崑山科技大學機械工程系實務專題報告

熱損失對熱力循環之影響

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摘要(中文)　　　　傳統熱力學的熱力循環之分析，係利用熱力平衡的觀念，但過程極緩慢且無摩擦，因此忽略熱力循環運行所需之時間，並將之視為無窮長。理想循環即不考慮熱損失，此方法所推算出來的數據有失精確性，所以此次研究我們將熱損失列入研究，而熱損失是利用燃料化學能的百分比(χ)來表示，並考慮空燃比(λ)，最高溫度(T_{max})α關係，進一步求出功與熱效率，希望能增加熱力循環分析的精確性。

目錄

一、前言：

傳統熱力學的熱力循環之分析，係利用熱力平衡的觀念，但過程極緩慢且無摩擦，因此忽略熱力循環運行所需之時間，並將之視為無窮長。理想循環即不考慮熱損失，此方法所推算出來的數據有失精確性，所以此次研究我們將熱損失列入研究，而熱損失是利用燃料化學能的百分比（χ）來表示，並考慮空燃比（λ），最高溫度（$T_{\max}$）α關係，進一步求出功與熱效率，希望能增加熱力循環分析的精確性。

對於一個標準空氣奧圖引擎奧圖或狄賽爾循環而言，隨著燃料的化學能量中某時間內任意的熱損失參數和平均溫度內熱增加的週期減少量，已被廣泛描述在文學上。熱損失參數和燃料的能量彼此有關聯。在文學上研究的標準空氣奧圖和狄賽爾循環其可行性必須要給訂一個明確的範圍。

如果任意地選擇，他們將提出不切實際的結果而且會讓標準空氣奧圖和狄賽爾循環變成不可行。因此，一個更加現實和更加精確的關係介於燃料的化學能量和熱損失之間需要獲得通過有效的溫度。本文的原始貢獻將描繪熱損失的總量作為燃料能量的百分比。因此，用熱損失參數和燃料能量來作為一內燃機的性能分析將可被適用於更現實和明確的範圍。明確的範圍都會在圖表和基本的等式內提出。

一個不可逆且無損失的理想循環。空氣被假設為表現成理想氣

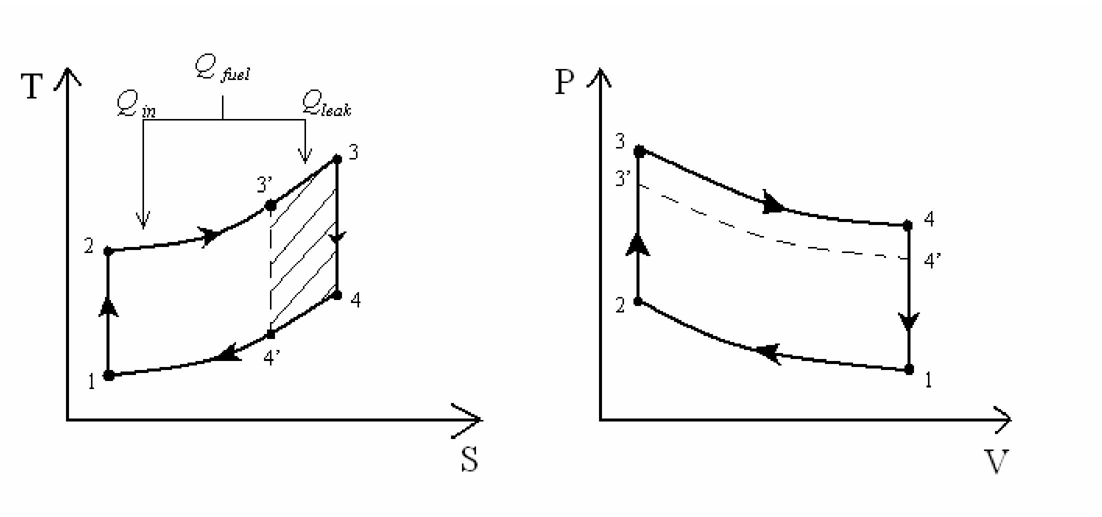
體，全部的過程都被考慮為可逆的且無熱損失的發生。眾所皆知在實際上循環的空氣標準分析對於引擎表現在熱力學上的說明是有用的。然而，對於真實的奧圖或是狄賽爾循環，熱傳遞的不可逆在工作流體與汽缸壁之間是不可忽略的。強烈的熱損失將會影響引擎性能的整體表現。如果將熱損失省略，則分析將只能合乎理想的空氣標準循環。當他們解釋真實過程和內燃機的情況時，空氣標準奧圖和狄賽爾循環是重要的工具。循環的差異是根據各個形式的能量輸入過程。一個空氣標準的奧圖循環(圖一)顯示在這理想的空氣標準奧圖循環中的加熱段為等容可逆過程2和3。假設比熱為常數，所以工作流體的能量增加可由熱力學第一定律來定義[1]：$Q_{in}=m_a c_{v0}\left(T_3-T_2\right)$

在這工作流體中，C_{vo} 是等容比熱係數，m_a 是空氣質量的總合，在能量增加的過程，由於燃燒顯然不被視為絕熱，因為循環中的最大溫度遠遠低於絕熱燃燒溫度，由各參考文獻所提出的性質上的初步模式可讓我們了解如何讓引擎的損失減少，分別對於理想的奧圖循環和理想的狄塞爾循環，他們是最理想的控制理論基礎，其損失是被考慮為由曲軸軸承和活塞環的摩擦阻力所組成，當燃氣通過進氣閥, 會造成壓降或差異, 熱損總數即為在某時間內其工作流體至活塞壁間之燃燒速度, 在參考文獻研究演說中有提出不同損失的比較關係之重要性, 假設在2和3的能量釋放期間, 介於平均燃汽和汽缸壁間的溫度, 其

熱傳遞到汽缸壁上的函數視為線性函數, 另外, 假設汽缸壁的溫度為常數, 而由燃燒產生的熱加至工作流體將由線性表示來描述。以下我們將以Otto、Diesel、Miller、Atkinson這四種循環為範例, 推導出它們的功與熱效率。

