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Robust stability of uncertain singular time-delay systems via LFT approach

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The robust stability problem of continuous-time singular systems with multiple state delays and bounded parametric uncertainties is considered. Both commensurate and non-commensurate delays are investigated. On the basis of the linear fractional transformations (LFTs) framework and μ-analysis, a systematic approach is derived to convert the robustness problem to a robust nonsingularity problem. Some explicit conditions are proposed to guarantee the uncertain singular time-delay system being regular, impulse-free and stable independent of delay. Two illustrative examples are given to show the feasibility of the proposed technique.

Keywords: uncertain singular time-delay systems; LFT; robust stability independent of delay; regularity; impulse-immunity

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1. Introduction

The system-theoretic problem of singular systems has been an active research area in the past years because singular systems are more general and natural in representing dynamic systems than regular (state-space) systems (Verghese, Levy, and Kailath 1981; Lewis 1986; Dai 1989). Singular systems are also referred to as descriptor systems, generalised state-space systems, differential-algebraic systems or implicit systems. Time delay is encountered in various dynamic systems owing to data computation, long transmission line, etc. Unfortunately, it is often a source of systems instability and performance degradation. Many stability criteria, which can be classified into delay-independent and delay-dependent, have been reported in Chen and Latchman (1995), Dugard and Verriest (1997), Hung and Zhou (2000) and references therein.

Recently, there has been a growing interest in the robust control problem of continuous singular time-delay systems due to inevitable uncertainties. The robust stability and stabilisation problems for singular systems with single state delay and parametric uncertainties were solved by the notations of generalised quadratic stability and stabilisation (Xu, Dooren, Stefan, and Lam 2002), where the strict linear matrix inequality (LMI) design approach was developed. The global exponential robust stability of singular and impulsive interval uncertain systems with time delay was presented in Guan and Lam (2001). The robust H_∞ control problem for singular time-delay systems was investigated in Ku, Lam, and Yang (2003), Zhong and Yang (2006) and Wu and Zhou (2007) and can be classified into delay-independent and delay-dependent types. On the other hand, the robust stability problem and optimal control problem of discrete-time singular systems with time delays have also attracted considerable interest in recent years (Xu and Zhang 2002; Chen and Chou 2004; Chen and Lin 2004; Ma and Zhang 2007).

It should be pointed out that, when the robust stability problem of singular time-delay systems is studied, the regularity and impulse-immunity for continuous-time cases (causality for discrete-time cases) must be considered simultaneously (Xu et al. 2002; Xu and Zhang 2002; Chen and Lin 2004), and the latter two properties do not appear in regular systems. Therefore, the stability robustness problem of uncertain singular systems with time delays is more complicated than that of uncertain regular time-delay systems.

The robust stability independent of delay for linear continuous-time singular systems with multiple state delays and parametric uncertainties is considered in this article. Singular systems with both commensurate and non-commensurate delays are studied. By transforming the problem into checking the nonsingularity of a class of uncertain matrices, μ-analysis is then involved in the approach. Sufficient conditions are obtained to guarantee the uncertain singular time-delay system being regular, impulse-free and stable independent of delay. The elegant linear

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fractional transformation (LFT) machinery, which is widely used in the robust control of standard state-space systems (Doyle, Packard, and Zhou 1991; Zbou, Doyle, and Glover 1996; Cockburn and Morton 1997), plays a key role for robustness analysis throughout this work. The LFT-framework approach was first applied to robust stability of discrete time-delay uncertain singular systems (Chen and Lin 2004); however, it has not been applied to the robust stability problem for the continuous singular time-delay systems in the literature.

In the sequel we denote the sets of real and complex number by $\boldsymbol{R}$ and $\boldsymbol{C}$. The open right half complex plane is denoted by $\boldsymbol{C}_+ := \{s\colon \operatorname{Re}(s) > 0\}$ and $\overline{\boldsymbol{C}}_+$ denotes its closure. For a matrix $\boldsymbol{A}$, we denote its transpose and complex conjugate transpose by $\boldsymbol{A}^{\boldsymbol{T}}$ and $\boldsymbol{A}^*$. Furthermore, we denote its spectral radius by $\rho(\boldsymbol{A})$ and the largest singular value by $\bar{\sigma}(\boldsymbol{A})$.

2. Problem formulation and preliminaries

Consider the uncertain singular system with multiple state delays and bounded parametric uncertainties described by

$$\boldsymbol{E}\dot{\boldsymbol{x}}(t) = (\boldsymbol{A} + \Delta\boldsymbol{A})\boldsymbol{x}(t) + \sum_{k=1}^{q} (\boldsymbol{A}_{dk} + \Delta\boldsymbol{A}_{dk})\boldsymbol{x}(t - \tau_k), \quad \tau_k \geq 0, \tag{1}$$

where $\boldsymbol{x}(t)$ is the state with a compatible continuous vector valued initial function. The matrix $\boldsymbol{E} \in \boldsymbol{R}^{n\times n}$ may be possible singular with $\operatorname{rank}(\boldsymbol{E}) = r \leq n \cdot \boldsymbol{A}$, $\boldsymbol{A}_{dk} \in \boldsymbol{R}^{n\times n}$ are given real constant matrices corresponding to the present and former states, respectively. The delay factors τ_k, $k = 1, 2, \ldots, q$, are nonnegative constants. If $\tau_k = k\tau$, $k = 1, 2, \ldots, q$, for some $\tau \geq 0$, then τ_k are said to be commensurate. Otherwise, the delays are said to be non-commensurate. $\Delta\boldsymbol{A}$ and $\Delta\boldsymbol{A}_{dk}$ represent the time-invariant parameter uncertainties bounded by

$$\bar{\sigma}(\Delta \mathrm{A}) \leq \alpha_A \quad \text{and} \quad \bar{\sigma}(\Delta\boldsymbol{A}_{dk}) \leq \alpha_{dk},\ k = 1, 2, \ldots, q. \tag{2}$$

The nominal case (free of uncertainties) of the uncertain singular time-delay system (1) can be written as

$$\boldsymbol{E}\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \sum_{k=1}^{q} \boldsymbol{A}_{dk}\boldsymbol{x}(t - \tau_k), \quad \tau_k \geq 0, \quad t \geq 0. \tag{3}$$

For simplicity, the system (3) is denoted by the triplet $(\boldsymbol{E}, \boldsymbol{A}, \boldsymbol{A}_{dk})$, and denoted by the pair $(\boldsymbol{E}, \boldsymbol{A})$ if there is no delay state in system (3).

Definition 1: (Lewis 1986; Dai 1989):

I. The pair $(\boldsymbol{E}, \boldsymbol{A})$ is said to be *regular* if $\det(s\boldsymbol{E} - \boldsymbol{A})$ is not identically zero.

