Real-Time Adjustment Underwater Positioning System for Submarines

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Real-Time Adjustment Underwater Positioning System for Submarines

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A rapid real-time adjustment scheme is proposed for improving the precision of the conventional short-base line (SBL) positioning fix system used by submarines and other underwater vehicles. In the proposed approach, an initial position estimate is obtained by solving the conventional SBL tracking equations of the submarine given the assumptions of a constant speed of sound in water and a straight-line propagation path. In the first stage of the real-time adjustment procedure, this initial estimate is corrected using an iterative computation scheme based on a 3D geometry model. The improved position estimate is then used to compute a new, more accurate value of the speed of sound in water. Finally, in the second stage of the real-time adjustment procedure, the corrected speed of sound in water and the discrepancy between the original and corrected position estimates obtained in the first adjustment procedure are applied to update the coordinates of the submarine based on the second signal received from the pinger. The numerical results show that the proposed real-time adjustment system yields a significant improvement in the accuracy of the positioning fix estimates compared to those obtained from the conventional SBL method or the SBL method with the first adjustment procedure only.

Keywords Submarines, real-time, SBL, pinger, iterative

1. Introduction

Acoustic positioning systems are not only a prerequisite for navigation purposes, but they are also essential in ensuring the safety of the submarine when performing maneuvers within confined waters (see Figure 1). Thus, the speed and precision of the positioning fix system have a fundamental effect on the ability of the submarine to complete its mission in a safe and timely manner.

The literature contains many proposals for enhancing the performance of traditional underwater sonar/acoustic positioning systems. For example, Young (1973) utilized the least-squares method to correct the position coordinates obtained from a traditional geometry model. Lee (1975) applied a discrete Kalman filtering approach to optimize the position estimates obtained from the conventional long-base line (LBL) system. Stockton et al. (1975) and Urick (1979) showed that the accuracy of the position fix estimates obtained...
from the short-base line (SBL) method is degraded by the ever-changing speed of sound underwater caused by variations in pressure, temperature, and so on. Sargent and Cowgill (1976) pointed out that the accuracy of the SBL positioning system also depends on errors in the submarine’s signal reception, which are affected by the sea waves. McCloskey and Evans (1977) applied a Kalman filtering scheme to compensate for errors in a positioning system used for underwater coal mining. Cestone et al. (1977) employed the least-squares method to minimize the error in the positioning estimates obtained from an LBL acoustic navigation system. Kemp and Pearson (1982) showed that the accuracy of SBL positioning systems is degraded by the limitations of the transponder system on the vessel and the movements of the vessel and used a Kalman filter to optimize the performance of various underwater acoustic positioning systems. Jackson et al. (1989) discussed the basic parameters involved in the design of sonar arrays for submarine navigation purposes. Murphy and Hannuksela (1990) compared and discussed the accuracy and range of various navigation systems. Gembarski (1992) argued that errors inevitably exist between the actual submarine position and that calculated using the traditional SBL method.

In general, the enhanced positioning systems proposed in the studies above are based on the use of nonreal-time hydrographic data. In practice, however, the underwater acoustic speed varies in real-time in accordance with changes in the geographic location, and water depth, respectively. Thus, the reliability of many of the enhanced positioning systems presented in the literature is inevitably somewhat limited. Furthermore, the potential for improving the speed of the positioning fix system by utilizing the corrected average speed of sound in water obtained in the previous positioning fix does not appear to have been addressed in the literature. As a consequence, both the accuracy and the speed of the positioning fix systems proposed in the literature are suboptimal for underwater missions, particularly those conducted in underwater environments characterized by rapid variations in the speed of sound, ray bending, and so forth. To resolve this problem, the present study proposes a real-time adjustment positioning system designed to enhance the performance
of the conventional SBL system. The two adjustment procedures in the proposed system can be summarized as follows:

a. **First adjustment procedure**: a 3D geometry model and an iterative computation scheme are used to improve the initial position estimates obtained from the conventional SBL method. The updated position coordinates are then used to compute a new, more accurate value of the speed of sound in water.

b. **Second adjustment procedure**: The updated value of the speed of sound in water and the discrepancy between the initial and final positioning fix estimates obtained in the first adjustment procedure are used to compensate the initial position estimates obtained in the next positioning fix process. As a result, the need for an iterative computation process is significantly reduced and the accuracy of the final position estimates is improved.

The real-time adjustment positioning procedure is based on three principal theorems/techniques, namely, the tracking equations proposed by Sargent and Cowgill (1976), basic 3D geometry relation theory, and an iterative correction technique. In the proposed approach, the tracking equations are used to obtain an initial position estimate in accordance with the conventional SBL method. 3D geometry theory is then applied to estimate the amount by which the initial position coordinates should be adjusted. Finally, an iterative scheme is applied to yield the final value of the position coordinates.

