{17}$$

and

$$Q_{34} = n\left[C - D(T_3 + T_4)\right] \tag{18}$$

where $A$ and $C$ are two constants related to combustion, and $B$ and $D$ are two constants related to heat transfer. In the numerical calculation, we take $A=C$ and $B=D$.

Employing the first law of thermodynamics, the work output without losses during the cycle's period is

$$W = Q_{in} - Q_{out} = \left\{n\left[b_v(T_3 + T_1 - T_2 - T_5) + a_p(T_4 - T_3) + 0.5k_1(T_4^2 + T_1^2 - T_2^2 - T_5^2)\right]\right\}. \tag{19}$$

Terefore, the power lost due to friction is

$$P_\mu = f_\mu v = -\mu\left(\frac{dx}{dt}\right)^2 = -\mu v^2 \tag{20}$$

Therefore, the net actual power output of the cycle can be written as:

$$\begin{aligned} P &= \frac{W}{\tau} - P_\mu \\ &= \left\{n\left[b_v(T_3 + T_1 - T_2 - T_5) + a_p(T_4 - T_3) + 0.5k_1(T_4{}^2 + T_1^2 - T_2{}^2 - T_5{}^2)\right]\right\} \\ &\quad \times\left[K_1(T_3 - T_2) + K_2(T_4 - T_3) + K_3(T_5 - T_1)\right]^{-1} - b(\gamma_c - 1)^2, \end{aligned} \tag{21}$$

and the efficiency of the cycle is expressed by:

$$\begin{aligned} \eta &= \frac{P\tau}{Q_{in}} = \frac{W - P_\mu \tau}{Q_{in}} \\ &= \left\{\left[b_v(T_3 + T_1 - T_2 - T_5) + a_p(T_4 - T_3) + 0.5k_1(T_4{}^2 + T_1^2 - T_2^2 - T_5^2)\right]\right. \\ &\quad \left. - b(\gamma_c - 1)^2[K_1(T_3 - T_2) + K_2(T_4 - T_3) + K_3(T_5 - T_1)] / n\right\} \\ &\quad \times\left[b_v(T_3 - T_2) + a_p(T_4 - T_3) + 0.5k_1(T_4^2 - T_2^2)\right]^{-1}. \end{aligned} \tag{22}$$

When $T_1$ and $\gamma_c$ are given, $T_2$ can be obtained from Eq. (15). Afterwards, $T_3$ can be got by substituting Eq. (9) into Eq. (17), and by the same way, the temperature $T_4$ can be found by substituting Eq. (10) into Eq. (18), then the temperature $T_5$ can be got from Eq. (16). Finally, by substituting $T_1$, $T_2$, $T_3$, $T_4$ and $T_5$ into Eqs. (24) and (26), respectively, the power output and the efficiency of the cycle can be obtained. Therefore, the relations between the power output, the efficiency and the compression ratio can be derived.

## III. RESULTS AND DISCUSSION

According to Ref. [5], the following constants and parameters are used in the numerical examples: $A$ = 60000-7000 $Jmol^{-1}$ , B = 20-30 $Jmol^{-1}K^{-1}$ , $b_v$ = 19.868-23.868 $Jmol^{-1}K^{-1}$ , $n$ = $1.57\times10^{-5}$ $kmol$ , $T_1$ = 350 $K$ , $k_1$ = 0.008244-0.009844 $Jmol^{-1}K^{-1}$ and $b$ = 32.5-78 $W$. Taking equal heating and cooling times, i.e., $t_1 = t_2 = \tau/2$ ($\tau$=33.33 ms), the constant temperature rates $K_1$ and $K_2$ are estimated as $K_1$ = 8.128×10-6 $sK^{-1}$ and $K_2$ = 18.67×10-6 $sK^{-1}$ [8]. Numerical examples are shown as follows.

The influence of combustion constant $A$ on the cycle performance is shown in Fig. 2. The power output given by Eq. (21) is a convex function with a single maximum for the optimum compression ratio, as shown in Fig. 2(a). Increase in compression ratio first leads to an increase in power output, and after reaching a peak, the net power output decreases dramatically with further increase in compression ratio. Fig. 2(b) indicates that the behavior of the efficiency versus compression ratio plot is similar to that for the power output shown in Fig. 2(a). Furthermore, Fig. 2 demonstrates that increasing $A$ corresponds to enlarging the amount of heat added to the engine due to combustion, and, therefore, $A$ has a positive effect on the P- $r_c$ and $\eta$-$r_c$ characteristic curves. In other words, for a given $r_c$, the power output and efficiency increase with the increase of $A$, and the maximum power output and its corresponding efficiency increase with the increase of $A$.