II. The pair $(\boldsymbol{E}, \boldsymbol{A})$ is said to be *impulse-free* if $\deg(\det(s\boldsymbol{E} - \boldsymbol{A})) = \operatorname{rank}(\boldsymbol{E})$.

III. The pair $(\boldsymbol{E}, \boldsymbol{A})$ is said to be *stable* if $\det(s\boldsymbol{E} - \boldsymbol{A}) \neq 0$, $\forall s \in \overline{\boldsymbol{C}}_+$. □

As shown in Dai (1989), the regularity is essential to guarantee the existence and uniqueness of all free response for a singular system. The impulse-immunity prevents the impulsive behaviour which is undesired in system control.

Lemma 1 (Xu et al. 2002)**:** *If the pair* $(\boldsymbol{E}, \boldsymbol{A})$ *is regular and impulse-free, then the solution to the singular time-delay system* (3) *exists and is impulse-free and unique on* $[0, \infty)$.

From Lemma 1, it is worth noting that the properties of regularity and impulse-immunity for the singular time-delay system (3) are only dependent on the pair $(\boldsymbol{E}, \boldsymbol{A})$, and independent of the matrices of delay states $\boldsymbol{A}_{dk}$, $k = 1, 2, \ldots, q$. Based on Lemma 1 and the notion of stability independent of delay (Dugard and Verriest 1997), the following definition for the singular time-delay systems (3) is introduced.

Definition 2:

I. The triplet $(\boldsymbol{E}, \boldsymbol{A}, \boldsymbol{A}_{dk})$ is said to be regular and impulse-free if the pair $(\boldsymbol{E}, \boldsymbol{A})$ is regular and impulse-free.

II. The triplet $(\boldsymbol{E}, \boldsymbol{A}, \boldsymbol{A}_{dk})$ is said to be stable independent of delay if

$$\det\left(s\boldsymbol{E} - \boldsymbol{A} - \sum_{k=1}^{q} \boldsymbol{A}_{dk} e^{-\tau_k s}\right) \neq 0, \quad \forall s \in \overline{\boldsymbol{C}}_+, \quad \forall \tau_k \geq 0.$$

Without loss of generality, the pair $(\boldsymbol{E}, \boldsymbol{A})$ is assumed to be regular, impulse-free and stable throughout this article. The purpose of this article is first to derive the conditions such that the nominal singular time-delay system (3) of commensurate and non-commensurate delays is stable independent of delay. Then we develop the sufficient conditions such that the robust regularity, impulse-immunity and stability independent of delay of the uncertain singular time-delay system (1) are guaranteed.

We conclude this section by presenting several preliminaries which will be employed in the main results.

Suppose that $\boldsymbol{M}$ is a complex matrix partitioned as $\boldsymbol{M} = \begin{bmatrix} \boldsymbol{M}_{11} & \boldsymbol{M}_{12} \\ \boldsymbol{M}_{21} & \boldsymbol{M}_{22} \end{bmatrix}$. The upper and lower LFTs (Doyle et al. 1991) are defined, respectively, as

$$F_u(\boldsymbol{M}, \Delta) := \boldsymbol{M}_{22} + \boldsymbol{M}_{21}\Delta(\boldsymbol{I} - \boldsymbol{M}_{11}\Delta)^{-1}\boldsymbol{M}_{12}, \tag{4}$$

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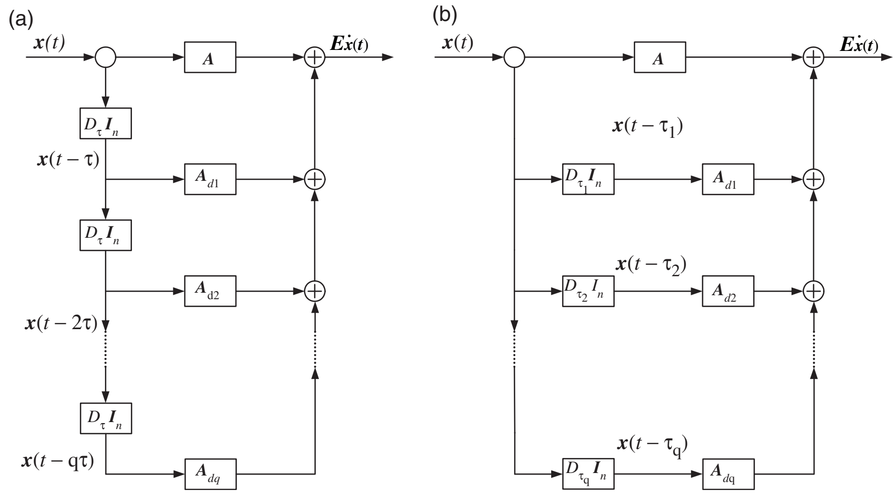

Figure 1. The block diagram representation for the nominal singular time-delay system: (a) the case with commensurate delays and (b) the case with non-commensurate delays.

$$F_l(\boldsymbol{M}, \Delta) := \boldsymbol{M}_{11} + \boldsymbol{M}_{12}\Delta(\boldsymbol{I} - \boldsymbol{M}_{22}\Delta)^{-1}\boldsymbol{M}_{21}. \quad (5)$$

Clearly, the existence of the LFTs depends on the invertibility of matrices $\boldsymbol{I} - \boldsymbol{M}_{11}\Delta$ and $\boldsymbol{I} - \boldsymbol{M}_{22}\Delta$, respectively.

The structured singular value μ introduced by Doyle (1982) has been well-known to be a powerful tool for the robustness analysis of linear systems with structured uncertainties. Let the set of block diagonal structure of repeated complex scalar and full block uncertainties be defined as

$$\Delta := \{\mathrm{diag}(\delta_1 \boldsymbol{I}_{r_1}, \delta_2 \boldsymbol{I}_{r_2}, \ldots, \delta_q \boldsymbol{I}_{r_q}, \Delta_1, \ldots, \Delta_k): \delta_i \in \boldsymbol{C}, \Delta_j \in \boldsymbol{C}^{m_j \times m_j}\} \quad (6)$$

with $\sum_{i=1}^{q} r_i + \sum_{j=1}^{k} m_j = n$.

Definition 3 (Doyle 1982)**:** For a complex matrix $\boldsymbol{M} \in \boldsymbol{C}^{n\times n}$, the structured singular value of $\boldsymbol{M}$ with respect to a block structure set Δ is defined by

$$\mu_\Delta(\boldsymbol{M}) = \frac{1}{\min_{\Delta \in \Delta}\{\bar{\sigma}(\Delta): \ \det(\boldsymbol{I} - \boldsymbol{M}\Delta) = 0\}}.$$

If there is no $\Delta \in \Delta$ such that $\det(\boldsymbol{I} - \boldsymbol{M}\Delta) = 0$, then $\mu_\Delta(\boldsymbol{M}) := 0$.

