

# Particle Swarm Optimization Techniques for the Traveling Routing Problem

Chung-Ling Yen<sup>1</sup> and Shih-Tang Lo<sup>2</sup>

Department of Real Estate Management, Department of Information Management,  
Kun Shan University, Taiwan

grace@mail.ksu.edu.tw<sup>1</sup>

edwardlo@mail.ksu.edu.tw<sup>2</sup>

Corresponding Author: edwardlo@mail.ksu.edu.tw

## Abstract

This research employs the meta-heuristic method of discrete particle swarm optimization (DPSO) to an application of traveling routing problems (TRP). The optimization procedure simulates the decision-making processes of swarm. And it is similar to other adaptive learning and artificial intelligence techniques such as ant colony optimization, simulated annealing and genetic algorithms. The objective is minimizing the total routing path and time of a trip. Experimentation results show that the algorithm is successful in finding solutions while applying discrete particle swarm. And the continuous variable particle swarm optimization (CPSO) also be implement in this paper, the simulation results also show CPSO is a better method in solving combination problems.

*Keywords:* Routing problem, Combination problem, Discrete particle swarm

## 1. Introduction

To find the efficient traveling routes has been studied for many years. Especially in the recent year, the mobile environments exist everywhere. The traveling The problem typically involves finding the minimum cost of the routes for a number of places or finding the minimum distance traveled distance. The process of selecting traveling routes allows the choice of any combination of places in determining the collection route. Therefore, the traveling routing problem is combinatorial optimization problems which the processing time for the problem will increase exponentially while more customers to be serviced. In addition, the TRP is closely related to the traveling salesman problem (TSP), the traveling routing problem is considered NP-hard [1]. The NP-hard problem cannot find the optimal solution within reasonable time; in fact there are no exact algorithms available that consistently solve problems with more than 50 to 75 cities [2]. For such problems, the heuristics method is considered a reasonable approach to the traveling routing problem and in this paper uses a particle swarm optimization (PSO) approach to find the solutions.

The particle swarm optimization is a meta-heuristic method, and can be applied extensively in solving many combinatorial problems. The PSO consists of a swarm of particles in the space; the position vector of a particle is indicated a solution. The PSO is initialized with a population of particles and searches for the best solution or schedule. In every generation or iteration, the local bests and global bests are determined through evaluating the performances, i.e., the fitness values of current population of particles [3]. Recently the inspired intelligence became increasingly popular through the development and utilization of intelligent paradigms in advanced information systems design. Among these approaches, there are some methods representing animal behavior, such as birds flocks or fish inspired particle swarm optimization [4], genetic algorithms [5][6], artificial immune systems [7][8] or ant colony system (ACS). ACS which are ants foraging behaviors to solve the optimization problem [9].

The rest of this paper is organized as follows: Section 2 derives the corresponding traveling routing problem according to the intrinsic constraints. Section 3 reviews discrete particle swarm optimization. The proposed discrete particle swarm approach will list in Section 4. The simulation examples and experimental results are presented in Section 5. The conclusion and future work is showed in Section 6.

## 2. Traveling Problem

The travel routing problem has been an interesting problem today. The travel routing problem can be described as finding the minimum distance or cost for a tourist. Mathematically, this system is described as a weighted graph  $G=(V, E)$  where the vertices are represented by  $V=\{v_0, v_1, \dots, v_n\}$ , and the edges are represented by  $E=\{(v_i, v_j), i \neq j\}$ . The  $v_0$  represent the starting address, a tourist starts and ending its route from starting place. The  $d_{ij}$

associated with each edge are represented distances of  $(v_i, v_j)$ , which is measured by Euclidean distance between these vertices. The  $t_{ij}$  associated with each edge are represented estimated traveling time of  $(v_i, v_j)$ , which is measured by the travel agency. Finally, There are some assumptions in this paper: (1) one place is visited only once by a tourist. (2) tourist traveling route must start and return to starting place,  $v_0$ . (3) The problem studied here is symmetrical where  $d_{ij}=d_{ji}$  for all  $i$  and  $j$ .

