

# Combined Effects of Variable Specific Heats and Heat Loss on the Performance of an Atkinson Cycle

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*Abstract*—The objective of this study is to analyze the effects of heat loss and variable specific heats of working fluid on the performance of an Atkinson engine by using finite-time thermodynamics. The variations in power output and thermal efficiency with compression ratio, and the relations between the power output and the thermal efficiency of the cycle are presented. The results show that the power output as well as the efficiency where maximum power occurs decrease with the increase of heat loss. The thermal efficiency increases with increasing compression ratio. The thermal efficiency is the greatest when the specific heat model proposed by Sonntag et al. [8] is used in the analysis. Both the constant model and the linear model of specific heat underpredict the thermal efficiency of the cycle.

*Keywords*- thermodynamics; Atkinson cycle; heat loss; specific heat

## I. INTRODUCTION

To make the analysis of the engine cycle much more manageable, air standard cycles are used to describe the major processes occurring in internal combustion engines. Air is assumed to behave as an ideal gas, and all processes are considered to be reversible. In practice, air standard analysis is useful for illustrating the thermodynamic aspects of an engine operation cycle. For an ideal engine cycle, the heat losses do not occur, however, for a real engine cycle, the heat losses indeed exist and should not be negligible. It is recognized that heat loss strongly affects the overall performance of the internal combustion engine. If it is neglected, the analysis will just depend on the ideal air standard cycle.

Some effort has been paid to analyze the effects of heat transfer losses on the performance of internal combustion engines [1-6]. Klein [1] examined the effect of heat transfer through a cylinder wall on work output of the Otto and Diesel cycles. Chen et al. [2, 3] derived the relations between net power output and the efficiency of the Diesel and Otto cycles with considerations of heat loss through the cylinder wall. Hou [4] studied the effect of heat transfer through a cylinder wall on performance of the Dual cycle.

The above studies [1-6] were done without considering the variable specific heats of working fluid. However, in the real engine cycle, the specific heat of working fluid is not a constant and should be considered in practice cycle analysis [7].

The objective of this study is to analyze the effects of heat loss and variable specific heats of working fluid on the net work output and the indicated thermal efficiency of an air standard Atkinson cycle. In the present study, we relax the assumptions that there are heat losses during combustion, and that specific heats of working fluid are constant. In other words, heat transfer between the working fluid and the environment through the cylinder wall is considered. Furthermore, we consider the variable specific heats of working fluid that is significant in practice cycle analysis. The results obtained in the study may offer guidance for the design and operation of the Atkinson engine.

## II. THERMODYNAMIC ANALYSIS

Figs. 1(a) and (b) show the pressure-volume (P-V) and temperature-entropy (T-S) diagrams for the thermodynamic processes of an air-standard Atkinson cycle. The compression process (1→2) is an isentropic process; the heat addition process (2→3) is an isochoric process; the expansion process (3→4) is an isentropic process. In the heat engine cycle models, we assume that the adiabatic processes (the compression process 1→2 and the expansion process 3→4) are taken as approximately instantaneous, and that the heat addition process 2→3 and the heat rejection process 4→1 proceed heat transfer according to constant temperature rates. That is,

$$\frac{dT}{dt} = \frac{1}{K_1} \quad (\text{for } 2 \rightarrow 3), \quad (1)$$

and

$$\frac{dT}{dt} = \frac{1}{K_2} \quad (\text{for } 4 \rightarrow 1), \quad (2)$$

where  $T$  is the absolute temperature and  $t$  is time,  $K_1$  and  $K_2$  are constants linked to the mean variation rate of the temperatures. Integrating Eq. (1) and Eq. (2), respectively, yield

$$t_1 = K_1(T_3 - T_2) \quad (3)$$

and

$$t_2 = K_2(T_4 - T_1) \quad (4)$$

where  $t_1$  and  $t_2$  denote the heating time and cooling time, respectively. In this way, the cycle's period ( $\tau$ ) is

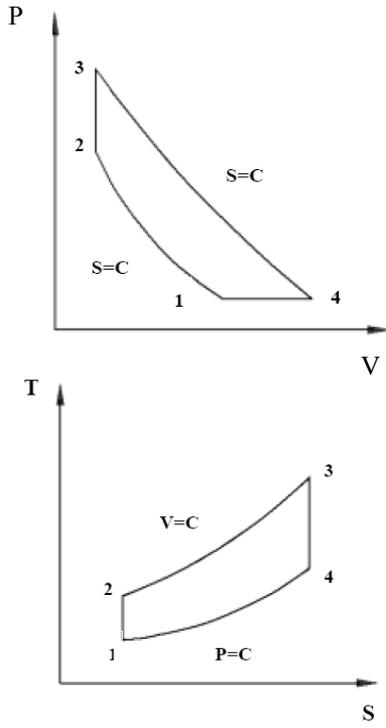


Fig. 1. (a) P-V diagram, and (b) T-S diagram for an the thermodynamic processes of an air-standard Atkinson cycle.