Lemma 2 (Packard and Doyle 1993)**:** *If* $\Delta = \{\delta \boldsymbol{I}_n: \delta \in \boldsymbol{C}\}$, *then* $\mu_\Delta(\boldsymbol{M}) = \rho(\boldsymbol{M})$.

Lemma 3 (Boyd and Desoer 1985)**:** *Let* $\boldsymbol{M}(s) \in \boldsymbol{C}^{n\times n}$ *be analytic in* $\overline{\boldsymbol{C}}_+$. *Then both* $\rho(\boldsymbol{M}(s))$ *and* $\mu_\Delta(\boldsymbol{M}(s))$ *are continuous and subharmonic in* $\overline{\boldsymbol{C}}_+$, *and it gives* $\sup_{s\in\overline{\boldsymbol{C}}_+} \mu_\Delta(\boldsymbol{M}(s)) = \sup_{\omega\in\boldsymbol{R}} \mu_\Delta(\boldsymbol{M}(j\omega))$.

The computation of exact structured singular value is not easy. Only upper and lower bounds are available for the real/mixed μ-problem in Fan, Tits, and Doyle (1991), Young and Doyle (1996) and references therein. The software used to calculate the real/mixed μ is currently available in the μ-Toolbox of MATLAB (Balas, Doyle, Glover, Packard, and Smith 1995).

3. Stability independent of delay

The nominal singular system (3) with commensurate and non-commensurate delays can be represented as the block diagrams in Figure 1(a) and (b), respectively, where D_τ is denoted as the delay operator such that $D_\tau \phi(t) = \phi(t-\tau)$ for any scalar function $\phi(t)$.

Based on Figure 1(a), one can pull out the time-delay blocks $D_\tau \boldsymbol{I}_n$ and an LFT-diagram can be redrawn with the time-delay blocks stacking diagonally as shown in Figure 2(a). Then, the LFT representation for the system (3) of commensurate delays can be represented as

$$\boldsymbol{E}\dot{\boldsymbol{x}}(t) = F_l(\boldsymbol{M}, D_c)\boldsymbol{x}(t) := F_l\left(\begin{bmatrix} \boldsymbol{M}_{11} & \boldsymbol{M}_{12} \\ \boldsymbol{M}_{21} & \boldsymbol{M}_{22} \end{bmatrix}, D_c\right)\boldsymbol{x}(t), \quad (7)$$

where

$$\boldsymbol{M}_{11} := \boldsymbol{A},$$
$$\boldsymbol{M}_{12} := [\boldsymbol{A}_{d1}, \boldsymbol{A}_{d2}, \ldots, \boldsymbol{A}_{dq}]_{n\times nq},$$
$$\boldsymbol{M}_{21} := [\boldsymbol{I}_n, 0_n, \ldots, 0_n]^T_{nq\times n},$$
$$\boldsymbol{M}_{22} := \begin{bmatrix} 0_n & 0_n & \cdots\cdots & 0_n \\ \boldsymbol{I}_n & 0_n & \cdots\cdots & 0_n \\ 0_n & \boldsymbol{I}_n & \ddots & \vdots \\ \vdots & \ddots\ddots & \ddots\vdots & \\ 0_n & \cdots 0_n & \boldsymbol{I}_n & 0_n \end{bmatrix}_{nq\times nq}$$

and $D_c = \mathrm{diag}(D_\tau \boldsymbol{I}_n, D_\tau \boldsymbol{I}_n, \ldots, D_\tau \boldsymbol{I}_n) = D_\tau \boldsymbol{I}_{nq}$.

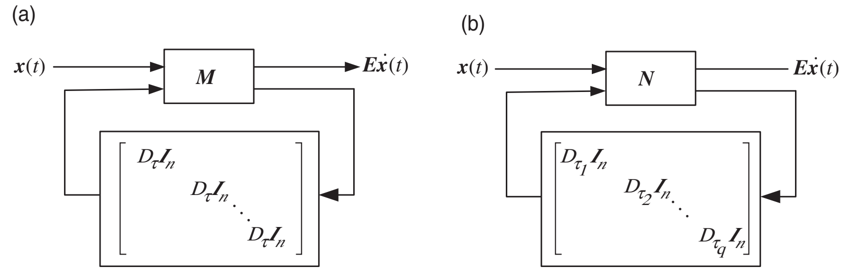

Figure 2. LFT representation of the nominal singular time-delay system: (a) the case with commensurate delays and (b) the case with non-commensurate delays.

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Similarly, the system (3) with non-commensurate delays, shown in Figure 1(b), can be described as another LFT representation of the form

$$\boldsymbol{E}\dot{\boldsymbol{x}}(t) = F_l(\boldsymbol{N}, D_{nc})\boldsymbol{x}(t) := F_l\left(\begin{bmatrix} \boldsymbol{N}_{11} & \boldsymbol{N}_{12} \\ \boldsymbol{N}_{21} & \boldsymbol{N}_{22} \end{bmatrix}, D_{nc}\right)\boldsymbol{x}(t), \tag{8}$$

where

$$\boldsymbol{N}_{11} := \mathbf{A}, \qquad \boldsymbol{N}_{12} := \left[\boldsymbol{A}_{d1}, \boldsymbol{A}_{d2}, \ldots, \boldsymbol{A}_{dq}\right]_{n\times nq},$$
$$\boldsymbol{N}_{21} := [\boldsymbol{I}_n, \boldsymbol{I}_n, \ldots, \boldsymbol{I}_n]^T_{nq\times n}, \quad \boldsymbol{N}_{22} := 0_{nq\times nq},$$

and $D_{nc} = \mathrm{diag}(D_{\tau_1}\boldsymbol{I}_n, D_{\tau_2}\boldsymbol{I}_n, \ldots, D_{\tau_q}\boldsymbol{I}_n)$, as shown in Figure 2(b).

In the subsequent development, we shall derive the condition of stability independent of delay for the cases of commensurate and non-commensurate delays based on the LFT representations in (7) and (8), respectively.

Theorem 1: *Suppose* $(\boldsymbol{E}, \boldsymbol{A})$ *is regular, impulse-free and stable, then the singular system* (3) *of commensurate delays is regular, impulse-free and stable independent of delay if*

$$\rho(\boldsymbol{M}_C(j\omega)) < 1, \quad \omega \ge 0,$$

where $\boldsymbol{M}_C(j\omega) = \boldsymbol{M}_{22} + \boldsymbol{M}_{21}(j\omega\boldsymbol{E} - \boldsymbol{M}_{11})^{-1}\boldsymbol{M}_{12}$.