The remainder of this paper is organized as follows. Section 2 describes the basic principles of the proposed real-time adjustment positioning system and presents the corresponding theoretical analysis. Section 3 evaluates the performance of the proposed system in the context of a hypothetical problem in which a search submarine is dispatched to look for the flight data recorder on a crashed plane lying on the seabed. Section 4 compares the speed and accuracy of the proposed real-time adjustment positioning system with that of the traditional SBL method and the SBL method with a single-adjustment procedure, respectively. Finally, section 5 draws some brief conclusions.

### 2. Theoretical Principles of Real-time Adjustment Underwater Positioning System

Most underwater positioning fix systems are based on the sound waves emitted by sonar devices and bounced back from underwater targets. However, the sound waves do not propagate along a perfectly straight line as they travel through the water, and thus an error is induced in the estimated distance between the submarine and the target with a pinger. As a result, the estimated position of the submarine is also subject to an inevitable degree of error. To resolve this problem, the present study integrates the conventional SBL positioning system with a dual-stage adjustment procedure. In the SBL positioning system, the known speed of sound in water, which is obtained from conventional hydrographic data, is used to calculate the distance between the submarine and a pinger on the seabed, and an initial estimate of the target coordinates is then obtained using conventional tracking equations. However, due to time-varying changes such as the speed of sound and ray bending in the ocean environment, the average speed of sound obtained from the hydrographic data may deviate from the actual speed, and thus the calculated target coordinates are subject to a certain amount of error. Accordingly, in the first stage of the proposed real-time adjustment process, the initial position estimate obtained from the SBL method is improved through the use of an iterative computation scheme based on a 3D geometry model, and the updated
target coordinates are then used to compute a new, more accurate value of the average speed of sound in water. The corrected value of the average speed of sound in water and the deviation between the initial and final estimates of the position coordinates obtained in the first adjustment procedure are used in the second stage of the real-time adjustment process to reduce the time required to compute the next estimate of the target coordinates and to improve the precision of the estimation results. The details of the initial position estimation procedure and the first and second adjustment procedures are described in the sections below.

2.1. First Step: Initial Position Estimation

The real-time adjustment positioning system proposed in this study is based on the asymmetrical SBL positioning method (Kemp and Pearson 1982). In implementing the proposed method, it is assumed that three or more hydrophones are installed on different planes at the bottom of the submarine, as shown in Figure 2. Therefore, the plane formed by the three hydrophones installed at the bottom of the submarine is not parallel with the water surface. Based on the distances from each hydrophone to the target with a pinger on the seabed, three independent tracking equations can be obtained. These equations can then be solved simultaneously to obtain the position of the target relative to that of the submarine.

For convenience, let the center of the submarine represent the origin of the coordinate system, that is, \( O(0,0,0) \), and let the coordinates of the three hydrophones installed at the base of the submarine be denoted as \( A_1(x_1, y_1, z_1) \), \( A_2(x_2, y_2, z_2) \), and \( A_3(x_3, y_3, z_3) \), respectively, as shown in Figure 2. In determining their distance from the seabed, submarines generally

![Figure 2. Arrangement of hydrophones.](image-url)
use a depth indicator composed of a bourdon tube pressure gauge to measure their distance $D_u(i)$ from the surface and then obtain the distance $D_d(i)$ to the seabed by subtracting $D_u(i)$ from the known sea depth $D(i)$ obtained from the published hydrographic data, as shown in Figure 3. In the present study, it is assumed that the underwater target is equipped with a pinger that broadcasts a signal periodically. Furthermore, the propagation times of the pinger signal to the three hydrophones installed on the submarine are denoted as $T_1(i)$, $T_2(i)$, and $T_3(i)$, respectively. Given the assumption that the pinger signal propagates along a straight line with a constant average speed of $\bar{v}(i)$ (Jackson et al. 1989), the distances between the three hydrophones and the pinger, that is, $r_1(i)$, $r_2(i)$, and $r_3(i)$, respectively, are given by

$$r_1(i) = \bar{v}(i)T_1(i) \ldots i = 1, 2, \ldots, n \ldots \tag{1}$$

$$r_2(i) = \bar{v}(i)T_2(i) \ldots i = 1, 2, \ldots, n \ldots \tag{2}$$

$$r_3(i) = \bar{v}(i)T_3(i) \ldots i = 1, 2, \ldots, n \ldots \tag{3}$$

where $i$ represents the $i$th signal emitted from the pinger, that is, the $i$th positioning signal, and $T_j(i)$ is the time interval between two consecutive signals received by the $j$th hydrophone, where $j = 1, 2, 3$.