二. Otto 循環之功與熱效率等方程式推導

Otto 循環之功與熱效率等方程式推導(常數)



(圖一) Otto cycle T-S 圖與 P-V 圖

令總燃料能量表示為:

$$Q_{fuel} = m_f \cdot Q_{LHV} \tag{1}$$

其中 m_f 為活塞中燃料的質量，Q_{LHV} 為燃料之低熱值

由封閉系統熱力學第一定律:

$$Q_{in} - W_{out} = \Delta U$$

取循環中加熱過程，且視氣體為理想氣體

$$Q_{in} = m\ C_V\ (T_3 - T_2)$$

$$Q_{in} = m_a\ C_{V0}\ (T_3' - T_2) \tag{2}$$

其中 m_a 為氣體的質量，C_{V0} 為在此文章中取代等容比熱係數之代號，T_3' 為實際情況之加熱過程末溫

由$T-S$圖可知

$Q_{fuel} = Q_{in} + Q_{leak}$

$Q_{in} = Q_{fuel} - Q_{leak}$

(3)

由$T-S$圖可知

$Q_{fuel} > Q_{leak}$

故Q_{leak}可視為χ倍的Q_{fuel} ，而χ介於(0~1)之間

$Q_{leak} = \chi\ Q_{fuel}$

將(1)代換Q_{fuel}後可得:

$Q_{leak} = \chi\ m_f\ Q_{LHV}$

(4)

由(3)可知$Q_{in} = Q_{fuel} - Q_{leak}$

將(1)(2)(4)代換後可得:

$m_a\ C_{V0}\ (T_3' - T_2) = m_f\ Q_{LHV} - \chi\ m_f\ Q_{LHV}$

(5)

$$C_{V0}\ (T_3' - T_2) = \frac{(1-\chi)Q_{LHV}}{(\frac{m_a}{m_f})_{ac}}$$

(6)

其中$(\frac{m_a}{m_f})$為空燃比，λ為實際空燃比和理論空燃比之比值

$$\lambda = \frac{(\frac{m_a}{m_f})_{ac}}{(\frac{m_a}{m_f})_s} \quad \Rightarrow (\frac{m_a}{m_f})_s \, \lambda = (\frac{m_a}{m_f})_{ac}$$

可將(6)寫成

$$C_{V0}\,(T_3' - T_2) = \frac{(1-\chi)Q_{LHV}}{\lambda(\frac{m_a}{m_f})_s} \tag{7}$$

$$T_2 = T_3' - \frac{(1-\chi)Q_{LHV}}{C_{VO}\lambda(\frac{m_a}{m_f})_s} \tag{8}$$

由$T-S$圖可知，$T_2 \le T_3'$（$T_3' = T_{\max}$）

將(8)的T_2代換可得:

$$T_3' - \frac{(1-\chi)Q_{LHV}}{C_{VO}\lambda(\frac{m_a}{m_f})_s} \le T_3' \tag{9}$$

由(9)可知，$\chi \le 1$，$\chi_{\max} = 1$ (10)

由$T-S$圖可知，$T_2 \ge T_1$（$T_1 = T_{\min}$）

將(8)的T_2代換可得:

$$T_3' - \frac{(1-\chi)Q_{LHV}}{C_{VO}\lambda(\frac{m_a}{m_f})_s} \ge T_1 \tag{11}$$

$$\chi \geq 1 - \frac{\lambda(\frac{m_a}{m_f})_s C_{VO}(T_3' - T_1)}{Q_{LHV}}$$

(12)

由$T-S$圖可知道：$T_3' = T_{\max}$ $T_1 = T_{\min}$，將(12)式代換後可得

$$\chi \geq 1 - \frac{\lambda(\frac{m_a}{m_f})_s C_{VO}(T_{\max} - T_{\min})}{Q_{LHV}}$$

(13)

而χ的最小值即為只有等號成立時，故(13)式可寫成

$$\chi_{\min} = 1 - \frac{\lambda(\frac{m_a}{m_f})_s C_{VO}(T_{\max} - T_{\min})}{Q_{LHV}}$$

(14)

由(7)：

$$C_{V0}(T_3' - T_2) = \frac{(1-\chi)Q_{LHV}}{\lambda(\frac{m_a}{m_f})_s}$$

$$\Rightarrow (1-\chi) = \frac{\lambda(\frac{m_a}{m_f})_s C_{VO}(T_3' - T_2)}{Q_{LHV}}$$

$$\Rightarrow \chi = 1 - \frac{\lambda(\frac{m_a}{m_f})_s C_{VO}(T_3' - T_2)}{Q_{LHV}}$$

(15)

由多變過程$T.V$關係式：$\frac{T_2}{T_1} = (\frac{V_1}{V_2})^{n-1}$

而[1→2]為等熵過程：$\frac{T_2}{T_1} = (\frac{V_1}{V_2})^{k-1}$

定義：$\frac{T_2}{T_1} = (\frac{V_1}{V_2})^{k-1} = r^{k-1} \Rightarrow T_2 = r^{k-1}\, T_1$

其中r代表壓縮比

故可將(15)的T_2代換掉，可得:

$$\chi = 1 - \frac{\lambda(\frac{m_a}{m_f})_s C_{VO}(T_3' - r^{k-1}T_1)}{Q_{LHV}} \tag{16}$$

由$T-S$圖可知$T_3' = T_{\max}$，$T_1 = T_{\min}$故可將(16)代換成:

$$\chi = 1 - \frac{\lambda(\frac{m_a}{m_f})_s C_{VO}(T_{\max} - T_{\min} r^{k-1})}{Q_{LHV}} \tag{17}$$