Proof: Since $(\boldsymbol{E}, \boldsymbol{A})$ is regular and impulse-free, the system $(\boldsymbol{E}, \boldsymbol{A}, \boldsymbol{A}_{dk})$ is also regular and impulse-free by Lemma 1. Taking Laplace transform to (7), by Definition 2 the singular time-delay system (3) is stable independent of delay if

$$\det(s\boldsymbol{E} - F_l(\boldsymbol{M}, \Delta_M(s))) \ne 0, \quad \forall s \in \overline{\boldsymbol{C}}_+, \tag{9}$$

where $\Delta_M(s) = \mathrm{diag}(e^{-\tau s}\boldsymbol{I}_n, e^{-\tau s}\boldsymbol{I}_n, \ldots, e^{-\tau s}\boldsymbol{I}_n) = e^{-\tau s}\boldsymbol{I}_{nq}$ with $\tau \ge 0$. It gives $\bar{\sigma}(\Delta_M(s)) \le 1$ for all $s \in \overline{\boldsymbol{C}}_+$ and $\tau \ge 0$. With the assumption of the system $(\boldsymbol{E}, \boldsymbol{A})$ being stable, which implies $\det(s\boldsymbol{E} - \boldsymbol{A}) = \det(s\boldsymbol{E} - \boldsymbol{M}_{11}) \ne 0$ for all $s \in \overline{\boldsymbol{C}}_+$, then (9) is equivalent to

$$\begin{aligned} &\det(s\boldsymbol{E} - (\boldsymbol{M}_{11} + \boldsymbol{M}_{12}\Delta_M(s)(\boldsymbol{I}_{nq} - \boldsymbol{M}_{22}\Delta_M(s))^{-1}\boldsymbol{M}_{21}) \ne 0, \\ &\qquad \forall s \in \overline{\boldsymbol{C}}_+, \\ &\Leftrightarrow \det(\boldsymbol{I}_n - (s\boldsymbol{E} - \boldsymbol{M}_{11})^{-1}\boldsymbol{M}_{12}\Delta_M(s)(\boldsymbol{I}_{nq} - \boldsymbol{M}_{22}\Delta_M(s))^{-1}\boldsymbol{M}_{21}) \\ &\qquad \ne 0, \quad \forall s \in \overline{\boldsymbol{C}}_+, \\ &\Leftrightarrow \det(\boldsymbol{I}_{nq} - (\boldsymbol{I}_{nq} - \boldsymbol{M}_{22}\Delta_M(s))^{-1}\boldsymbol{M}_{21}(s\boldsymbol{E} - \boldsymbol{M}_{11})^{-1}\boldsymbol{M}_{12}\Delta_M(s)) \\ &\qquad \ne 0, \quad \forall s \in \overline{\boldsymbol{C}}_+. \end{aligned} \tag{10}$$

For the last equivalency we use the fact $\det(\boldsymbol{I}_m + \boldsymbol{X}\boldsymbol{Y}) = \det(\boldsymbol{I}_n + \boldsymbol{Y}\boldsymbol{X})$ for any $m \times n$ matrix $\boldsymbol{X}$ and $n \times m$ matrix $\boldsymbol{Y}$. It is easy to verify the nonsingularity of $\boldsymbol{I}_{nq} - \boldsymbol{M}_{22}\Delta_M(s)$ for all $s \in \overline{\boldsymbol{C}}_+$. Hence the equivalency is continued to be

$$\begin{aligned} &\det(\boldsymbol{I}_{nq} - (\boldsymbol{M}_{22} + \boldsymbol{M}_{21}(s\boldsymbol{E} - \boldsymbol{M}_{11})^{-1}\boldsymbol{M}_{12})\Delta_M(s)) \ne 0, \\ &\quad \forall s \in \overline{\boldsymbol{C}}_+, \Leftrightarrow \det(\boldsymbol{I}_{nq} - \boldsymbol{M}_C(s)\Delta_M(s)) \ne 0, \quad \forall s \in \overline{\boldsymbol{C}}_+, \end{aligned} \tag{11}$$

where $\boldsymbol{M}_C(s)$ is a stable transfer matrix and $\bar{\sigma}(\Delta_M(s)) \le 1$ for all $s \in \overline{\boldsymbol{C}}_+$ and $\tau \ge 0$. Let the uncertainty structure set $\Omega_1 = \left\{\mathrm{diag}(\delta\boldsymbol{I}_{nq}), \delta \in \boldsymbol{C}, |\delta| \le 1\right\}$. By the definition of structured singular value, Lemmas 2 and 3, it is concluded that the singular system (3) of commensurate delays is stable independent of delay if

$$\sup_{s\in\overline{\boldsymbol{C}}_+} \mu_{\Omega_1}(\boldsymbol{M}_C(s)) = \sup_{\omega\ge0} \mu_{\Omega_1}(\boldsymbol{M}_C(j\omega)) = \sup_{\omega\ge0} \rho(\boldsymbol{M}_C(j\omega)) < 1. \tag{12}$$

This completes the proof. □

To proceed, a sufficient condition of stability independent of delay for the case of non-commensurate delays is given by the following theorem.

Theorem 2: *Suppose* $(\boldsymbol{E}, \boldsymbol{A})$ *is regular, impulse-free and stable, then the singular system* (3) *of non-commensurate delays is regular, impulse-free and stable independent of delay if*

$$\mu_{\Omega_2}(\boldsymbol{N}_{nc}(j\omega)) < 1, \quad \forall \omega \ge 0,$$

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where $\Omega_2 = \{\mathrm{diag}(\delta_1 \boldsymbol{I}_n, \delta_2 \boldsymbol{I}_n, \ldots, \delta_q \boldsymbol{I}_n), \delta_i \in \boldsymbol{C}, |\delta_i| \le 1\}$ *is the set of block diagonal structure of repeated complex scalar uncertainties and* $\boldsymbol{N}_{nc}(j\omega) = \boldsymbol{N}_{21}(j\omega \boldsymbol{E} - \boldsymbol{N}_{11})^{-1}\boldsymbol{N}_{12}$.