According to Cheng (2006), the tracking equations for the distances between the hydrophones and a pinger with coordinates $(x(i), y(i), z(i))$ relative to the center of the submarine are formulated as follows:

$$(x(i) - x_1)^2 + (y(i) - y_1)^2 + (z(i) - z_1)^2 = r_1^2(i) \tag{4}$$

$$(x(i) - x_2)^2 + (y(i) - y_2)^2 + (z(i) - z_2)^2 = r_2^2(i) \tag{5}$$

Figure 3. Three-dimensional configuration of underwater acoustic propagation.
Since the depth of the underwater target \( z(i) \) is much greater than the depth of the three hydrophones \( z_1, z_2, \) and \( z_3 \), it can be assumed for simplicity that \( z(i) - z_1 \approx z(i), \) \( z(i) - z_2 \approx z(i), \) and \( z(i) - z_3 \approx z(i). \) The following equations can then be obtained by subtracting Eqs. (5) and (6), respectively, from Eq. (4):

\[
2x(i) + 2y(i)(y_2 - y_1)/(x_2 - x_1) = (r_1^2(i) - r_2^2(i) + y_2^2 - y_1^2 + x_2^2 - x_1^2)/(x_2 - x_1) \tag{7}
\]

\[
2x(i) + 2y(i)(y_3 - y_1)/(x_3 - x_1) = (r_1^2(i) - r_3^2(i) + y_3^2 - y_1^2 + x_3^2 - x_1^2)/(x_3 - x_1) \tag{8}
\]

For convenience, let the following notations be introduced:

\[
a = (r_1^2(i) - r_2^2(i) + y_2^2 - y_1^2 + x_2^2 - x_1^2)/(x_2 - x_1) \tag{9}
\]

\[
b = (r_1^2(i) - r_3^2(i) + y_3^2 - y_1^2 + x_3^2 - x_1^2)/(x_3 - x_1) \tag{10}
\]

\[
c = (y_2 - y_1)/(x_2 - x_1) \tag{11}
\]

\[
d = (y_3 - y_1)/(x_3 - x_1) \tag{12}
\]

Solving Eqs. (7) and (8) simultaneously, the position coordinates \((x(i), y(i), z(i))\) of the target relative to the center of the submarine are derived as follows:

\[
x(i) = (ad - bc)/(2(d - c)) \tag{13}
\]

\[
y(i) = (a - b)/(c - d) \tag{14}
\]

\[
z(i) = (r_1^2(i) - (x(i) - x_1)^2 - (y(i) - y_1)^2)^{1/2} \tag{15}
\]

In deriving equations (13) to (15) above, the speed of sound in water is easily obtained from the published hydrographic data and the signal emitted by the pinger is assumed to propagate along a straight line. However, in practice, the speed of sound in water is time-varying and differs slightly from that given in the hydrographic data. In addition, sound does not propagate in a perfectly straight line when traveling through water. As a result, the position coordinates of the pinger relative to the center of the submarine calculated in Eqs. (13) through (15) deviate slightly from the true coordinates. Therefore, the present study proposes the following adjustment procedure to correct the calculated coordinates and to accelerate the positioning fix procedure, respectively.

### 2.2. Second Step: Real-time Adjustment Positioning Procedure

The real-time adjustment positioning procedure involves two stages. In the first stage, a 3D geometry model and an iterative calculation procedure are used to correct the target coordinates obtained in Eqs. (13) through (15) and to compute a new, improved estimate of the actual underwater speed of sound. In the second stage, the time required to update the positioning fix estimate based on the second positioning signal received from the target is reduced by using the error between the initial and final estimates of the target coordinates obtained in the first positioning fix procedure to compensate the new estimated distances between the hydrophones and the target.

#### 2.3.1. First Adjustment Procedure: Correction of Underwater Acoustic Propagation Errors in Real-time

In correcting for the overly simplistic assumptions in the SBL method of a
Underwater Positioning System for Submarines

constant speed of sound in water and a straight-line propagation path, the depth $D_d(i)$ between the submarine and the pinger on the seabed is taken as a reference to verify the calculated depth $z(i)$ obtained in Eq. (15) and to correct the coordinates $x(i)$ and $y(i)$ obtained in Eqs. (13) and (14), respectively. Let the difference between the calculated depth, $z(i)$, and the known depth, $D_d(i)$, be defined as

$$|z(i) - D_d(i)| = \delta$$  \hspace{1cm} (16)

In implementing Eq. (16), two conditions may arise, namely:

1. $\delta \leq 0.001$ m; that is, the calculated depth $z(i)$ is virtually identical to the known depth $D_d(i)$. The criterion given in Eq. (16) is used as an initial convergence criterion to determine whether further processing is required. In other words, when meeting this condition, $\delta \leq 0.001$ m, the calculated position of the target relative to the submarine is very close to the true position and the corresponding positioning fix is completed.