在文章中有給定：$(\frac{m_a}{m_f})_s$ ＝14.6　　C_{V0} ＝0.72　　k＝1.4

Q_{LHV} ＝44000KJ/Kg　　$T_{\min}$ ＝300K

故再給定不同的 λ，$T_{\min}$ ，r，即可求得不同的χ

文章中給定

$C_{pm} = a_p + k_1 T$　　　$C_{vm} = b_v + k_1 T$

其中C_{pm}為等壓比熱之變數表示式，C_{vm}為等容比熱之變數表示式

由封閉系統熱力學第一定律配合循環散熱過程

$$Q_{out} = m\int_{T_1}^{T_4'} C_{vm}\,dT$$

$$= m_a\left[b_v(T_4' - T_1) + \frac{1}{2}k_1(T_4'^2 - T_1^2)\right]$$

其中的m_a為氣體之質量，T_4'為實際情況中散熱過程之初溫

由封閉系統熱力學第一定律配合循環加熱過程

$$Q_{in} = m\int_{T_2}^{T_3'} C_{vm}\,dT$$
$$= m_a\left[b_v(T_3' - T_2) + \frac{1}{2}k_1(T_3'^2 - T_2^2)\right]$$

其中的m_a為氣體之質量，T_3'為實際情況中加熱過程之末溫

由多變過程$T \cdot V$關係式配合等熵狀態

$T \cdot V^{k-1} = C$　取$\ln \Rightarrow \ln T + (k-1)\ln V = \ln C$

(18)

將第(18)式微分得到 $\frac{1}{T}\,dT + (k-1)\,\frac{1}{V}\,dV = 0$

(19)

其中熱力學中有定義等熵狀態之多變指數為

$k = \frac{Cp}{Cv} = \frac{Cv + R}{Cv} \quad \Rightarrow (k-1) = \frac{R}{b_v + k_1 T}$

(20)

將第(20)式代回第(19)式，得到: $(b_v + k_1 T)\frac{dT}{T} = (-R\frac{dv}{v})$

(21)

將第(21)式積分得到 $b_v \ln\frac{T_j}{T_i} + K_1(T_j - T_i) = -R\ln\frac{V_j}{V_i}$

帶入循環[$T_1 \to T_2$]: $b_v \ln\frac{T_2}{T_1} + K_1(T_2 - T_1) = R\ln\frac{V_1}{V_2} = R\ln r \Rightarrow T_2$

帶入循環[$T_3 \to T_4$]: $b_v \ln\frac{T_4}{T_3} + K_1(T_4 - T_3) = R\ln\frac{1}{r} \Rightarrow T_4'$

之前文章中有定義

$$Q_{in} = Q_{fuel} - Q_{leak}$$

$$Q_{fuel} = m_f \; Q_{LHV}$$

$$Q_{leak} = \chi \; m_f \; Q_{LHV}$$

將其用變數求出之值代換可得

$$\Rightarrow m_a\left[b_v(T_3' - T_2) + \frac{1}{2}k_1(T_3'^2 - T_2^2)\right] = m_f \; Q_{LHV} - \chi \; m_f \; Q_{LHV}$$

之前文章中有定義 $\lambda = \dfrac{(\frac{m_a}{m_f})_{ac}}{(\frac{m_a}{m_f})_s}$

故上式可代換為

$$\Rightarrow b_v(T_3' - T_2) + \frac{1}{2}k_1(T_3'^2 - T_2^2) = \frac{(1-\chi)Q_{LHV}}{\lambda(\frac{m_a}{m_f})_s} \Rightarrow T_3'$$

由熱力學第一定律(能量守衡)可知

W＝ $Q_{in} - Q_{out}$

代換可得

W＝（ $m_f \; Q_{LHV} - \chi \; m_f \; Q_{LHV}$ ） $- m_a\left[b_v(T_4' - T_1) + \frac{1}{2}k_1(T_4'^2 - T_1^2)\right]$

⇒W＝［（ $\frac{m_f}{m_a}$ ） Q_{LHV} －（ $\frac{m_f}{m_a}$ ） $\chi \; Q_{LHV}$ ］ $-\left[b_v(T_4' - T_1) + \frac{1}{2}k_1(T_4'^2 - T_1^2)\right]$

⇒W＝ $\dfrac{(1-\chi)Q_{LHV}}{\lambda(\frac{m_a}{m_f})_s} - \left[b_v(T_4' - T_1) + \frac{1}{2}k_1(T_4'^2 - T_1^2)\right]$

由熱效率表示式可知

$$\eta = \frac{W}{Q_{in}}$$

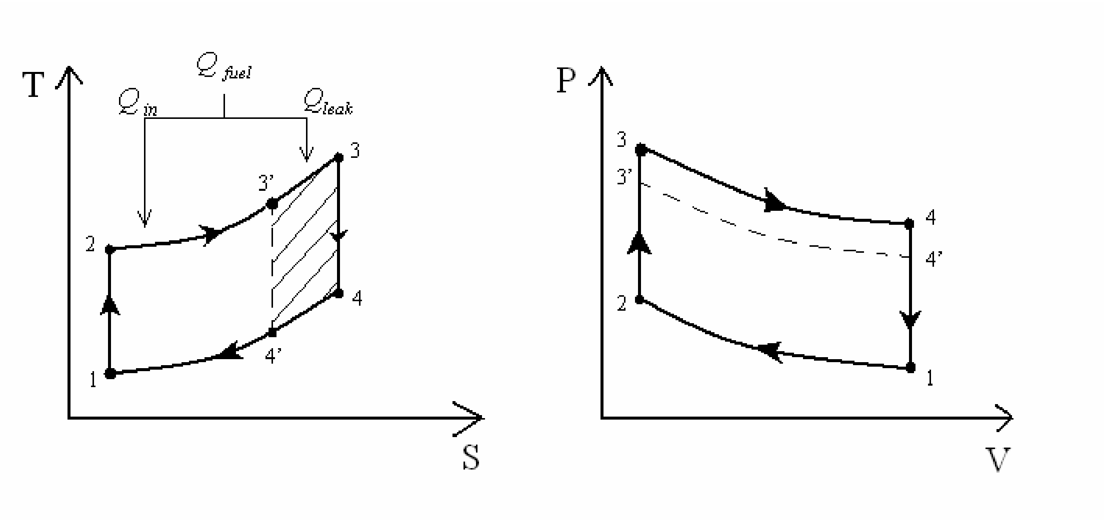
$$= \frac{(m_f Q_{LHV} - \chi m_f Q_{LHV}) - m_a[b_v(T_4' - T_1) + \frac{1}{2}k_1(T_4'^2 - T_1^2)]}{(m_f Q_{LHV} - \chi m_f Q_{LHV})}$$

$$= 1 - \frac{m_a[b_v(T_4' - T_1) + \frac{1}{2}k_1(T_4'^2 - T_1^2)]}{(m_f Q_{LHV} - \chi m_f Q_{LHV})}$$

$$= 1 - \frac{b_v(T_4' - T_1) + \frac{1}{2}k_1(T_4'^2 - T_1^2)}{(\frac{m_f}{m_a})Q_{LHV} - \chi(\frac{m_f}{m_a})Q_{LHV}}$$

$$= 1 - \frac{b_v(T_4' - T_1) + \frac{1}{2}k_1(T_4'^2 - T_1^2)}{(1-\chi)\frac{Q_{LHV}}{(\frac{m_a}{m_f})_{ac}}}$$

