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變形化多相取樣之重建

楊慶祥

崑山科技大學電子工程系 副教授

摘要

本文推導出變形化多相取樣之重建公式。此取樣方法為一簡單且有效率之方法，而且其重建公式對於其設置具有非常大之幫助。本文最後結果顯示，所有平行處理的程序工作在一個比較低的頻率，但是需要處理取樣極限高頻的能力。

關鍵字：取樣、薛能取樣定理、類比數位轉換器、變形、變形化多相取樣

INTRODUCTION

A band-limited continuous-time signal, with bandwidth f_b, can be recovered from its sampled one provided that the sampling frequency f_s is larger than $2f_b$. This is the celebrated Shannon's sampling theorem [1,2]. It sets a lower limit of sampling frequencies for more and more high frequencies of modern signal processing. In the area of ultra-wide-band communications, the sampling frequencies are generally required to be in the range of GHz [3]. The analog-to-digital converters (ADC) therefore encounter great difficulties in the situations of similar constraints.

Numerous attempts have been reported to overcome these conditions. Non-uniform sampling can do a favor for anti-aliasing [4,5]. Randomized sampling can add another aid [6]. Phase-shifted sampling has been understood to be very helpful to recover aliased signals [7]. Filter banks consist of many sub-bands so that everyone can be sampled at a lower frequency [3]. Hybrid sampling [8] improves pseudo-randomized sampling and avoids fuzzy aliasing [9]. The time interleaved method [10] takes advantage of an array of samplers which operate concurrently at a lower frequency. This is a parallel computing approach. Another one is the aliased polyphase sampling (APS) [11].

The next section will show the reconstruction formula for the APS and then a conclusion section follows.

II. RECONSTRUCTION OF THE APS

Let x(t) be a band-limited continuous-time signal with bandwidth f_b, and X(f) be its Fourier transform. The APS form of sampled x(t) [11] is

$$x^*(t) = \sum_{n=-\infty}^{\infty}\sum_{m=0}^{M-1} x(t)\delta\left(t - nT_s - \frac{m}{M}T_s\right) \ , \tag{1}$$

where δ is the impulse function. Using the symbol $\mathcal{F}$ for Fourier transform and the symbol $\mathcal{F}^{-1}$ for inverse Fourier transform, one takes Fourier transform on (1) and obtains

$$\begin{aligned} X^*(f) &= \mathcal{F}\{x^*(t)\} = \\ &\sum_{n=-\infty}^{\infty}\sum_{m=0}^{M-1} \mathcal{F}\{x(t)\}◎\mathcal{F}\{\delta\left(t - nT_s - \frac{m}{M}T_s\right)\} = \\ &\sum_{n=-\infty}^{\infty}\sum_{m=0}^{M-1} \int_{-\infty}^{\infty} X(v)e^{-j2\pi(f-v)(nT_s+\frac{m}{M}T_s)}\, dv = \\ &\sum_{n=-\infty}^{\infty}\sum_{m=0}^{M-1} x(nT_s + \frac{m}{M}T_s)\, e^{-j2\pi f(nT_s+\frac{m}{M}T_s)}, \end{aligned} \tag{2}$$

where the "◎" on the second line in (2) represents Fourier convolution. The X*(f) is a Fourier series and a periodic function on the variable f with period Mfs (fs = 1/Ts). One can derive

$$X^*(f) = x(f), -Mf_s \le f \le Mf_s, \tag{3}$$

$$X^*(f \pm Mf_s) = X^*(f), \tag{4}$$

In order to recover the original signal, one utilizes the following low-pass filter, which is shown in Fig. 1.

$$H(f) = \begin{cases} 1, -f_b \le f \le f_b \\ 0,\ \text{otherwise} \end{cases} , \tag{5}$$

By (3), (4), and (5), the original signal can be recovered as

$$\begin{aligned} x(t) &= \mathcal{F}^{-1}\{X(f)\} = \mathcal{F}^{-1}\{X^*(f)H(f)\} \\ &= x^*(t)◎\sin(2\pi f_b\, t)/\pi t = \\ &\sum_{m=0}^{M-1}\sum_{n=-\infty}^{\infty} x\left(nT_s + \frac{m}{M}T_s\right) \times \\ &\qquad \mathrm{sinc}(2\pi f_b(t - \left(nT_s + \frac{m}{M}T_s\right))), \end{aligned} \tag{6}$$

where sinc(u) = sin(u)/u.