2. $\delta \geq 0.001$ m: that is, the calculated depth $z(i)$ differs from the known depth $D_d(i)$. In other words, the calculated position of the target relative to the submarine differs from the true position. In practice, two scenarios may exist here, namely:

   (a) $z(i) < D_d(i)$, that is, the depth is underestimated; see error point $E$ in Figure 3.

   (b) $z(i) > D_d(i)$, that is, the depth is overestimated; see error point $G$ in Figure 3.

Since in both cases the error distance is much less than the distance between the origin of the coordinate frame, $O$, and the target at point $F$, the segment $OF$ is much greater than either $EF$ or $GF$. Therefore, an assumption can be made that points $O$, $E$, $F$, and $G$ all lie along the same straight line such that three similar triangles, $\Delta OPE$, $\Delta OQF$ and $\Delta ORG$, are formed. In accordance with the principles of the geometrical structure, the error amounts in the calculated values of the x- and y-coordinates of the target, i.e., $\Delta x(i)$ and $\Delta y(i)$, respectively, can be derived as follows:

1. When $z(i)$ is less than $D_d(i)$,

   $$\Delta x(i) = |OF - OE| \sin \alpha(i) \frac{x(i)}{\sqrt{x^2(i) + y^2(i)}}$$  \hspace{1cm} (17)

   $$\Delta y(i) = |OF - OE| \sin \alpha(i) \frac{y(i)}{\sqrt{x^2(i) + y^2(i)}}$$  \hspace{1cm} (18)

   where the angle $\alpha(i)$ is the included angle between the z-axis and $OE$.

2. When $z(i)$ is greater than $D_d(i)$,

   $$\Delta x(i) = |OF - OG| \sin \alpha(i) \frac{x(i)}{\sqrt{x^2(i) + y^2(i)}}$$

   $$\Delta y(i) = |OF - OG| \sin \alpha(i) \frac{y(i)}{\sqrt{x^2(i) + y^2(i)}}$$

   where the angle $\alpha(i)$ is the included angle between the z-axis and $OG$.

Having computed these error amounts, the initial estimates of the target coordinates obtained from the SBL method can be corrected as follows.
1. If $z(i)$ is less than $D_d(i)$, the error amounts are added to the coordinates of point $E$, that is,

$$
x'(i) = x(i) + \Delta x(i) \\
y'(i) = y(i) + \Delta y(i)
$$

(19)

2. If $z(i)$ is greater than $D_d(i)$, the error amounts are subtracted from the coordinates of point $G$, that is,

$$
x'(i) = x(i) - \Delta x(i) \\
y'(i) = y(i) - \Delta y(i)
$$

(20)

Coordinates $x'(i)$ and $y'(i)$ are then used to calculate the corrected coordinate $z'(i)$ in accordance with

$$
 z'(i) = (x'^2(i) + y'^2(i))^{1/2} \tan \alpha(i)
$$

(21)

Eqs. (19) through (21) yield an updated (improved) set of coordinates $(x'(i), y'(i), z'(i))$ for the underwater sound-emitting device located at point $F$. Substituting these coordinates into Eq. (4), the new value of the distance between the target and hydrophone $A_1$ is obtained as

$$
 r'_1(i) = [(x'(i) - x_1)^2 + (y'(i) - y_1)^2 + (z'(i) - z_1)^2]^{1/2}
$$

(22)

From Eq. (22), the average speed of underwater sound can then be corrected as follows:

$$
 \bar{v}'(i) = r'_1(i)/T_1(i)
$$

(23)

The modified average speed of sound $\bar{v}'(i)$ is then used to repeat all the calculation formulae described above until the following convergence condition is achieved:

$$
 \left| \frac{\bar{v}_f(i) - \bar{v}'_f(i)}{\bar{v}_f(i)} \right| \leq 0.001
$$

(24)

where $\bar{v}_f(i)$ denotes the corrected average speed of sound obtained in the final iteration while $\bar{v}'_f(i)$ denotes the corrected average speed of sound obtained in the final iteration but one. As shown in the lower-right corner of Figure 4, the first adjustment procedure outputs both an improved estimate of the position of the target relative to the submarine and a more accurate value of the average speed of sound in water.