Otto 循環之功與熱效率等方程式推導(變數)



(圖一) Otto cycle T-S 圖與 P-V 圖

文章中給定

$C_P = a_p + k_1 T$

$C_V = b_v + k_1 T$

$Q_{fuel} = m_f \cdot Q_{LHV}$

(1)

由封閉系統熱力學第一定律推導加熱過程$[2 \rightarrow 3]$

$Q_{in} - W_{out} = \Delta U$

$\Rightarrow Q_{in} = (U_3 - U_2)$

$\Rightarrow Q_{in} = m\ C_V\ dT$

$\Rightarrow Q_{in} = \int_{T_2}^{T_3'} m_a (b_v + k_1 T) dT$

$\Rightarrow Q_{in} = m_a (b_v T + k_1 \frac{1}{2} T^2) \Big|_{T_2}^{T_3'}$

$\Rightarrow Q_{in} = m_a \left[b_v (T_3' - T_2) + \frac{1}{2} k_1 (T_3'^2 - T_2^2) \right]$

(2)

由$T-S$圖可知

$Q_{fuel}=Q_{in}+Q_{leak}$

$\Rightarrow Q_{in}=Q_{fuel}-Q_{leak}$

(3)

由$T-S$圖可知$Q_{fuel}>Q_{leak}$

故Q_{leak}可視為χ倍的Q_{fuel} ，而χ介於(0~1)之間

$\Rightarrow Q_{leak}=\chi\ Q_{fuel}$

將(1)代換Q_{fuel}後可得

$\Rightarrow Q_{leak}=\chi\ m_f\cdot Q_{LHV}$

(4)

由(3)可知：$Q_{in}=Q_{fuel}-\ Q_{leak}$

將(1)(2)(4)代換後可得：

$$m_a\left[b_v(T_3'-T_2)+\frac{1}{2}k_1(T_3'^2-T_2^2)\right]=m_f\cdot Q_{LHV}-\chi\ m_f\cdot Q_{LHV}$$

(5)

將(5)的m_a除到等式右邊：

$$\left[b_v(T_3'-T_2)+\frac{1}{2}k_1(T_3'^2-T_2^2)\right]=\frac{m_f}{m_a}Q_{LHV}-\ \chi\frac{m_f}{m_a}Q_{LHV}$$

$$\Rightarrow b_v(T_3'-T_2)+\frac{1}{2}k_1(T_3'^2-T_2^2)=(1-\chi)\ \frac{m_f}{m_a}Q_{LHV}$$

而$(\frac{m_a}{m_f})$：空燃比 $\lambda = \dfrac{(\frac{m_a}{m_f})_{ac}}{(\frac{m_a}{m_f})_s}$

代入可得$b_v(T_3' - T_2) + \frac{1}{2}k_1(T_3'^2 - T_2^2) = \dfrac{(1-\chi)Q_{LHV}}{\lambda(\frac{m_a}{m_f})_s}$

(6)

由(6)：$b_v(T_3' - T_2) + \frac{1}{2}k_1(T_3'^2 - T_2^2) = \dfrac{(1-\chi)Q_{LHV}}{\lambda(\frac{m_a}{m_f})_s}$

$$\Rightarrow b_v T_3' - b_v T_2 + \frac{1}{2}k_1 T_3'^2 - \frac{1}{2}k_1 T_2^2 = \frac{(1-\chi)Q_{LHV}}{\lambda(\frac{m_a}{m_f})_s}$$

$$\Rightarrow -b_v T_2 - \frac{1}{2}k_1 T_2^2 = \frac{(1-\chi)Q_{LHV}}{\lambda(\frac{m_a}{m_f})_s} - b_v T_3' - \frac{1}{2}k_1 T_3'^2$$

$$\Rightarrow \frac{1}{2}k_1 T_2^2 + b_v T_2 + [\frac{(1-\chi)Q_{LHV}}{\lambda(\frac{m_a}{m_f})_s} - b_v T_3' - \frac{1}{2}k_1 T_3'^2] = 0$$

由公式解：$\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$\therefore T_2 = \frac{-b_v \pm \sqrt{b_v^2 - 2k_1[(1-\chi)\dfrac{Q_{LHV}}{\lambda(\frac{m_a}{m_f})_s} - b_v T_3' - \frac{1}{2}k_1 T_3'^2]}}{k_1}$$

(7)

由$T-S$圖可知：$T_2 \leq (T_3' = T_{\max})$，故(7)的$T_2$可代換成：

$$\frac{-b_v \pm \sqrt{b_v^2 - 2k_1[(1-\chi)\dfrac{Q_{LHV}}{\lambda(\dfrac{m_a}{m_f})_s} - b_v T_3' - \dfrac{1}{2}k_1 T_3'^2]}}{k_1} \leq T_3'$$

(8)

由$T-S$圖可知：$T_2 \geq (T_1 = T_{\min})$，將(7)的$T_2$代換可得：

$$\frac{-b_v \pm \sqrt{b_v^2 - 2k_1[(1-\chi)\dfrac{Q_{LHV}}{\lambda(\dfrac{m_a}{m_f})_s} - b_v T_3' - \dfrac{1}{2}k_1 T_3'^2]}}{k_1} \geq T_1$$

(9)