Proof: The system $(\boldsymbol{E}, \boldsymbol{A}, \boldsymbol{A}_{dk})$ is stable independent of delay by Definition 3 if

$$\det(s\boldsymbol{E} - F_l(\boldsymbol{N}, \Delta_N(s))) \neq 0, \quad \forall s \in \overline{\boldsymbol{C}}_+, \tag{13}$$

where $\Delta_N(s) = \mathrm{diag}(e^{-\tau_1 s}\boldsymbol{I}_n, e^{-\tau_2 s}\boldsymbol{I}_n, \ldots, e^{-\tau_q s}\boldsymbol{I}_n)$. It gives $\bar{\sigma}(\Delta_N(s)) \le 1$ for all $s \in \overline{\boldsymbol{C}}_+$ and $\tau_k \ge 0$, $k = 1, 2, \ldots, q$. As a similar process to that of Theorem 1, (13) holds if

$$\det(\boldsymbol{I}_{nq} - \boldsymbol{N}_{nc}(s)\Delta_N(s)) \neq 0, \quad \forall s \in \overline{\boldsymbol{C}}_+, \tag{14}$$

where $\boldsymbol{N}_{nc}(s) = \boldsymbol{N}_{21}(s\boldsymbol{E} - \boldsymbol{N}_{11})^{-1}\boldsymbol{N}_{12}$. By the definition of structured singular value and Lemma 3, it is concluded that the singular system (3) of non-commensurate delays is stable independent of delay if

$$\sup_{s \in \overline{\boldsymbol{C}}_+} \mu_{\Omega_2}(\boldsymbol{N}_{nc}(s)) = \sup_{\omega \ge 0} \mu_{\Omega_2}(\boldsymbol{N}_{nc}(j\omega)) < 1. \tag{15}$$

This completes the proof. □

4. Robust stability independent of delay

Consider the continuous-time uncertain singular time-delay system (1). We first tackle the problem of robust regularity and impulse-immunity. Based on Lemma 1, the regularity and impulse-immunity of the system $(\boldsymbol{E}, \boldsymbol{A}, \boldsymbol{A}_{dk})$ and the system $(\boldsymbol{E}, \boldsymbol{A})$ are equivalent. Consequently, the condition of preserving robust regularity and impulse-immunity for the system (1) will be derived directly from $(\boldsymbol{E}, \boldsymbol{A} + \Delta\boldsymbol{A})$.

By carrying out a singular value decomposition of the matrix $\boldsymbol{E}$, there exist the unitary matrices $\boldsymbol{U}$ and $\boldsymbol{V}$ (Dai 1989) such that

$$\boldsymbol{E} = \boldsymbol{U}\boldsymbol{E}_D\boldsymbol{V}^* = \boldsymbol{U}\begin{bmatrix} \boldsymbol{E}_1 & 0 \\ 0 & 0 \end{bmatrix}\boldsymbol{V}^*, \quad \boldsymbol{A} = \boldsymbol{U}\begin{bmatrix} \boldsymbol{A}_{11} & \boldsymbol{A}_{12} \\ \boldsymbol{A}_{21} & \boldsymbol{A}_{22} \end{bmatrix}\boldsymbol{V}^*,$$
$$\Delta\boldsymbol{A} = \boldsymbol{U}\begin{bmatrix} \Delta\boldsymbol{A}_{11} & \Delta\boldsymbol{A}_{12} \\ \Delta\boldsymbol{A}_{21} & \Delta\boldsymbol{A}_{22} \end{bmatrix}\boldsymbol{V}^*, \tag{16}$$

where $\boldsymbol{E}_1$ is a diagonal matrix with positive diagonal elements and $\mathrm{rank}(\boldsymbol{E}_1) = r$.

The following two lemmas are helpful to derive the condition of robust regularity and impulse-immunity for the uncertain singular time-delay system (1).

Lemma 4 (Chen 1989)**:** *For an* $n \times n$ *matrix* $\boldsymbol{G}$, *if* $\rho(\boldsymbol{G}) < 1$, *then* $\det(\boldsymbol{I}_n \pm \boldsymbol{G}) \neq 0$.

Lemma 5 (Bender and Laub 1987)**:** *The singular system* $(\boldsymbol{E}, \boldsymbol{A})$ *is impulse-free if and only if the submatrix* $\boldsymbol{A}_{22}$ *in* (16) *is invertible.*

Lemma 6: *Suppose* $(\boldsymbol{E}, \boldsymbol{A}, \boldsymbol{A}_{dk})$ *is regular and impulse-free, then the uncertain singular time-delay system* (1) *is robustly regular and impulse-free if* $\rho(\boldsymbol{A}_{22}^{-1}\Delta\boldsymbol{A}_{22}) < 1$.

Proof: With the assumption of $(\boldsymbol{E}, \boldsymbol{A}, \boldsymbol{A}_{dk})$ being regular and impulse-free, the submatrix $\boldsymbol{A}_{22}$ is non-singular by Lemmas 1 and 5. The degree of the characteristic polynomial

$$\det(s\boldsymbol{E} - (\boldsymbol{A} + \Delta\boldsymbol{A})) = \det\left(\boldsymbol{U}\begin{bmatrix} s\boldsymbol{E}_1 - \boldsymbol{A}_{11} - \Delta\boldsymbol{A}_{11} & -\boldsymbol{A}_{12} - \Delta\boldsymbol{A}_{12} \\ -\boldsymbol{A}_{21} - \Delta\boldsymbol{A}_{21} & -\boldsymbol{A}_{22} - \Delta\boldsymbol{A}_{22} \end{bmatrix}\boldsymbol{V}^*\right) \tag{17}$$

for the system $(\boldsymbol{E}, \boldsymbol{A} + \Delta\boldsymbol{A})$ results from the diagonal entries of the block matrix $s\boldsymbol{E}_1 - \boldsymbol{A}_{11} - \Delta\boldsymbol{A}_{11}$. The highest order coefficient of this polynomial is not equal to zero if and only if $\det(\boldsymbol{A}_{22} + \Delta\boldsymbol{A}_{22}) \neq 0$. Hence, the system (1) is robustly impulse-free if

$$\begin{aligned} &\deg(\det(s\boldsymbol{E} - (\boldsymbol{A} + \Delta\boldsymbol{A}))) = \mathrm{rank}(\boldsymbol{E}) = r \\ &\Leftrightarrow \det(\boldsymbol{A}_{22} + \Delta\boldsymbol{A}_{22}) \neq 0 \\ &\Leftrightarrow \det(\boldsymbol{I}_{n-r} + \boldsymbol{A}_{22}^{-1}\Delta\boldsymbol{A}_{22}) \neq 0. \end{aligned} \tag{18}$$

It is directly from Lemma 4 if $\rho(\boldsymbol{A}_{22}^{-1}\Delta\boldsymbol{A}_{22}) < 1$, then (18) holds. Moreover, the result of $\deg(\det(s\boldsymbol{E} - (\boldsymbol{A} + \Delta\boldsymbol{A}))) = r$ implies that $\det(s\boldsymbol{E} - (\boldsymbol{A} + \Delta\boldsymbol{A}))$ is not identically zero, which in turn $(\boldsymbol{E}, \boldsymbol{A} + \Delta\boldsymbol{A})$ is robustly regular by the Definition 1. This completes the proof. □

Let us proceed to tackle the robust stability independent of delay for the uncertain singular time-delay system (1). Let

$$\Delta\boldsymbol{A} = \alpha_A\Delta_A, \; \Delta\boldsymbol{A}_{dk} = \alpha_{dk}\Delta_{dk}, \; k = 1, 2, \ldots, q \tag{19}$$

with $\bar{\sigma}(\Delta_A) \le 1$ and $\bar{\sigma}(\Delta_{dk}) \le 1$. The uncertain singular system (1) of commensurate and non-commensurate delays are represented as block diagrams in Figure 3(a) and (b), respectively.