Clearly, the relative distance between the submarine and the target changes over time as the submarine completes its mission. Therefore, the initial positioning fix and adjustment procedures described above should be repeated each time a signal is received from the pinger. Although the adjustment procedure described above improves the precision of the initial positioning estimate, the need for an iterative calculation procedure delays the positioning fix process and may therefore degrade the accuracy of the positioning estimates due to instantaneous changes in the ocean environment. Accordingly, in the second adjustment procedure, the positioning error associated with the initial position estimate and correction procedure is used to compensate the estimated distances between the submarine and the target based on the second signal received from the pinger.
2.4.2 Second Adjustment Procedure: Correction of Underwater Acoustic Propagation Errors per Unit Time in Real-time. From the discussions above, it is clear that the number of iterations required to compute the updated values of the target position depends fundamentally upon the magnitude of the error between the initial estimate of the target coordinates and the actual target coordinates. Consequently, the number of iterations required to obtain the new position of the target based on the second signal received from the pinger is reduced by using the error amount in the initial estimated distances between the pinger and the three hydrophones obtained at the beginning of the positioning fix to modify the calculated distances between the pinger and the submarine in the second (and all subsequent) fix procedures. Let the following unit-time error $e_j(i)$ (referred to hereafter as the error coefficient) be defined as:

$$e_j(i) = \frac{r_j(i) - \tilde{r}_j(i)}{T_j(i)}, \quad j = 1, 2, 3$$

(25)
where $j$ indexes the hydrophones installed on the submarine, $T_j(i)$ denotes the propagation time from the pinger to the $j$th hydrophone in the $i$th fix, $r_j'(i)$ denotes the corrected relative distance between the pinger and the $j$th hydrophone obtained at the end of the $i$th fix and $r_j(i)$ represents the initial estimate of the relative distance between the pinger and the $j$th hydrophone obtained at the beginning of the $i$th fix. Thus, for the second fix (and all successive fixes), the initial estimates of the distances between the underwater pinger and the three hydrophones can be obtained as

\begin{align}
    r_1(i+1) &= \bar{v}(i+1)T_1(i+1) - T_1(i+1)e_1(i) \\
    r_2(i+1) &= \bar{v}(i+1)T_2(i+1) - T_2(i+1)e_2(i) \\
    r_3(i+1) &= \bar{v}(i+1)T_3(i+1) - T_3(i+1)e_3(i)
\end{align}

Eqs. (26), (27), and (28) can then be used in place of Eqs. (1), (2), and (3), respectively, in the first step of the second (and later) positioning fix. The remaining fix procedures are then the same as those described above for the initial position estimation and first adjustment procedure, respectively (see Figure 4). The second adjustment procedure reduces the number of iterations required to obtain the corrected target position since the initial estimate of the target position is closer to the actual position than in the original position estimation and correction procedure. Consequently, the iteration time is reduced, and all of the position fixes other than the initial fix are not only more accurate than those obtained using the first adjustment procedure only but also are obtained more rapidly.

### 3. Numerical Evaluation

To verify the performance of the proposed real-time adjustment positioning system, this section considers a hypothetical case in which a submarine equipped with the proposed positioning system is dispatched to search for an airplane that has crashed and sunk to the bottom of the ocean. The relative positions of the flight data recorder, the submarine, the hydrophones installed on the bottom of the submarine, and the ocean environment are illustrated in Figure 5. The position estimation process commences the moment the rescue submarine receives the first signal from the flight data recorder. Assume that the relative coordinates between the center of the submarine and the three hydrophones are given by $A_1(5.0 \text{ m}, 1.0 \text{ m}, 0.1 \text{ m}), A_2(-5.0 \text{ m}, 1.5 \text{ m}, 0.2 \text{ m})$ and $A_3(-8.0 \text{ m}, -2.0 \text{ m}, 0.1 \text{ m})$, respectively. Assume also that the propagation times of the first signal transmitted from the flight recorder and received at the three hydrophones are measured as $T_1(1) = 0.1259s$, $T_2(1) = 0.1274s$, and $T_3(1) = 0.1283s$, respectively. In addition, in determining the initial estimate of the target position, the average speed of sound is assigned a value of $1465 \text{ ms}^{-1}$ in accordance with the hydrographic data. Finally, the initial submergence depth $D_u(1)$ of the rescue submarine is measured as 84.0m from a depth indicator while the distance between the rescue submarine and the seabed $D_d(1)$ is calculated as 180.0 m by subtracting $D_u(i)$ from the known sea depth $D(i)$. Note that all the initial parameter settings considered in this hypothetical example are summarized in Table 1a. In accordance with the dual-step positioning system described in the previous section, the relative position between the rescue submarine and the flight data recorder is determined as follows.
Table 1
Numerical evaluation of proposed real-time adjustment positioning system

(a) First positioning \((i = 1)\)