One can define the one-component signal

$$\begin{aligned} x_m(t) = &\sum_{n=-\infty}^{\infty} x\left(nT_s + \frac{m}{M}T_s\right) \times \\ &\qquad \mathrm{sinc}(2\pi f_b(t - \left(nT_s + \frac{m}{M}T_s\right))) \end{aligned} \tag{7}$$

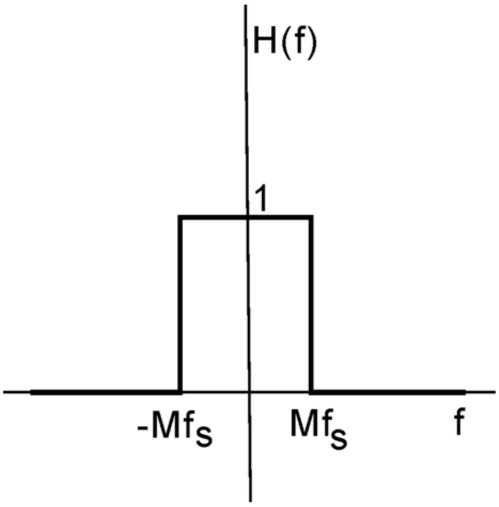

Fig. 1. The low-pass filter for reconstruction.

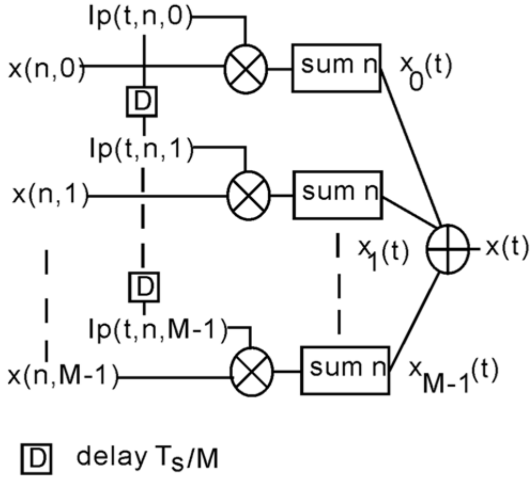

Fig. 2. The block diagram of the reconstruction processes

The original signal now becomes

$$x(t) = \sum_{m=0}^{M-1} x_m(t) \tag{8}$$

This reconstruction process, formulated from (3) to (8), is depicted in Fig. 2. The x(n,m) is defined in (9), and the Ip(t,n,m) is in (10).

$$x(n,m) = x\left(nT_s + \frac{m}{M}T_s\right) \tag{9}$$

$$Ip(t,n,m) = \mathrm{sinc}(2\pi f_b(t - \left(nT_s + \frac{m}{M}T_s\right))) \tag{10}$$

The sampled discrete data x(n,m) is interpolated by Ip(t,n,m). This interpolation is the key process to obtain the original analog signal. However, the condition (11) for being free of aliasing is still implicitly implied by (2) to (5). This is exactly what the sampling theorem says.

$$Mf_s \geq 2f_b \tag{11}$$

For the one-component recovery (7), the sampling frequency f_s may not exceed $2f_b$, but the summation (8) can eliminate aliasing and regain the original signal on condition that (11) is satisfied. Nevertheless, every one-component recovery must be able to recognize the time interval T_s/M, which means the capability to deal with the sampling rate Mf_s.