$$\tau = t_1 + t_2 = K_1(T_3 - T_2) + K_2(T_4 - T_1) \quad (5)$$

In practical internal combustion engine cycle, constant-pressure and constant-volume specific heats of the working fluid are variable and will greatly affect the performance of the cycle. According to Ref. [8], it can be assumed that the specific heats of the working fluid are functions of temperature alone and have the following forms:

$$C_p = C_1 T^2 + C_2 T^{1.5} + C_3 T + C_4 T^{0.5} + C_5 + C_6 T^{-1.5} + C_7 T^{-2} + C_8 T^{-3} \quad (6)$$

and

$$C_v = C_p - R \quad (7)$$

where  $C_p$  and  $C_v$  are, respectively, the molar specific heats with constant pressure and volume.  $C_1 \sim C_8$  are constants, which can be expressed as:

$$\begin{aligned} C_1 &= 2.506 \times 10^{-11}, \quad C_2 = 1.45 \times 10^{-7}, \quad C_3 = -4.246 \times 10^{-7}, \\ C_4 &= 3.162 \times 10^{-5}, \quad C_5 = 1.3303, \quad C_6 = -1.512 \times 10^4, \\ C_7 &= 3.063 \times 10^5, \quad C_8 = -2.212 \times 10^7. \end{aligned}$$

Accordingly, the molar gas constant (R) of the working fluid can be expressed as

$$R = 8.3143 \text{ kJ}/(\text{kmole} \cdot \text{K}) \quad (8)$$

The heat added to the working fluid ( $Q_{in}$ ) during process 2→3 is

$$\begin{aligned} Q_{in} &= n \int_{T_2}^{T_3} C_v dT \\ &= n \left[ C_1 \frac{T^3}{3} + C_2 \frac{T^{2.5}}{2.5} - C_3 \frac{T^2}{2} + C_4 \frac{T^{1.5}}{1.5} + C_5 - R - C_6 \left( -\frac{T^{-0.5}}{0.5} \right) + C_7 (-T^{-1}) - C_8 \left( \frac{T^{-2}}{2} \right) \right]_{T_2}^{T_3} \end{aligned} \quad (9)$$

The heat rejected by working fluid during process 4→1 is

$$\begin{aligned} Q_{out} &= n \int_{T_4}^{T_1} C_p dT \\ &= n \left[ C_1 \frac{T^3}{3} + C_2 \frac{T^{2.5}}{2.5} - C_3 \frac{T^2}{2} + C_4 \frac{T^{1.5}}{1.5} + C_5 - C_6 \left( -\frac{T^{-0.5}}{0.5} \right) + C_7 (-T^{-1}) - C_8 \left( \frac{T^{-2}}{2} \right) \right]_{T_4}^{T_1} \end{aligned} \quad (10)$$

Where  $n$  is the molar number of the working fluid. The adiabatic exponent  $k = C_p/C_v$  will vary with temperature since both  $C_p$  and  $C_v$  are dependent on temperature. Accordingly, the equation often used in reversible adiabatic process with constant  $k$  cannot be used in reversible adiabatic process with variable  $k$ . However, a suitable engineering approximation for reversible adiabatic process with variable  $k$  can be made, i.e., this process can be divided into infinitesimally small processes, for each of these processes, adiabatic exponent  $k$  can be regarded as constant. For instance, for any reversible adiabatic process between states  $i$  and  $j$ , we can regard the process as that it consists of numerous infinitesimally small processes with constant  $k$ . For any of these processes, when small changes in temperature  $dT$ , and in volume  $dV$  of the working fluid take place, the equation for reversible adiabatic process with variable  $k$  can be written as follows:

$$TV^{k-1} = (T + dT)(V + dV)^{k-1} \quad (11)$$

Form Eq.(11), we get the following equation

$$\begin{aligned} & \left[ \frac{C_1}{2} T^2 + \frac{C_2}{1.5} T^{1.5} + C_3 T + \frac{C_4 T^{0.5}}{0.5} + C_5 \ln T + \frac{C_6 T^{-1.5}}{(-1.5)} + \frac{C_7 T^{(-2)}}{(-2)} + \frac{C_8 T^{(-3)}}{(-3)} \right]_{T_1}^{T_2} - R_g \ln \gamma_c = 0 \end{aligned} \quad (12)$$