<table>
<thead>
<tr>
<th>Hydrophone coordinates</th>
<th>No.A₁</th>
<th>No.A₂</th>
<th>No.A₃</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x_{1})</td>
<td>5.0 m</td>
<td>(x_{2})</td>
<td>5.0 m</td>
</tr>
<tr>
<td>(y_{1})</td>
<td>1.0 m</td>
<td>(y_{2})</td>
<td>1.5 m</td>
</tr>
<tr>
<td>(z_{1})</td>
<td>0.1 m</td>
<td>(z_{2})</td>
<td>0.2 m</td>
</tr>
</tbody>
</table>

| Time of received signal | \(T_{1}(1) = 0.1259s\) | \(T_{2}(1) = 0.1274s\) | \(T_{3}(1) = 0.1283s\) |

| Assumed average underwater speed | \(\bar{v}(1) = 1465.0\text{ ms}^{-1}\) |

| Distance between submarine and seabed | \(D_{d}(1) = 180.0\text{ m}\) |

| First adjustment | |

<table>
<thead>
<tr>
<th>Solution</th>
<th>First iteration</th>
<th>Second iteration</th>
<th>Third iteration</th>
<th>Fourth iteration</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x(1) = 43.1058\text{ m})</td>
<td>(x'(1) = 43.9182)</td>
<td>(x'(1) = 43.9326)</td>
<td>(x'(1) = 43.9938)</td>
<td>(x'(1) = 43.9994)</td>
</tr>
<tr>
<td>(y(1) = 29.2834\text{ m})</td>
<td>(y'(1) = 29.9344)</td>
<td>(y'(1) = 29.9460)</td>
<td>(y'(1) = 29.9955)</td>
<td>(y'(1) = 29.9992)</td>
</tr>
<tr>
<td>(z(1) = 178.3517\text{ m})</td>
<td>(z'(1) = 179.8501)</td>
<td>(z'(1) = 179.8766)</td>
<td>(z'(1) = 179.9887)</td>
<td>(z'(1) = 179.9997)</td>
</tr>
<tr>
<td>(\bar{v}(1) = 1478.634\text{ ms}^{-1})</td>
<td>(\bar{v}'(1) = 1478.875)</td>
<td>(\bar{v}'(1) = 1479.896)</td>
<td>(\bar{v}'(1) = 1479.990)</td>
<td>(\bar{v}'(1) = 1479.999)</td>
</tr>
<tr>
<td>(\Delta z(1) = 1.6493\text{ m})</td>
<td>(\Delta z(1) = 0.1499\text{ m})</td>
<td>(\Delta z(1) = 0.1234\text{ m})</td>
<td>(\Delta z(1) = 0.0113\text{ m})</td>
<td>(\Delta z(1) = 0.0003\text{ m})</td>
</tr>
</tbody>
</table>

| Error Coefficient | |
| \(e_{1}(1) = 14.9324\text{ ms}^{-1}\) | |
| \(e_{2}(1) = 14.9952\text{ ms}^{-1}\) | |
| \(e_{3}(1) = 14.9984\text{ ms}^{-1}\) | |

(Continued on next page)
Table 1
Numerical evaluation of proposed real-time adjustment positioning system *(Continued)*

<table>
<thead>
<tr>
<th>Time of received signal</th>
<th>( T_1(2) = 0.1142\text{s} )</th>
<th>( T_2(2) = 0.1156\text{s} )</th>
<th>( T_3(2) = 0.1164\text{s} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relative distance</td>
<td>( r_1(2) = 170.7865 \text{m} )</td>
<td>( r_2(2) = 172.8662 \text{m} )</td>
<td>( r_3(2) = 174.054 \text{m} )</td>
</tr>
<tr>
<td>Measured depth</td>
<td>( D_d(2) = 165.0 \text{m} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Error coefficient</td>
<td>( e_1(1) = 14.9324 )</td>
<td>( e_2(1) = 14.9952 )</td>
<td>( e_3(1) = 14.9984 )</td>
</tr>
<tr>
<td>Corrected relative</td>
<td>( r_1'(2) = 169.0739 \text{m} )</td>
<td>( r_2'(2) = 171.1323 \text{m} )</td>
<td>( r_3'(2) = 172.3079 \text{m} )</td>
</tr>
<tr>
<td>distance</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Error &lt; 0.001 m</td>
<td>( \delta = 0.00015 \text{m} )</td>
<td>( y(2) = 20.8 \text{m} )</td>
<td>( z(2) = 164.99985 \text{m} \approx 165.0 \text{m} )</td>
</tr>
<tr>
<td>Solution</td>
<td>( x(2) = 36.1 \text{m} )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Second adjustment procedure obtains accurate solutions without need for further iterative processing.*
3.1. First Step: Determine Initial Position of Flight Data Recorder Relative to Submarine

Substituting the coordinates of the three hydrophones and the initial estimated distances obtained from Eqs. (1) through (3) into Eqs. (4) through (6), the tracking equations are obtained as follows:

\[
\begin{align*}
(x(1) - 5)^2 + (y(1) - 1)^2 + (z(1) - 0.1)^2 &= (118.215)^2 \\
(x(1) + 5)^2 + (y(1) - 1.5)^2 + (z(1) - 0.2)^2 &= (118.665)^2 \\
(x(1) + 8)^2 + (y(1) + 2)^2 + (z(1) - 0.1)^2 &= (116.925)^2
\end{align*}
\]

Solving the tracking equations above using Eqs. (7) through (16), the initial coordinates of the flight data recorder relative to the center of the submarine are found to be (43.1058 m, 29.2834 m, 178.3517 m). The depth indicator and the published hydrographic data have determined that the submarine is positioned at a perpendicular distance of \( D(1) = 180 \) m above the seabed. The depth obtained by the depth indicator and the published hydrographic data can be considered to be the true depth. Therefore, the error in the initial estimate of the target depth is equal to 1.65 m. This degree of error is unacceptable for most underwater positioning missions and thus an improved positioning fix is obtained by executing the real-time adjustment procedure, as described in the following section.