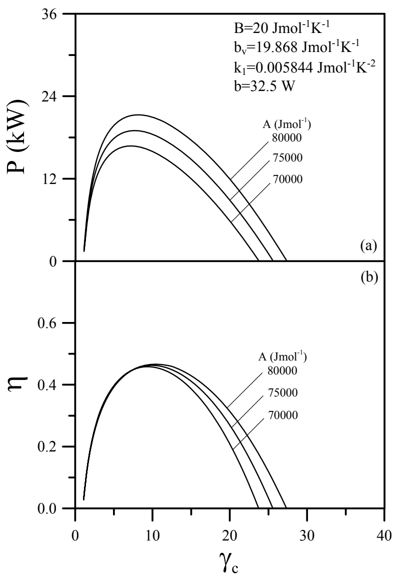


Fig. 2. (a)The influence of A on the variation of the power output with compression ratio;(b)The influence of A on the variation of the efficiency with compression ratio.

On the other hand, heat loss has a negative effect on the performance of the cycle. As can be found in Fig. 3, enlarging $B$ corresponds to increasing heat loss and, thus, decreasing the amount of heat added to the engine. Accordingly, $B$ has an opposite effect on the P - $r_c$ and $\eta$ - $r_c$ characteristic curves to $A$, that is, the maximum power output and its corresponding efficiency decrease with increasing $B$.

Figure 4 shows the influence of the parameter $b_v$ related to the variable specific heats of the working fluid on the performance of the Dual cycle. For a fixed $k_1$, a larger $b_v$ corresponds to a greater value of the molar specific heat with constant volume ($C_{vm}$) or the molar specific heat with constant pressure ($C_{pm}$). Figure 4(a) demonstrates that for a given $r_c$, the

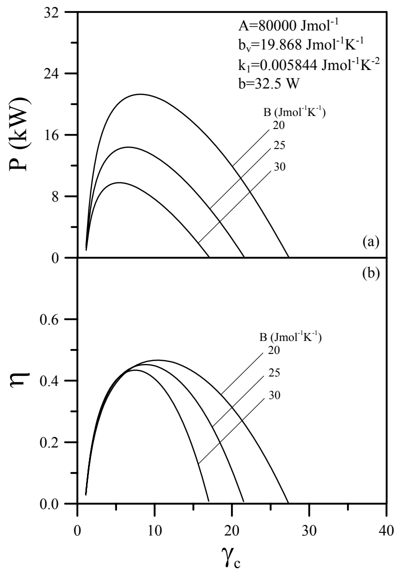


Fig. 3. (a)The influence of B on the variation of the power output with compression ratio;(b)The influence of B on the variation of the efficiency with compression ratio.

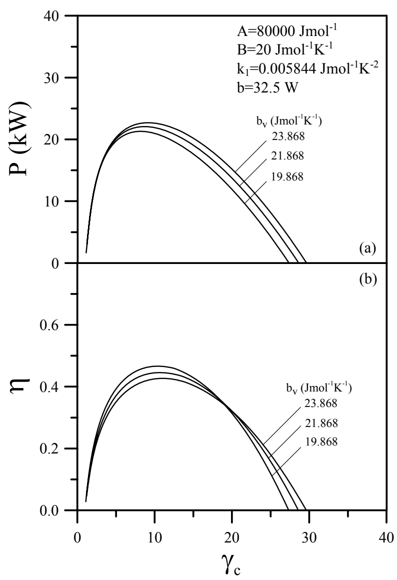


Fig. 4. (a)The influence of $b_v$ on the variation of the power output with compression ratio;(b)The influence of $b_v$ on the variation of the efficiency with compression ratio.

maximum power output and the working range of the cycle increase with the increase of $b_v$. Nevertheless, Fig. 4(b) shows that the maximum efficiency decreases with the increase of $b_v$. It is noteworthy that the parameter $b_v$ has a great influence on the compression ratio where the maximum power or efficiency occurs, as shown in Fig. 4.

Figure 5 represents the influence of the parameter $k_1$ related to the variable specific heats of the working fluid on the performance of the Dual cycle. For a given $b_v$, a larger $k_1$ corresponds to a greater value of the molar specific heats with constant volume ($C_{vm}$) or the molar specific heat with constant pressure ($C_{pm}$). Figure 5 shows that $k_1$ has the same influence as $b_v$ (shown in Fig. 4) on the performance of the cycle. That is, for a given $r_c$, the power output and the working range of the cycle increase with increasing $k_1$, as shown in Fig. 5(a), whereas, the efficiency decreases with the increase of k1, as depicted in Fig. 5(b).