The compression ratio ( $\gamma_c$ ) is defined as  $\gamma_c = V_1/V_2$ . Therefore, the equations for processes 1→2 and 3→4 are shown, respectively, by the following equations

$$\left[ \begin{array}{l} \frac{C_1}{2}T^2 + \frac{C_2}{1.5}T^{1.5} + C_3T + \frac{C_4T^{0.5}}{0.5} \\ + C_5 \ln T + \frac{C_6T^{-1.5}}{(-1.5)} + \frac{C_7T^{(-2)}}{(-2)} + \\ \frac{C_8T^{(-3)}}{(-3)} \end{array} \right]_{T_1}^{T_2} - R \ln \gamma_c = 0 \quad (13)$$

and

$$\left[ \begin{array}{l} \frac{C_1}{2}T^2 + \frac{C_2}{1.5}T^{1.5} + C_3T + \frac{C_4T^{0.5}}{0.5} + \\ C_5 \ln T + \frac{C_6T^{-1.5}}{(-1.5)} + \frac{C_7T^{(-2)}}{(-2)} + \frac{C_8T^{(-3)}}{(-3)} \end{array} \right]_{T_3}^{T_4} + R \ln \left( \frac{V_4}{V_3} \right) = 0 \quad (14)$$

For an ideal Atkinson cycle, all the processes are reversible and thus no irreversible losses occur. However, there are heat losses in the cycle of a real engine that are neglected in air-standard analysis. Therefore, irreversibility due to heat transfer between the working fluid and the cylinder wall occurs, which is considered in this paper. The heat loss through the cylinder wall is assumed to be proportional to the average temperature of both working fluid and cylinder wall. Accordingly, the heats added to the working fluid during the constant-volume combustion processes can be given in the following linear expressions [5]:

$$Q_{in} = n[A - B(T_2 + T_3)] \quad (15)$$

where  $A$  and  $B$  are two constants related to combustion and heat transfer.

Therefore, the power output of the cycle is

$$P = \frac{W}{\tau} = \frac{Q_{in} - Q_{out}}{\tau} \quad (16)$$

and the efficiency of the cycle is

$$\eta = \frac{W}{Q_{in}} \quad (17)$$

When  $T_1$  and  $\gamma_c$  are given,  $T_2$  can be obtained from Eq. (13). Afterwards,  $T_3$  can be got by substituting Eq. (9) into Eq. (15), and then the temperature  $T_4$  can be found from Eq. (14). Finally, by substituting  $T_1$ ,  $T_2$ ,  $T_3$  and  $T_4$  into Eqs. (16) and (17), respectively, the power output and the efficiency of the cycle can be obtained. Therefore, the relations between the power output, the efficiency and the compression ratio can be derived.

### III. RESULTS AND DISCUSSION

The following constants and ranges of parameters are used in the calculations:  $T_1 = 350 \text{ K}$ ,  $A = 60000 \sim 70000$

J/mol,  $B = 20 \sim 30 \text{ J/mol-K}$ , and  $n = 4.57 \times 10^{-5} \text{ kmole}$ . The linear model [9] is shown in Eqs. (18) and (19)

$$C_p = a_p + k_1 T \quad (18)$$

$$C_v = b_v + k_1 T \quad (19)$$

where  $a_p$ ,  $b_v$ , and  $k_1$  are constants. The values of the constants are as follows:  $a_p = 28 \text{ J/mol-K}$ ,  $b_v = 20 \text{ J/mol-K}$ , and  $k_1 = 0.004 \text{ J/mol-K}^2$ . The constant specific heats in the constant model are taken as  $C_p = 29.11 \text{ kJ/kmol-K}$  and  $C_v = 20.8 \text{ kJ/kmol-K}$ . Numerical examples are presented as follows.

Figs. 2~5 depict the influence of combustion constant  $A$  and heat transfer constant  $B$  on the cycle performance as the specific heat proposed by Sonntag et al. [8] is used. The curves, shown in Figs. 2 and 3, are concave downward. Figures 2 and 3 illustrate that increasing  $A$  corresponds to enlarging the amount of heat added to engine due to combustion, and, therefore,  $A$  has a positive effect on the  $P-\gamma_c$  and  $P-\eta$  characteristic curves. In other words, for a given  $\gamma_c$ , the power output and efficiency increase with the increase of  $A$ , and the maximum power output and its corresponding efficiency increase with the increase of  $A$ .

On the other hand, increasing  $B$  corresponds to enlarging heat loss and, thereby, decreasing the amount of heat added to the engine. Accordingly,  $B$  has an opposite effect to  $A$  on the  $P-\gamma_c$  and  $P-\eta$  characteristic curves, i.e., the maximum power output and its corresponding efficiency decrease with increasing  $B$ .