3.2. Second Step: Real-time Adjustment Procedure

3.2.1. First Adjustment Procedure: Correction of Underwater Acoustic Propagation Errors in Real-time. From Eqs. (17) and (18), the adjustment amounts for the initial estimates of the \( x \)- and \( y \)-coordinates of the target are determined to be \( \Delta x(1) = 0.3984 \) m and \( \Delta y(1) = 0.2706 \) m, respectively. Note that angle \( \alpha(1) \) in Eqs. (17) and (18) is equal to \( \cos^{-1} \left( \frac{178.35}{185.8} \right) = 0.2826 \). Since \( z(1) = 178.35 \) m is less than \( D_\text{D}(1) = 180.0 \) m, it follows that Eq. (19) should be applied to adjust the initial estimates of the target coordinates. Accordingly, the relative coordinates of the target in the \( x \)- and \( y \)-axis directions are modified to \( x'(1) = 43.50 \) m and \( y'(1) = 29.55 \) m, respectively. Substituting \( x'(1) \) and \( y'(1) \) into...
Eq. (21), the updated value of the z-coordinate is obtained as $z'(1) = 179.8501$ m. From Eq. (22), the distance between hydrophone $A_1$ and the flight data recorder is adjusted to $r_1'(1) = 186.16$ m. Finally, from Eq. (23), the updated value of the average speed of sound in water is obtained as $\bar{v}'(1) = 1478.63$ ms$^{-1}$. Since $\Delta z(1) = |z'(1) - D_d(1)| = 0.1499$ m does not satisfy the convergence constraint of 0.001 m defined in Eq. (16), the modified value of the average speed of sound, i.e., $\bar{v}(1) = 1478.63$ ms$^{-1}$, is substituted into Eq. (1), and the correction process is repeated. After a total of four iterations, the adjustment value $\Delta z(1)$ reduces from $1.6493$ m to 0.0003 m. Under these conditions, the modified value of the average speed of sound in water (1479.999 ms$^{-1}$) satisfies the constraint given in Eq. (24), and thus the corresponding values of the target coordinates relative to the submarine, that is, $(x(1), y(1), z(1)) = (43.9994$ m, 29.9992 m, 179.9997 m) (see Table 1a) are taken as the final estimated coordinates in the first adjustment procedure. Figure 6 illustrates the four solution points obtained during the iterative procedure, and Figure 7 summarizes the corresponding convergence histories of the x-, y-, and z-components of the target position and the average speed of sound, $\bar{v}(i)$, respectively. From Table 1a it can be seen that the x-, y-, and z-coordinates of the target relative to the submarine are evolved from initial values of (43.1058 m, 29.2834 m, 178.3517 m) before the first iteration of the adjustment procedure to final values of (43.9994 m, 29.9992 m, 179.9997 m) after the fourth (i.e., final) iteration. From Eq. (25), the error coefficients associated with the first positioning fix ($i = 1$) are obtained as $e_1(1) = 14.9324$ ms$^{-1}$, $e_2(1) = 14.9952$ ms$^{-1}$, $e_3(1) = 14.9984$ ms$^{-1}$, respectively. These coefficients are then used to compensate the initial estimates of the distances between the three hydrophones and the flight data recorder in the second positioning fix ($i = 2$) in accordance with Eqs. (26)–(28).