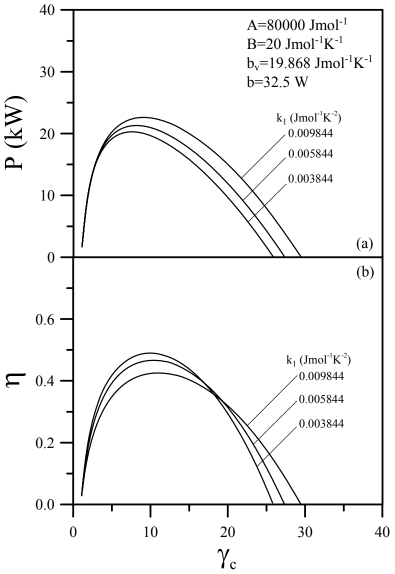


Fig. 5 (a)The influence of $k_1$ on the variation of the power output with compression ratio;(b)The influence of $k_1$ on the variation of the efficiency with compression ratio.

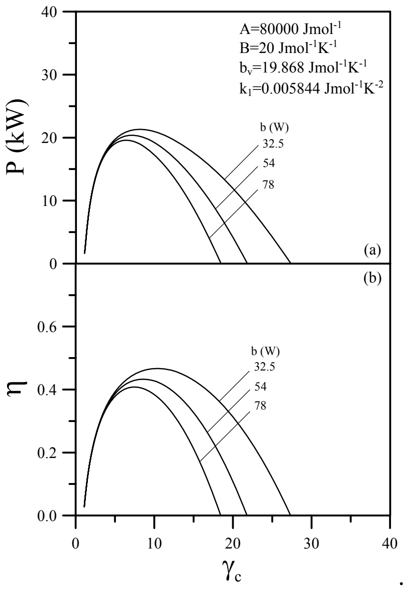


Fig. 6. (a)The influence of $b$ on the variation of the power output with compression ratio;(b)The influence of $b$ on the variation of the efficiency with compression ratio.

Figure 6 shows the influence of the parameter $b$ related to the friction loss on the performance of the Dual cycle. It is clear that the parameter $b$ has a negative effect on the performance. Therefore, as is seen in Fig. 6, the power output and the efficiency of the cycle will decrease with increasing $b$. Moreover, Fig. 6 shows that the points of maximum power and maximum efficiency of the cycle will decrease with an increase of $b$.

## IV. CONCLUSIONS

The power output and efficiency increase with the increase of combustion constant A, and, meanwhile, the maximum power output and its corresponding efficiency increase with the increase of A. Conversely, increasing heat loss constant B corresponds to enlarging heat loss and, thus, decreasing the amount of heat added to the engine. Therefore, the maximum power output and its corresponding efficiency decrease with increasing B.

The parameters $b_v$ and $k_1$ related to the variable specific heats of the working fluid have a significant influence on the performance of the Dual cycle. For a fixed $k_1$ (or $b_v$), a larger $b_v$ (or $k_1$) corresponds to a greater value of the molar specific heats with constant volume ($C_{vm}$) or the molar specific heat with constant pressure ($C_{pm}$). For a given compression ratio $r_c$, the power output and the working range of the cycle increase with the increase of the parameter $b_v$ or $k_1$. Nevertheless, the efficiency decreases with the increase of $b_v$ or $k_1$. Moreover, it is noteworthy that the parameter $b_v$ or $k_1$ has a great influence on the efficiency where the maximum power output occurs or the power output where the maximum efficiency output takes place.

The influence of the parameter b related to the friction loss has a negative effect on the performance. Accordingly, the power output and efficiency of the cycle decrease with increasing b. In addition, the points of maximum power and maximum efficiency of the cycle decrease with an increase of b.

## REFERENCES

[1] L. Chen, C. Wu, F. Sun and S. Cao, Energy Convers. Mgnt. 1998; 39:643-648.

[2] B. A. Akash, Int. Commun. Heat Mass Transfer 2001;28: 87-95.

[3] S. S. Hou, Energy Convers. Mgnt. 2004;45:3003-3015.

[4] Y. Ge, L. Chen, F. Sun and C. Wu, Int. Commun. Heat Mass Transfer 2005;32:1045-1056.

[5] Y. Ge, L. Chen, F. Sun and C. Wu, International Journal of Thermal Sciences 2005;44:506-511.

[6] .A. Al-Sarkhi, J. O. Jaber, M. Abu-Qudais and S. D. Probert, Applied Energy 2006;83:153-165.

[7] A. Al-Sarkhi, J. O. Jaber and S. D. Probert, Applied Energy 2006;83:343-351.

[8] M. Mozurkewich and R. S. Berry, J. Appl. Phys. 1982;53: 34-42.