由(9)可知：$-b_v \pm \sqrt{b_v^2 - 2k_1[(1-\chi)\dfrac{Q_{LHV}}{\lambda(\dfrac{m_a}{m_f})_s} - b_v T_3' - \dfrac{1}{2}k_1 T_3'^2]} \geq k_1 T_1$

$$\Rightarrow \sqrt{b_v^2 - 2k_1[(1-\chi)\dfrac{Q_{LHV}}{\lambda(\dfrac{m_a}{m_f})_s} - b_v T_3' - \dfrac{1}{2}k_1 T_3'^2]} \geq k_1 T_1 + b_v$$

$$\Rightarrow b_v^2 - 2k_1[\frac{(1-\chi)Q_{LHV}}{\lambda(\dfrac{m_a}{m_f})_s} - b_v T_3' - k_1 T_3'^2] \geq (k_1 T_1 + b_v)^2$$

$$\Rightarrow -2k_1[\frac{(1-\chi)Q_{LHV}}{\lambda(\dfrac{m_a}{m_f})_s} - b_v T_3' - k_1 T_3'^2] \geq (k_1 T_1 + b_v)^2 - b_v^2$$

$$\Rightarrow 2k_1\left[\frac{(1-\chi)Q_{LHV}}{\lambda(\frac{m_a}{m_f})_s}-b_v T_3'-k_1 T_3'^2\right]\leq b_v^2-(k_1 T_1+b_v)^2$$

$$\Rightarrow \frac{(1-\chi)Q_{LHV}}{\lambda(\frac{m_a}{m_f})_s}-b_v T_3'-k_1 T_3'^2\leq\frac{b_v^2-(k_1T_1+b_v)^2}{2k_1}$$

$$\Rightarrow \frac{(1-\chi)Q_{LHV}}{\lambda(\frac{m_a}{m_f})_s}\leq\frac{b_v^2-(k_1T_1+b_v)^2}{2k_1}+b_v T_3'-k_1 T_3'^2$$

$$\Rightarrow (1-\chi)\leq\left[\frac{b_v^2-(k_1T_1+b_v)^2}{2k_1}+b_v T_3'-k_1 T_3'^2\right]\frac{\lambda(\frac{m_a}{m_f})_s}{Q_{LHV}}$$

$$\Rightarrow -\chi\leq\left[\frac{b_v^2-(k_1T_1+b_v)^2}{2k_1}+b_v T_3'-k_1 T_3'^2\right]\frac{\lambda(\frac{m_a}{m_f})_s}{Q_{LHV}}-1$$

$$\Rightarrow \chi\geq 1-\left[\frac{b_v^2-(k_1T_1+b_v)^2}{2k_1}+b_v T_3'-k_1 T_3'^2\right]\frac{\lambda(\frac{m_a}{m_f})_s}{Q_{LHV}}$$

$$\Rightarrow \chi\geq 1-\left[\frac{-k_1T_1(k_1T_1+2b_v)}{2k_1}+b_v T_3'-k_1 T_3'^2\right]\frac{\lambda(\frac{m_a}{m_f})_s}{Q_{LHV}}$$

$$\Rightarrow \chi\geq 1-\left[\frac{-T_1(k_1T_1+2b_V)}{2}+b_v T_3'-k_1 T_3'^2\right]\frac{\lambda(\frac{m_a}{m_f})_s}{Q_{LHV}}$$

(10)

由之前有提到：$T_3'=T_{\max}$ ，$T_1=T_{\min}$

將(10)代換後可得：$\chi \geq 1 - \frac{\lambda(\frac{m_a}{m_f})_s}{Q_{LHV}}[\frac{-T_{\min}(k_1 T_{\min} + 2b_v)}{2} + b_v T_{\max} + \frac{1}{2}k_1 T_{\max}{}^2]$

而χ的最小值即為只有等號成立時，故(10)可變為：

$$\chi_{\min} = 1 - \frac{\lambda(\frac{m_a}{m_f})_s}{Q_{LHV}}[\frac{-T_{\min}(k_1 T_{\min} + 2b_v)}{2} + b_v T_{\max} + \frac{1}{2}k_1 T_{\max}{}^2]$$

(11)

由第(6)式可知：$b_v(T_3' - T_2) + \frac{1}{2}k_1(T_3'^2 - T_2^2) = \frac{(1-\chi)Q_{LHV}}{\lambda(\frac{m_a}{m_f})_s}$

$$\Rightarrow (1-\chi) = \frac{\lambda(\frac{m_a}{m_f})_s}{Q_{LHV}}\left[b_v(T_3' - T_2) + \frac{1}{2}k_1(T_3'^2 - T_2^2)\right]$$

$$\Rightarrow \chi = 1 - \frac{\lambda(\frac{m_a}{m_f})_s}{Q_{LHV}}\left[b_v(T_3' - T_2) + \frac{1}{2}k_1(T_3'^2 - T_2^2)\right]$$

(12)

由$TV^{k-1} = C$取$\ln$

$$\Rightarrow \ln T + \ln V^{k-1} = \ln C$$

$$\Rightarrow \ln T + (k-1)\ln V = \ln C$$

$$\Rightarrow \frac{1}{T}dT + (k-1)\frac{1}{V}dV = 0$$

(13)

由$k = \frac{C_P}{C_V} = \frac{C_V + R}{C_V} = \frac{R + (b_v + k_1 T)}{(b_v + k_1 T)}$

$$\Rightarrow k = 1 + \frac{R}{b_v + k_1 T}$$

$$\Rightarrow (k-1) = \frac{R}{b_v + k_1 T}$$

故(13)可變為：$\frac{1}{T}\, dT + \frac{R}{b_v + k_1 T}\, \frac{1}{V}\, dV = 0$

$$\Rightarrow (b_v + k_1 T)\, \frac{1}{T}\, dT + R\, \frac{1}{V}\, dV = 0$$

$$\Rightarrow b_v\, \frac{1}{T}\, dT + k_1\, dT + R\, \frac{1}{V}\, dV = 0$$

$$\Rightarrow b_v\, \frac{1}{T}\, dT + k_1\, dT = -R\, \frac{1}{V}\, dV$$

積分可得：$b_v \int_{T_I}^{T_J} \frac{1}{T}\, dT + k_1 \int_{T_I}^{T_J} dT = -R \int_{T_I}^{T_J} \frac{1}{V}\, dV$

$$\Rightarrow b_v \ln\frac{T_J}{T_I} + k_1 (T_J - T_I) = -R \ln\frac{V_J}{V_I}$$

過程$[1 \rightarrow 2]$：$b_v \ln\frac{T_2}{T_1} + k_1 (T_2 - T_1) = R \ln\frac{V_1}{V_2} = R \ln r \Rightarrow T_2$（給定$T_1$，r，R 可求出） (14)