Based on Figure 3(a), we can pull out the time-delay blocks $D_\tau \boldsymbol{I}_n$ and uncertainty blocks $(\Delta_A, \Delta_{dk}, k = 1, 2, \ldots, q)$ to construct an LFT diagram. Then, the uncertain singular system (1) of commensurate delays can be represented by the following LFT description:

$$\boldsymbol{E}\dot{\boldsymbol{x}}(t) = F_l(\boldsymbol{P}, \tilde{D}_c)\boldsymbol{x}(t) := F_l\left(\begin{bmatrix} \boldsymbol{P}_{11} & \boldsymbol{P}_{12} \\ \boldsymbol{P}_{21} & \boldsymbol{P}_{22} \end{bmatrix}, \tilde{D}_c\right)\boldsymbol{x}(t), \tag{20}$$

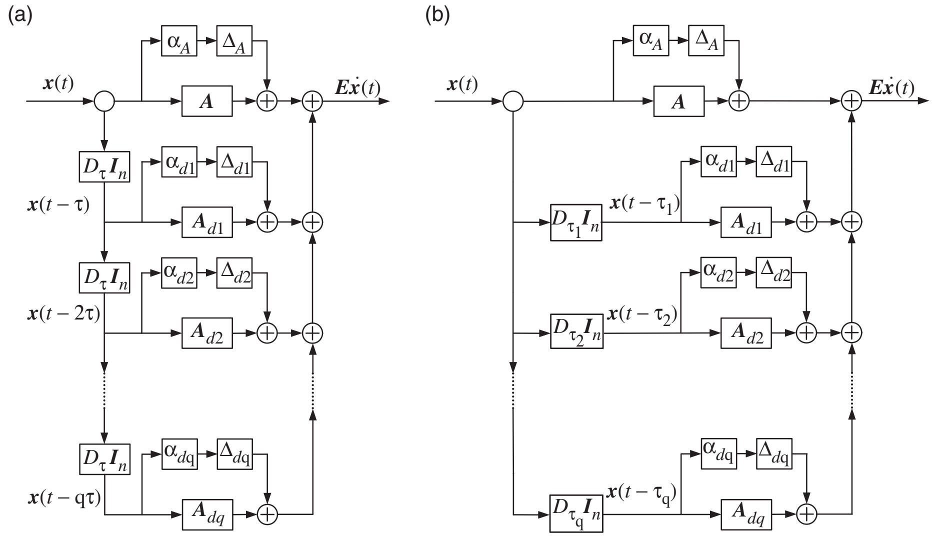

Figure 3. The block diagrams representation for the uncertain singular time-delay system (1): (a) the case of commensurate delays and (b) the case of non-commensurate delays.

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where

$$\begin{aligned}
\boldsymbol{P}_{11} &:= \boldsymbol{A},\\
\boldsymbol{P}_{12} &:= \left[\boldsymbol{A}_{d1}, \boldsymbol{A}_{d2}, \ldots, \boldsymbol{A}_{dq}, \boldsymbol{I}_n, \boldsymbol{I}_n, \ldots, \boldsymbol{I}_n\right]_{n\times n(2q+1)},\\
\boldsymbol{P}_{21} &:= [\boldsymbol{I}_n, 0_n, \ldots, 0_n, \alpha_A \boldsymbol{I}_n, 0_n, \ldots, 0_n]^T_{n(2q+1)\times n},\\
\boldsymbol{P}_{22} &:= \begin{bmatrix} \boldsymbol{M}_{22} & 0_{nq\times n(q+1)} \\ \Lambda & 0_{n(q+1)\times n(q+1)} \end{bmatrix}_{n(2q+1)\times n(2q+1)},
\end{aligned}$$

$$\Lambda := \begin{bmatrix} 0_n & & & \\ \alpha_{d1}\boldsymbol{I}_n & 0_n & & \\ & \alpha_{d2}\boldsymbol{I}_n & \ddots & \\ & & \ddots & 0_n \\ & & & \alpha_{dq}\boldsymbol{I}_n \end{bmatrix}_{n(q+1)\times nq}$$

and

$$\begin{aligned}
\tilde{D}_c &= \mathrm{diag}(D_\tau \boldsymbol{I}_n, \ldots, D_\tau \boldsymbol{I}_n, \Delta_A, \Delta_{d1}, \ldots, \Delta_{dq})\\
&= \mathrm{diag}(D_\tau \boldsymbol{I}_{nq}, \Delta_A, \Delta_{d1}, \ldots, \Delta_{dq}).
\end{aligned}$$

The submatrix $\boldsymbol{M}_{22}$ in $\boldsymbol{P}_{22}$ is shown in (7). Then, the condition for robust stability independent of delay for the system (1) of commensurate delays can be developed by Theorem 3.

Theorem 3: *Suppose* $(\boldsymbol{E}, \boldsymbol{A}, \boldsymbol{A}_{dk})$ *is regular, impulse-free and stable independent of delay. The uncertain singular system* (1) *of commensurate delays is robustly regular, impulse-free and stable independent of delay if*

$$\rho(\boldsymbol{A}_{22}^{-1}\Delta\boldsymbol{A}_{22}) < 1 \quad \textit{and} \quad \mu_{\Omega_3}(\boldsymbol{P}_C(j\omega)) < 1, \quad \forall\omega \geq 0,$$

where

$$\Omega_3 = \left\{\mathrm{diag}(\delta \boldsymbol{I}_{nq}, \Delta_1, \Delta_2, \cdots, \Delta_{q+1}), \quad \delta \in \boldsymbol{C},\ \Delta_j \in \boldsymbol{C}^{n\times n}, \quad |\delta| \leq 1, \quad \bar{\sigma}(\Delta_j) \leq 1\right\}$$

is the set of block diagonal uncertainty structure and $\boldsymbol{P}_C(j\omega) = \boldsymbol{P}_{22} + \boldsymbol{P}_{21}(j\omega\boldsymbol{E} - \boldsymbol{P}_{11})^{-1}\boldsymbol{P}_{12}$.