For example, there are 6 places to be visited, Table 1 shows the distance between these cities. The place 1 to place 2 the traveling distance is 70. Table 2 shows the traveling time between these places. These two tables' data show the real circumstance which there is different traffic condition. Our object is to minimize the total traveling time and distance.

Table 1: The distance matrix between the customers

	1	2	3	4	5	6
1	0	70	65	30	20	75
2	70	0	20	25	40	55
3	65	20	0	20	20	45
4	30	25	20	0	30	40
5	20	40	20	30	0	35
6	75	55	45	40	35	0

Table 2: The distance matrix between the places

	1	2	3	4	5	6
1	0	40	40	20	15	45
2	40	0	15	20	30	35
3	40	15	0	17	17	37
4	20	20	17	0	28	33
5	15	30	17	28	0	35
6	45	35	37	33	35	0

The traveling sequence can be represented as Table 3, the row represented the city number, and the column represented the visited sequence. The first place is the first place to be visited, following is the forth place, and so on.

Table 3: The solution representation

		Visited sequence					
		1	2	3	4	5	6
Place	1	1	0	0	0	0	0
	2	0	0	0	1	0	0
	3	0	0	1	0	0	0
	4	0	1	0	0	0	0
	5	0	0	0	0	0	1
	6	0	0	0	0	1	0

Table 4: The Place to be visited

Place#	1	2	3	4	5	6
Place name	hotel	Store market	Sea shore	Museum	Part	Culture center

A lot of research work has been presented about the vehicle routing problem [11][12] including approaches such as Tabu search [13][14], simulated annealing [15] and ant colony system[16][17]. The vehicle routing problem is a general problem of traveling problem. A limited amount of research is presented addressing the traveling routing problem solved by DPSO to solve the TRP.

### 3. Discrete Particle Swarm Optimization Method

The traditional particle swarm optimization is based on the continuous variables, called CPSO. There are a lot of related works using the CPSO to solve the combination problem. In this paper, The discrete particle swarm optimization is adopted in this work, which the position variable is binary value, 0 or 1. In this paper, we assumed there are  $N_p$  particles, and each particle searches for  $D = N \times N$  dimension space ( $N$ , the number of places). For the  $h$ th particle ( $h=1, \dots, N_p$ ), the position consists of  $N \times N$  components  $X_h = \{ X_{h11}, \dots, X_{hNN} \}$ ,  $X_{hij} \in \{0,1\}$ .  $X_{hij}=1$  mean that the  $i$ th place is  $j$ th traveling sequence ( $i, j=1, \dots, N$ ) of a trip. The velocity  $V_h = \{ V_{h11}, \dots, V_{hNN} \}$ , where  $V_{hij}$  is the velocity of component  $X_{hij}$ , and the individual experience is a position  $L_h = \{ L_{h11}, \dots, L_{hNN} \}$ , the local best solution for the  $h$ th particle. Additionally,  $G = \{ G_{11}, \dots, G_{NN} \}$  represents the global best experience shared among all the population of particles achieved so far. The mentioned parameters in above are used to calculate the updating of the  $V_h$ . component of the position. The velocity for the  $i$ th particle is shown in Eq. (1).

$$V_{hij}^{new} = wV_{hij} + c_1r_1(L_{hij} - X_{hij}) + c_2r_2(G_{ij} - X_{hij}) \tag{1}$$

where  $w$  is an inertia weight used to determine the influence of the previous velocity to the new velocity. The  $c_1$  and  $c_2$  are learning factors used to derive how the  $i$ th particle

approaching the position closes to the individual experience position or global experience position respectively. Furthermore, the  $r_1$  and  $r_2$  are the random numbers uniformly distributed in  $[0, 1]$ , influencing the tradeoff between the global and local exploration abilities during search.