由(14)求出T_2後代入(12)而$T_3' = T_{\max}$，可求出χ

文章中給定

$$C_{pm} = a_p + k_1 T \qquad C_{vm} = b_v + k_1 T$$

其中C_{pm}為等壓比熱之變數表示式，C_{vm}為等容比熱之變數表示式

由封閉系統熱力學第一定律配合循環散熱過程

$$Q_{out} = m \int_{T_1}^{T_4'} C_{vm}\, dT$$

$$= m_a\left[b_v(T_4' - T_1) + \frac{1}{2}k_1(T_4'^2 - T_1^2)\right]$$

其中的m_a為氣體之質量，T_4'為實際情況中散熱過程之初溫

由封閉系統熱力學第一定律配合循環加熱過程

$$Q_{in} = m\int_{T_2}^{T_3'} C_{vm}\,dT$$

$$= m_a\left[b_v(T_3' - T_2) + \frac{1}{2}k_1(T_3'^2 - T_2^2)\right]$$

其中的m_a為氣體之質量，T_3'為實際情況中加熱過程之末溫

由多變過程$T \cdot V$關係式配合等熵狀態

$T \cdot V^{k-1} = C$　取$\ln \Rightarrow \ln T + (k-1)\ln V = \ln C$

(18)

將第(18)式微分得到 $\dfrac{1}{T}\,dT + (k-1)\,\dfrac{1}{V}\,dV = 0$

(19)

其中熱力學中有定義等熵狀態之多變指數為

$$k = \frac{C_P}{C_V} = \frac{C_V + R}{C_V} \quad \Rightarrow (k-1) = \frac{R}{b_v + k_1 T}$$

(20)

將第(20)式代回第(19)式，得到: $(b_v + k_1 T)\dfrac{dT}{T} = (-R\dfrac{dv}{v})$

(21)

將第(21)式積分得到 $b_v \ln\dfrac{T_j}{T_i} + K_1(T_j - T_i) = -R\ln\dfrac{V_j}{V_i}$

帶入循環[$T_1 \rightarrow T_2$]: $b_v \ln\dfrac{T_2}{T_1} + K_1(T_2 - T_1) = R\ln\dfrac{V_1}{V_2} = R\ln r \Rightarrow T_2$

帶入循環[$T_3 \rightarrow T_4$]: $b_v \ln\frac{T_4}{T_3} + K_1(T_4 - T_3) = R\ln\frac{1}{r} \Rightarrow T_4'$

之前文章中有定義

$$Q_{in} = Q_{fuel} - Q_{leak}$$

$$Q_{fuel} = m_f\ Q_{LHV}$$

$$Q_{leak} = \chi\ m_f\ Q_{LHV}$$

將其用變數求出之值代換可得

$$\Rightarrow m_a\left[b_v(T_3' - T_2) + \frac{1}{2}k_1(T_3'^2 - T_2^2)\right] = m_f\ Q_{LHV} - \chi\ m_f\ Q_{LHV}$$

之前文章中有定義 $\lambda = \dfrac{(\frac{m_a}{m_f})_{ac}}{(\frac{m_a}{m_f})_s}$

故上式可代換為

$$\Rightarrow b_v(T_3' - T_2) + \frac{1}{2}k_1(T_3'^2 - T_2^2) = \frac{(1-\chi)Q_{LHV}}{\lambda(\frac{m_a}{m_f})_s} \Rightarrow T_3'$$

由熱力學第一定律(能量守衡)可知

W $= Q_{in} - Q_{out}$

代換可得

W $= (m_f\ Q_{LHV} - \chi\ m_f\ Q_{LHV}) - m_a\left[b_v(T_4' - T_1) + \frac{1}{2}k_1(T_4'^2 - T_1^2)\right]$

$\Rightarrow$ W $= [(\frac{m_f}{m_a})\ Q_{LHV} - (\frac{m_f}{m_a})\ \chi\ Q_{LHV}] - \left[b_v(T_4' - T_1) + \frac{1}{2}k_1(T_4'^2 - T_1^2)\right]$

$$\Rightarrow\ W=\frac{(1-\chi)Q_{LHV}}{\lambda(\frac{m_a}{m_f})_s}-\left[b_v(T_4'-T_1)+\frac{1}{2}k_1(T_4'^2-T_1^2)\right]$$

由熱效率表示式可知

$$\eta=\frac{W}{Q_{in}}$$

$$=\frac{(m_fQ_{LHV}-\chi m_fQ_{LHV})-m_a[b_v(T_4'-T_1)+\frac{1}{2}k_1(T_4'^2-T_1^2)]}{(m_fQ_{LHV}-\chi m_fQ_{LHV})}$$