Proof: Based on Lemma 6, the condition $\rho(\boldsymbol{A}_{22}^{-1}\Delta\boldsymbol{A}_{22}) < 1$ guarantees the system (1) being robustly regular and impulse-free. Taking Laplace transform to the LFT system (20), then the system (1) is robustly stable independent of delay if and only if

$$\det(s\boldsymbol{E} - F_l(\boldsymbol{P}, \Delta_P(s))) \neq 0, \quad \forall s \in \overline{\boldsymbol{C}}_+, \tag{21}$$

where $\Delta_P(s) = \mathrm{diag}(e^{-\tau s}\boldsymbol{I}_{nq}, \Delta_A, \Delta_{d1}, \ldots, \Delta_{dq})$ with $\bar{\sigma}(\Delta_p(s)) \leq 1$, for all $s \in \overline{\boldsymbol{C}}_+$ and $\tau \geq 0$. It follows that (21) is equivalent to

$$\det(s\boldsymbol{E} - (\boldsymbol{P}_{11} + \boldsymbol{P}_{12}\Delta_P(s)(\boldsymbol{I} - \boldsymbol{P}_{22}\Delta_P(s))^{-1}\boldsymbol{P}_{21}) \neq 0, \quad \forall s \in \overline{\boldsymbol{C}}_+, \tag{22}$$

where $\boldsymbol{I} - \boldsymbol{P}_{22}\Delta_P(s)$ is nonsingular. Since $(\boldsymbol{E}, \boldsymbol{A}) = (\boldsymbol{E}, \boldsymbol{P}_{11})$ is stable, it gives $(s\boldsymbol{E} - \boldsymbol{P}_{11})$ invertible for all $s \in \overline{\boldsymbol{C}}_+$. Equation (22) is thereby equivalent to

$$\det(\boldsymbol{I} - (s\boldsymbol{E} - \boldsymbol{P}_{11})^{-1}\boldsymbol{P}_{12}\Delta_P(s)(\boldsymbol{I} - \boldsymbol{P}_{22}\Delta_P(s))^{-1}\boldsymbol{P}_{21}) \neq 0, \quad \forall s \in \overline{\boldsymbol{C}}_+. \tag{23}$$

Using the fact that $\det(\boldsymbol{I} + \boldsymbol{XY}) = \det(\boldsymbol{I} + \boldsymbol{YX})$, we have

$$\det(\boldsymbol{I} - (\boldsymbol{I} - \boldsymbol{P}_{22}\Delta_P(s))^{-1}\boldsymbol{P}_{21}(s\boldsymbol{E} - \boldsymbol{P}_{11})^{-1}\boldsymbol{P}_{12}\Delta_P(s)) \neq 0, \quad \forall s \in \overline{\boldsymbol{C}}_+. \tag{24}$$

As a result

$$\det(\boldsymbol{I}-(\boldsymbol{P}_{22}+\boldsymbol{P}_{21}(s\boldsymbol{E}-\boldsymbol{P}_{11})^{-1}\boldsymbol{P}_{12}\Delta_P(s)))\neq 0,\quad \forall s\in\overline{\boldsymbol{C}}_+. \tag{25}$$

Hence

$$\det(\boldsymbol{I}-\boldsymbol{P}_C(s)\Delta_P(s))\neq 0,\quad \forall s\in\overline{\boldsymbol{C}}_+. \tag{26}$$

By the definition of structured singular value and Lemma 3, we conclude that if

$$\sup_{s\in\overline{\boldsymbol{C}}_+}\mu_{\Omega_3}(\boldsymbol{P}_C(s))=\sup_{\omega\geq 0}\mu_{\Omega_3}(\boldsymbol{P}_C(j\omega))<1,$$

then (26) holds. This completes the proof. □

Similarly, we can derive the following LFT description for the uncertain singular system (1) of non-commensurate delays from Figure 3(b):

$$\boldsymbol{E}\dot{\boldsymbol{x}}(t)=F_l(\boldsymbol{Q},\tilde{D}_{nc})\boldsymbol{x}(t):=F_l\left(\begin{bmatrix}\boldsymbol{Q}_{11} & \boldsymbol{Q}_{12}\\ \boldsymbol{Q}_{21} & \boldsymbol{Q}_{22}\end{bmatrix},\tilde{D}_{nc}\right)\boldsymbol{x}(t), \tag{27}$$

where

$$\boldsymbol{Q}_{11}:=\boldsymbol{A},$$
$$\boldsymbol{Q}_{12}:=\left[\boldsymbol{A}_{d1},\boldsymbol{A}_{d2},\ldots,\boldsymbol{A}_{dq},\boldsymbol{I}_n,\boldsymbol{I}_n,\ldots,\boldsymbol{I}_n\right]_{n\times n(2q+1)},$$
$$\boldsymbol{Q}_{21}:=[\boldsymbol{I}_n,\ldots,\boldsymbol{I}_n,\alpha_A\boldsymbol{I}_n,0_n,\ldots,0_n]^T_{n(2q+1)\times n},$$
$$\boldsymbol{Q}_{22}:=\begin{bmatrix}0_{nq\times nq} & 0_{nq\times n(q+1)}\\ \Lambda & 0_{n(q+1)\times n(q+1)}\end{bmatrix}_{n(2q+1)\times n\times(2q+1)},$$

and

$$\tilde{D}_{nc}=\operatorname{diag}(D_{\tau_1}\boldsymbol{I}_n,D_{\tau_2}\boldsymbol{I}_n,\ldots,D_{\tau_q}\boldsymbol{I}_n,\Delta_A,\Delta_{d1},\ldots,\Delta_{dq})$$
$$=\operatorname{diag}(D_{nc},\Delta_A,\Delta_{d1},\ldots,\Delta_{dq}).$$

Analogously, the condition for robust stability of the uncertain singular systems (1) of non-commensurate delays can be developed by following theorem.