Fig. 6 demonstrates the effects of specific heat under various models on the performance of the Atkinson cycle for  $A=70000 \text{ J/mol}$ , and  $B=25 \text{ J/mol-K}$ . The effect of specific heat on the efficiency is clear in this figure. As can be seen, the thermal efficiency increases with increasing compression ratio. The thermal efficiency (denoted by the solid line) is the greatest when the specific heat model proposed by Sonntag et al. [8] is used. Obviously, both the constant specific heat model (designated by the dashed line) and the linear specific heat model (demonstrated by the dotted-dashed line) underestimate the thermal efficiency of the air-standard Atkinson cycle.

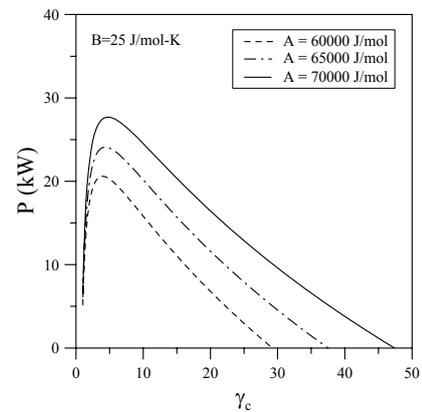


Fig. 2. Effect of  $A$  on  $P$ .

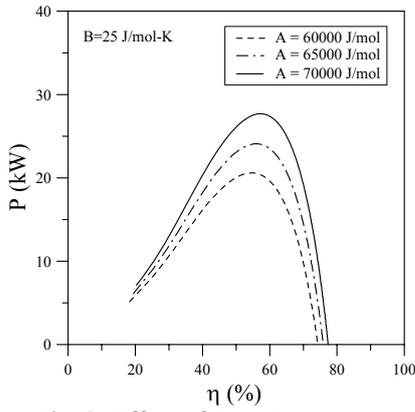


Fig. 3. Effect of A on P-  $\eta$  curve.

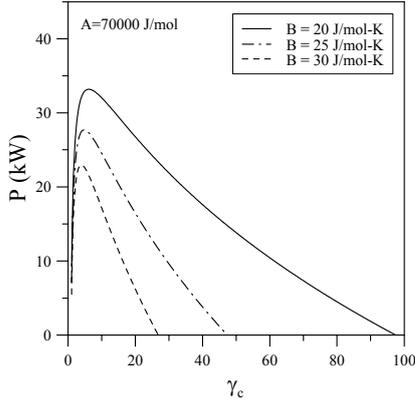


Fig. 4. Effect of B on P.

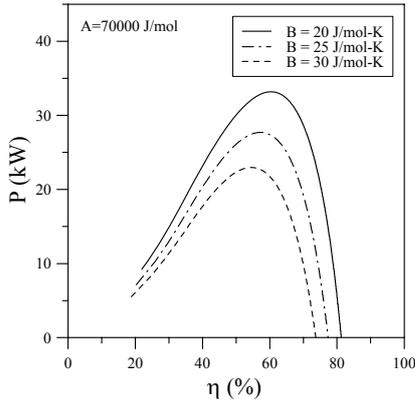


Fig. 5. Effect of B on P-  $\eta$  curve.

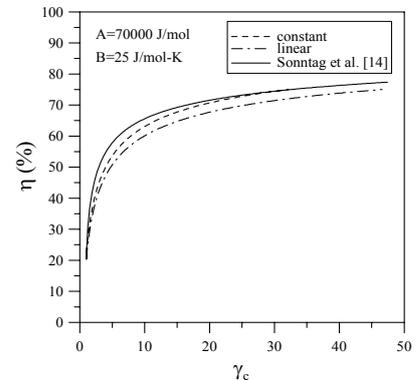


Fig. 6. Effect of specific heat on  $\eta$ .

#### IV. CONCLUSIONS

The effects of heat loss and variable specific heats of working fluid on the performance of an Atkinson engine are presented in this study. The results are summarized as follows.

- (1) For a given compression ratio, the power output and efficiency increase with the increase of the constant A related to combustion and the maximum power output and its corresponding efficiency increase with the increase of A.
- (2) Increasing the constant B related to heat loss corresponds to enlarging heat loss and, thus, decreasing the amount of heat added to the engine. Accordingly, B has an opposite effect on the performance to A, i.e., the maximum power output and its corresponding efficiency decrease with increasing B.
- (3) The thermal efficiency is the greatest when the specific heat model proposed by Sonntag et al. [8] is used. Both the constant specific heat model and the linear specific heat model underpredict the thermal efficiency of the cycle.

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