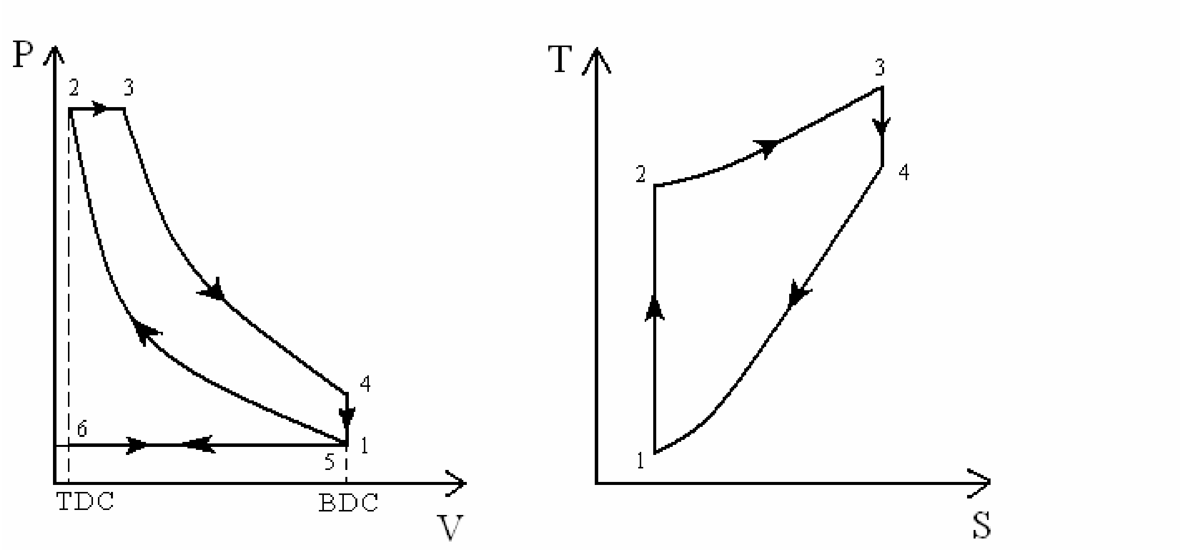
$$=1-\frac{m_a[b_v(T_4'-T_1)+\frac{1}{2}k_1(T_4'^2-T_1^2)]}{(m_fQ_{LHV}-\chi m_fQ_{LHV})}$$

$$=1-\frac{b_v(T_4'-T_1)+\frac{1}{2}k_1(T_4'^2-T_1^2)}{(\frac{m_f}{m_a})Q_{LHV}-\chi(\frac{m_f}{m_a})Q_{LHV}}$$

$$=1-\frac{b_v(T_4'-T_1)+\frac{1}{2}k_1(T_4'^2-T_1^2)}{(1-\chi)\frac{Q_{LHV}}{(\frac{m_a}{m_f})_{ac}}}$$

三. 完成 Diesel 循環之功與熱效率等方程式推導

Diesel 循環之功與熱效率等方程式推導(常數)



圖(二)Diesel cycle T-S 圖與 P-V 圖

$$Q_{fuel} = m_f \cdot Q_{LHV}$$

$$Q_{leak} = \chi \cdot Q_{fule} = \chi \cdot m_f \cdot Q_{LHV}$$

由等壓加熱過程可知輸入熱只為焓的變化量

$$Q_{in} = \Delta H$$

$$Q_{in} = m_a C_{p0} (T_3' - T_2)$$

(1)

由$T-S$圖可知:

$$Q_{fuel} = Q_{in} + Q_{leak}$$

$$Q_{in} = Q_{fuel} - Q_{leak}$$

代換可得:

$$m_a C_{p0} (T_3' - T_2) = m_f \cdot Q_{LHV} - \chi \cdot m_f \cdot Q_{LHV}$$

$$\Rightarrow C_{P0} (T_3' - T_2) = (1-\chi)\left(\frac{m_f}{m_a}\right) Q_{LHV}$$

$$\Rightarrow T_3' - T_2 = (1-\chi)\frac{Q_{LHV}}{C_{p0}(\frac{m_a}{m_f})_{ac}}$$

$$\Rightarrow T_2 = T_3' - (1-\chi)\frac{Q_{LHV}}{C_{p0}(\frac{m_a}{m_f})_{ac}}$$

將$\lambda = \frac{(\frac{m_a}{m_f})_{ac}}{(\frac{m_a}{m_f})_s}$代入式中:

$$\Rightarrow T_2 = T_3' - (1-\chi)\frac{Q_{LHV}}{C_{pO}\lambda(\frac{m_a}{m_f})_s}$$

(2)

由$T-S$圖可知 :$T_2 \le T_3'$　故(2)式可變為:

$$T_3' - (1-\chi)\frac{Q_{LHV}}{C_{pO}\lambda(\frac{m_a}{m_f})_s} \le T_3'$$

(3)

由(3)可知$\chi \le 1, \chi_{\max} = 1$

(4)

由$T-S$圖可知，$T_2 \ge T_1 (T_1 = T_{\min})$　故(2)是可變為:

$$T_3' - (1-\chi)\frac{Q_{LHV}}{C_{pO}\lambda(\frac{m_a}{m_f})_s} \ge T_1$$

$$\Rightarrow (1-\chi)\frac{Q_{LHV}}{C_{pO}\lambda(\frac{m_a}{m_f})_s} \le T_3' - T_1$$

$$\Rightarrow (1-\chi)\frac{Q_{LHV}}{\lambda(\frac{m_a}{m_f})_s} \le C_{p0}(T_3'-T_1)$$

$$\Rightarrow (1-\chi) \le \frac{\lambda(\frac{m_a}{m_f})_s}{Q_{LHV}} C_{p0}(T_3'-T_1)$$

$$\Rightarrow \chi \ge 1-\frac{\lambda(\frac{m_a}{m_f})_s}{Q_{LHV}} C_{p0}(T_3'-T_1)$$

(5)

由$T-S$圖可知：$T_3'=T_{\max}$ $T_1=T_{\min}$

故會變成：$\chi \ge 1-\frac{\lambda(\frac{m_a}{m_f})_s}{Q_{LHV}} C_{p0}(T_{\max}-T_{\min})$

(6)

而χ的最小值即為只有等號成立時，故(6)可變為：

$$\chi_{\min} = 1-\frac{\lambda(\frac{m_a}{m_f})_s}{Q_{LHV}} C_{p0}(T_{\max}-T_{\min})$$

(7)

由(2)：$C_{p0}(T_3'-T_2) = (1-\chi)\frac{Q_{LHV}}{\lambda(\frac{m_a}{m_f})_s}$

$$\Rightarrow (1-\chi) = \frac{\lambda(\frac{m_a}{m_f})_s}{Q_{LHV}} C_{p0}(T_3'-T_2)$$

$$\Rightarrow \chi = 1-\frac{\lambda(\frac{m_a}{m_f})_s}{Q_{LHV}} C_{p0}(T_3'-T_2)$$