Recall that particles are represented by binary variables. Kennedy and Eberhart claim that the higher velocity value is more likely to choose 1, while lower value favors the value of 0. Furthermore, they restrict the velocity values to the interval  $[0, 1]$  by using the following sigmoid function:

$$s(V_{hij}) = \frac{1}{1 + \exp(-V_{hij})} \quad (2)$$

where  $s(V_{hij})$  is defined as representing the probability of  $X_{hij} = 1$ . To avoid the value of  $s(V_{hij})$  approaching 0 or 1, a constant  $V_{max}$  is used to limit the range of  $V_{hij}$ . In practice,  $V_{max}$  and  $V_{min}$  is often set in discrete PSO, i.e.,  $V_{hij} \in [V_{min}, V_{max}]$ . For TRP, each place can only be visited once. In the proposed algorithm, each particle  $h$  places the unschedule place  $i$  to the sequence  $j$  according to the following probability equation [18]:

$$q_h(i, j) = \frac{s(V_{hij})}{\sum_{j \in U} s(V_{hij})} \quad (3)$$

$U$  is the set of unscheduled place

The discrete PSO is given by the Kennedy et al. [19], and the proposed algorithm is showed in Figure 3. The computation steps of the proposed algorithm in the simulation system can be summarized as: (1) Initialize the parameters and input the problem data. (2) Generate the initial particle solution, including velocity matrix ( $V_{NpNN}$ ), and then transform the velocity to a matrix of  $s(V_{hij})$ , and use Eq. (3) to generate the solution matrix ( $X_{NpNN}$ ), and update the local best and global best solution. (3) use Eq. (1) to generate next generation particles velocity until a specified stopping criterion is reached.

#### 4. The Proposed Discrete Particle Swarm Optimization

The DPSO is utilized to schedule the traveling place sequence for a tourist. For each tourist is to determine the shortest routing path and time based on the Eq. (3). The proposed method is showed as follow:

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Initialize and generate each particle solution  $X_h$  matrix and velocity  $V_h$ 
Set  $L_h = X_h$ ,  $h=1, N_p$ 
Find the global best solution of  $L_h$  and  $g$  is the index of the global best G
Loop
  For  $h= 1$  to  $N_p$ 
    Update the velocity matrix  $V_h$  on Eq. (1)
      subject to  $V_{hij} \in [-Vmin, +Vmax]$ 
    Update the matrix  $X_h$  based on the velocity of  $V_{hij}$ 
  Next  $h$ 
//Find the best routing tour by Eq.(3)
  For each sequence  $j$ 
    To schedule the place route
  Next  $h$ 
// find the local best solution
  For  $h = 1$  to  $N_p$ 
    If  $Z(X_h) < Z(L_h)$  then //  $Z( )$  objective function
       $L_h = X_h$  //  $L_h$  is the best so far for particle  $h$ 
    End if
  Next  $h$ 
//fing the global best solution
  If  $Z(X_h) < Z(G)$  then
     $g = h$  //  $g$  is the index of the global best G
  End if
Until end criterion met
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Figure 2: The proposed PSO

## 5. Simulation Results

In the simulation results show the CPSO and DPSO can solve the traveling problem. The simulation case problem is shown in Table 1 and 2. The distance between these places are list in table 1. The traveling (traffic) time is shown in Table 2. And the  $Vmax = 2$ ,  $Vmin = 0$  of these simulations. The simulations using the number of particle from are 10, 20 and 100. The computation iteration is set from 200 to 5000. In the suggest problem, the best solution of traveling distance is 137, and the shortest traveling traffic time is 180 minutes. The best travelling sequence is 1-4-3-2-6-5-1, that is the sequence form hotel→Museum→Sea shore→Store market→Culture center→Part, then return to hotel. The best traveling cost is 317. The proposed method is compared to the CPSO. The DPSO can solve the problem with small size problem. It is suitable for the traveling problem which the

problem size is limited. But for the large problem, the DPSO is not guarantee can solve the problem. And the computation time for CPSO is faster than DPSO. The DPSO is not a good method in solving the combination problem.

## 6. Conclusion

This research employs the meta-heuristic method of discrete particle swarm optimization (DPSO) to an application of routing problems. In the simulation results we don't show that the different inertia weight and different learning factors in the DPSO. The DPSO is utilized to schedule the all places and to find the total shortest route path and time. The DPSO is easy to implement for the traveling routing problem. The DPSO is also convenient to combine other methods, such as genetic algorithm or local search method. Following, we will continue to solve such problems combined with other methods in the future. And we also want to solve the problems in DPSO.

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