III. DEMONSTRATION

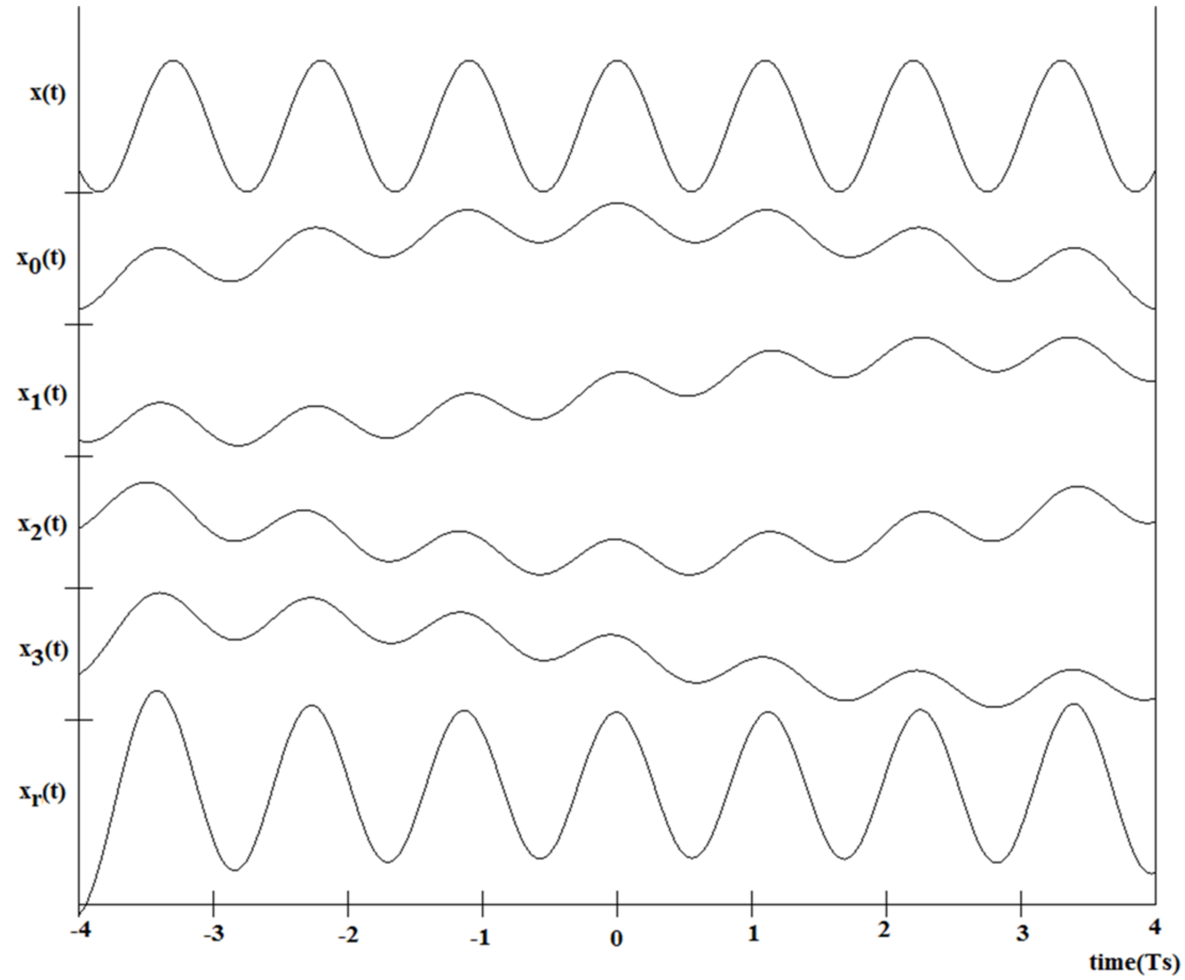

Fig. 3 X(t) is the original signal. $X_m(t)$ (M = 4) is defined in (7). $X_r(t)$ is the reconstructed signal.

In Fig. 3, T_b = 1.1 ms. T_s = 1.0 ms. x(t) = cos(6.2832t/T_b). $x_m(t)$ (m = 0, 1, 2, 3), defined in (7), is aliased. But the reconstructed signal $x_r(t)$ is not aliased.

IV. CONCLUSION

Many methods have been developed to handle the sampling of more and more high frequencies. Utilizing the parallel computing spirit, the aliased polyphase sampling is a simple and efficient solution. The reconstruction formula is deduced and formulated explicitly, which can lead to an easy implementation of it. The final results show that the capability to deal with the high frequencies is needed, but every process of the parallel processes is indeed in a lower frequency. This paper provides a different theory on the aliased polyphase sampling in addition to that in [11] and a new reconstruction formula for the aliased polyphase sampling.

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Reconstruction of the Aliased Polyphase Sampling

C. S. Yang

Department of Electronic Engineering, Kun Shan University

Abstract

This paper deduces the reconstruction formula for the aliased polyphase sampling. This sampling method is a simple and efficient one, and its reconstruction formula can do a great favor with its implementation. The final results show that every process of the parallel processes is working in a lower frequency, but needs the capability to deal with the high frequency of sampling limit.

Keywords: Sampling, Shannon's sampling theorem, ADC, Aliasing, Aliased polyphase sampling