Figure 6. Identification of crashed airplane on seabed using iterative procedure with four correction steps.
3.2.2. Second Adjustment Procedure: Correction of Underwater Acoustic Propagation Errors per Unit Time in Real-time. Having completed the first positioning fix procedure, the submarine moves to new coordinates \((x(1), y(1), z(1)) = (43.9994\, \text{m}, 29.9992\, \text{m}, 179.9997\, \text{m})\) and commences the second positioning fix process based on the new signal received from the flight data recorder. Assume that the propagation times of the new signal received at the three hydrophones on the base of the submarine are measured as \(T_1(2) = 0.1142\, \text{s}\), \(T_2(2) = 0.1156\, \text{s}\), and \(T_3(2) = 0.1164\, \text{s}\), respectively, and the distance \(D_d(2)\) between the rescue submarine and the seabed is found from the depth indicator and the published hydrographic data to be 165 m (see Table 1b). In performing the second positioning fix...
procedure (and all subsequent fixes), Eqs. (26)–(28) are used in place of Eqs. (1)–(3) to obtain an initial estimate of the distances between the three hydrophones and the flight data recorder. Substituting the error coefficients $e_1(1) = 14.9324 \text{ ms}^{-1}$, $e_2(1) = 14.9952 \text{ ms}^{-1}$, and $e_3(1) = 14.9984 \text{ ms}^{-1}$ into Eqs. (26)–(28), the initial distances are found to be $r_1(2) = 170.7865 \text{ m}$, $r_2(2) = 172.8662 \text{ m}$, and $r_3(2) = 174.0540 \text{ m}$, respectively. From Eqs. (4) through (15), the relative coordinates of the flight data recorder are determined to be $x(2) = 36.1 \text{ m}$, $y(2) = 20.8 \text{ m}$, and $z(2) = 164.99985 \text{ m}$, respectively. The error between the reference depth and the calculated depth, that is, $\delta = 0.00015 \text{ m}$, is less then 0.001 m. Thus, the estimated depth position is sufficiently close to the actual depth position so that no further iterations are required to correct the estimated values of the target position. In other words, the use of the error coefficients obtained in the first positioning fix yields a significant improvement in both the precision and the speed of the second (and subsequent) positioning fix procedures.

4. Performance Comparison

Table 2 compares the performance of the proposed real-time adjustment positioning system with that of the traditional SBL method and the SBL method with a single-adjustment procedure, respectively, for the illustrative example considered in the previous section. It is observed that the traditional SBL system yields a significant depth error of 1.6483 m, i.e., $z(1) = 178.3517$. The relatively poor performance of the SBL system is to be expected due to its overly simplistic treatment of the propagation speed and direction of sound in water. By contrast, the two adjustment-based positioning fix procedures yield a notable reduction in the positioning error, namely 0.0003 m (first adjustment procedure only) and 0.00015 m (dual adjustment procedure), respectively. It can be seen that the single-adjustment procedure requires four iteration loops to obtain an improved estimate of the target position. However, having completed the first adjustment procedure, the second adjustment process requires no further iterations. In other words, the real-time adjustment procedure improves both the precision and the speed of the target estimation process in the second (and all subsequent) positioning fix processes.

Table 2

Comparison of conventional SBL positioning system, real-time adjustment SBL system with first-adjustment procedure only, and real-time adjustment SBL system with dual-adjustment procedure

<table>
<thead>
<tr>
<th>SBL positioning system</th>
<th>Conventional SBL system without adjustment</th>
<th>Real-time adjustment SBL system with only the first adjustment</th>
<th>Real-time adjustment SBL system with dual adjustment</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of iterative calculations</td>
<td>—</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>Calculated depth minus the known depth ($</td>
<td>Z - D_d</td>
<td>$)</td>
<td>1.6483 m</td>
</tr>
</tbody>
</table>
5. Summary

The ability of submarines to accomplish their underwater missions in a safe and timely manner fundamentally depends on the performance of the onboard positioning fix system. While submarines commonly utilize the traditional SBL method to evaluate their distance from a designated target, the SBL method not only assumes that sound propagates in a straight line through water, but also considers the speed of sound in water to be a nonvariable parameter. In practice, however, neither assumption is strictly true, and thus the position estimates obtained using the SBL method are subject to an inevitable degree of error. Accordingly, the present study has improved the positioning performance of the SBL method through the introduction of a real-time adjustment procedure. The major contributions of this study can be summarized as follows.

1. To compensate for the effects of the overly simplistic assumptions in the SBL method, the first adjustment process in the proposed real-time adjustment procedure corrects the errors in the initial estimates of the target position obtained using the SBL method utilizing a 3D geometry model and an iterative computation scheme. Having obtained an updated (improved) estimate of the target position relative to the submarine, the new target coordinates are then used to compute a new, improved value of the speed of sound in water. It has been shown that the position estimates obtained from the first adjustment procedure are significantly more accurate than those obtained from the conventional SBL method.

2. Although the first adjustment procedure achieves a notable improvement in the accuracy of the positioning fix results, the iterative computation process results in an inevitable delay in the positioning process. Consequently, a slight degradation in the positioning results may occur as a result of transient changes in the ocean environment. To resolve this problem, the second adjustment process utilizes the error between the initial and final position estimates obtained in the first adjustment procedure to compensate the initial position estimate when updating the submarine’s position based upon the second signal received from the target. The results have shown that the compensation procedure avoids the requirement for an iteration procedure in the second (and all subsequent) positioning fix processes and yields a significant improvement in the accuracy of the position estimates compared to those obtained when using the SBL method and the first adjustment procedure only.

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References