Theorem 4: *Suppose* $(\boldsymbol{E},\boldsymbol{A},\boldsymbol{A}_{dk})$ *is regular, impulse-free and stable independent of delay. The uncertain singular system* (1) *of non-commensurate delays is robustly regular, impulse-free and stable independent of delay if*

$$\rho(A_{22}^{-1}\Delta A_{22})<1\quad \textit{and}\quad \mu_{\Omega_4}(\boldsymbol{Q}_{nc}(j\omega))<1,\quad \forall\omega\geq 0,$$

where

$$\Omega_4=\left\{\operatorname{diag}(\delta_1\boldsymbol{I}_n,\ldots,\delta_q\boldsymbol{I}_n,\Delta_1,\ldots,\Delta_{q+1}),\ \delta_i\in\boldsymbol{C},\Delta_j\in\boldsymbol{C}^{n\times n},\ |\delta_i|\leq 1,\ \bar{\sigma}(\Delta_j)\leq 1\right\}$$

is the set of block diagonal uncertainty structure and $\boldsymbol{Q}_{nc}(j\omega)=\boldsymbol{Q}_{22}+\boldsymbol{Q}_{21}(j\omega\boldsymbol{E}-\boldsymbol{Q}_{11})^{-1}\boldsymbol{Q}_{12}$.

Proof: The proof can be followed by Lemma 6 and the analysis of a similar process to that of Theorem 3, and hence omitted. □

5. Numerical examples

In this section, we consider two numerical examples for illustration.

Example 1: Consider the uncertain singular system of commensurate delays in (1) with

$$\boldsymbol{E}=\begin{bmatrix}1&0&0\\0&0&1\\0&0&0\end{bmatrix},\quad \boldsymbol{A}=\begin{bmatrix}-1&0&-3\\3&0&-1\\0&-1&1\end{bmatrix},$$
$$\boldsymbol{A}_{d1}=\begin{bmatrix}0.4&0.3&0.1\\0.1&0&-0.5\\0&0&0.3\end{bmatrix},\quad \boldsymbol{A}_{d2}=\begin{bmatrix}0.1&0&0.2\\0.1&0&0\\0&0&0.1\end{bmatrix},$$
$$\Delta\boldsymbol{A}=\begin{bmatrix}0&0&0.25\\0.1&0&0.2\\0&0.18&0.1\end{bmatrix},\quad \Delta\boldsymbol{A}_{d1}=\begin{bmatrix}0.05&0&0\\0&0&-0.05\\0&0&0.04\end{bmatrix},$$
$$\Delta\boldsymbol{A}_{d2}=\begin{bmatrix}0.01&0&0\\0.02&0&0\\0&0&0.01\end{bmatrix},$$

It is easy to show that the system $(\boldsymbol{E},\boldsymbol{A})$ is regular, impulse-free and stable with the set of generalised eigenvalues as $\{-1\pm 3j\}$. Consequently, the nominal singular time-delay system $(\boldsymbol{E},\boldsymbol{A},\boldsymbol{A}_{dk})$ is regular and impulse-free by Lemma 1. With the LFT representation in (7), the spectral radius of the frequency dependent matrix of interest is $\sup_{\omega\geq 0}\rho(\boldsymbol{M}_C(j\omega))=0.5252$. By Theorem 1, $(\boldsymbol{E},\boldsymbol{A},\boldsymbol{A}_{dk})$ is stable independent of delay. As to robustness analysis, two unitary matrices

$$\boldsymbol{U}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix},\quad \boldsymbol{V}=\begin{bmatrix}1&0&0\\0&0&-1\\0&1&0\end{bmatrix}$$

are used to perform a singular value decomposition for matrix $\boldsymbol{E}$. Then we can obtain $\rho(A_{22}^{-1}\Delta A_{22})=0.18$. Hence, the considered system is robustly regular, impulse-free by Lemma 6. Moreover, we can get $\bar{\sigma}(\Delta\boldsymbol{A})\leq\alpha_A=0.3462$, $\bar{\sigma}(\Delta\boldsymbol{A}_{d1})\leq\alpha_{d1}=0.0640$ and $\bar{\sigma}(\Delta\boldsymbol{A}_{d2})\leq\alpha_{d2}=0.0224$, which are needed in constructing the LFT representation in (20). In Theorem 3, the structured singular value of the frequency dependent matrix of interest is $\sup_{\omega\geq 0}\mu_{\Omega_3}(\boldsymbol{P}_C(j\omega))=0.9688<1$ where Ω_3 is the set of block diagonal uncertainties $\{\operatorname{diag}(\delta\boldsymbol{I}_6,\Delta_1,\Delta_2,\Delta_3)\colon \delta\in\boldsymbol{C},\Delta_j\in\boldsymbol{C}^{3\times 3},|\delta|\leq 1,|\Delta_j|\leq 1\}$. Hence, the uncertain singular time-delay system is robustly regular, impulse-free and stable independent of delay.

Example 2: Consider the case of non-commensurate delays in (1) with the same parameters of system

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matrices in Example 1. As shown in Example 1, the nominal singular time-delay system $(\boldsymbol{E}, \boldsymbol{A}, \boldsymbol{A}_{dk})$ is regular and impulse-free by Lemma 1, and the considered uncertain system is robustly regular and impulse-free based on Lemma 6.

By Theorem 2, the structured singular value of interest is $\sup_{\omega \geq 0} \mu_{\Omega_2}(\boldsymbol{N}_{nc}(j\omega)) = 0.4528 < 1$, where Ω_2 is the block diagonal uncertainty set $\{\text{diag}(\delta_1 \boldsymbol{I}_3, \delta_2 \boldsymbol{I}_3)\text{: } \delta_i \in \boldsymbol{C}, |\delta_i| \leq 1\}$. Hence, the nominal singular time-delay system $(\boldsymbol{E}, \boldsymbol{A}, \boldsymbol{A}_{dk})$ is regular, impulse-free and stable independent of delay. The structured singular value of interest in Theorem 4 is $\sup_{\omega \geq 0} \mu_{\Omega_4}(\boldsymbol{Q}_{nc}(j\omega)) = 0.9898 < 1$ where Ω_4 is the set of block diagonal uncertainties $\{\text{diag}(\delta_1 \boldsymbol{I}_3, \delta_2 \boldsymbol{I}_3, \Delta_1, \Delta_2, \Delta_3)\text{: } \delta_i \in \boldsymbol{C}, \Delta_j \in \boldsymbol{C}^{3\times 3}, |\delta_i| \leq 1, \bar{\sigma}(\Delta_j) \leq 1\}$. Therefore, the considered uncertain singular system of non-commensurate delays is robustly regular, impulse-free and stable independent of delay.

6. Conclusions

Based on LFT representations of linear continuous-time singular systems with state delays and bounded uncertainties, this article contributes to linking the LFT framework, robust nonsingular and μ-analysis to the robustness analysis of regularity, impulse-immunity and stability independent of delay. The systematic approach can be applied to deal with the uncertain singular systems of commensurate and non-commensurate delays and derive the conditions for preserving the considered properties. A further investigation of extending the proposed method to delay-dependent conditions is ongoing.

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