(8)

由多變過程 $T.V.$關係式：$\dfrac{T_2}{T_1}=(\dfrac{V_1}{V_2})^{n-1}$

而$[1\to 2]$為$S=C$過程：$\dfrac{T_2}{T_1}=(\dfrac{V_1}{V_2})^{K-1}$ ，$K=\dfrac{C_P}{C_V}=1.4$

定義：$\dfrac{V_1}{V_2}=r$ (壓縮比)代換入 $T.V.$關係式可得：

$$\Rightarrow \frac{T_2}{T_1}=r^{K-1}\Rightarrow T_2=r^{K-1}\cdot T_1$$

故可將(8)的T_2代換掉，可得：

$$\chi=1-\frac{\lambda(\dfrac{m_a}{m_f})_s}{Q_{LHV}}C_{p0}(T_3'-T_1\cdot r^{k-1})$$

由$T-S$圖可知$T_3'=T_{\max}$ ，$T_1=T_{\min}$故可換成：

$$\chi=1-\frac{\lambda(\dfrac{m_a}{m_f})_s}{Q_{LHV}}C_{p0}(T_{\max}-T_{\min}\cdot r^{k-1}) \tag{9}$$

給定：$(\dfrac{m_a}{m_f})_s=14.5$ $C_{p0}=1.008\ {}^{kj}\!/_{kg-k}$ $K=1.4$

$Q_{LHV}=43200\ {}^{kj}\!/_{kg}$ $T_{\min}=300K$

變係數推導熱效率和功：

$$C_{pm} = a_p + k_1 T$$

$$C_{vm} = b_v + k_1 T$$

$$Q_{out} = m \int_{T1}^{T4'} C_{vm} dT \quad = m_a \int_{T1}^{T4'} (b_V + k_1 T) dT$$

$$= m_a (b_V T + k_1 \cdot \tfrac{1}{2} T^2) \Big|_{T1}^{T4'} \quad = m_a \left[b_v (T_4' - T_1) + \tfrac{1}{2} k_1 (T'_4{}^2 - T_1{}^2) \right]$$

$$Q_{in} = m \int_{T2}^{T3'} C_{pm} dT = m_a \int_{T2}^{T3'} (a_p + k_1 T) dT = m_a (a_p T + k_1 \cdot \tfrac{1}{2} T^2) \Big|_{T2}^{T3'}$$

$$= m_a \left[a_P (T_3' - T_2) + \tfrac{1}{2} k_1 (T_3{}'^2 - T_2{}^2) \right]$$

由 $T \cdot V^{k-1} = C$ 取 $\ln \Rightarrow \ln T + \ln V^{k-1} = \ln C$

$$\Rightarrow \ln T + (K-1) \ln V = \ln C$$

將上式微分得到 ： $\frac{1}{T} dT + (K-1) \frac{1}{V} dV = 0$ （常數微分為 0）

(1)

由 $k = \frac{C_p}{C_v} = \frac{C_v + R}{C_v} = \frac{R + (b_v + k_1 T)}{(b_v + k_1 T)}$

$$\Rightarrow k = 1 + \frac{R}{b_v - k_1 T}$$

$$\Rightarrow (k-1) = \frac{R}{b_v + k_1 T}$$

故(1)式可為 ： $\frac{dT}{T}+\frac{R}{b_v+k_1T}\cdot\frac{dV}{V}=0$

$$\Rightarrow (b_v+k_1T)\cdot\frac{dT}{T}+R\frac{dV}{V}=0$$

$$\Rightarrow b_v\frac{dT}{T}+k_1dT+R\frac{dV}{V}=0$$

$$\Rightarrow b_v\frac{dT}{T}+k_1dT=-R\frac{dV}{V}$$

積分 $\Rightarrow b_v\int_{Ti}^{Tj}\frac{1}{T}dT+k_1\int_{Ti}^{Tj}dT=-R\int_{Vi}^{Vj}\frac{1}{V}dV$

$$\Rightarrow b_v\ln\frac{Tj}{Ti}+k_1(Tj-Ti)=-R\ln\frac{Vj}{Vi}$$

代入過程[$T_1\rightarrow T_2$]: $b_v\ln\frac{T_2}{T_1}+k_1(T_2-T_1)=R\ln\frac{V_1}{V_2}=R\ln r\Rightarrow T_2$

(給定 T_1,r,R 可求出)

代入過程[[$T_3\rightarrow T_4$]: $b_v\ln\frac{T_4}{T_3}+k_1(T_4-T_3)=R\ln\frac{V_3}{V_4}$

由 $P-V$ 圖可知: $R\ln\frac{V_3}{V_4}=R\ln(\frac{V_3}{V_2}\frac{V_2}{V_1})=R\ln(r_C\frac{1}{r})\Rightarrow T_4'$

($T_3\rightarrow T_3'$ *已知* T_3',R,r,r_C 可求出 $T_4\rightarrow T_4'$)

之前文章有定義

$$Q_{in}=Q_{fuel}-Q_{leak},Q_{fuel}=m_fQ_{LHV},Q_{leak}=\chi m_fQ_{LHV}$$

將其用變數求出之值代換可得

$$\Rightarrow m_a\left[a_p\left(T_3'-T_2\right)+\frac{1}{2}k_1\left(T_3'^2-T_2^{\,2}\right)\right]=m_f Q_{LHV}-\chi m_f Q_{LHV},\lambda=\frac{\left(\frac{m_a}{m_f}\right)_{ac}}{\left(\frac{m_a}{m_f}\right)_s}$$

$$\Rightarrow a_p\left(T_3'-T_2\right)+\frac{1}{2}k_1\left(T_3'^2-T_2^{\,2}\right)=(1-\chi)\frac{Q_{LHV}}{\lambda\left(\frac{m_a}{m_f}\right)_s}$$

$\Rightarrow T_3'$(*已知*$T_2.Q_{LHV}.\lambda.\left(\frac{m_a}{m_f}\right)$可求出)

$$Q_{out}=m_a\left[b_v\left(T_4'-T_1\right)+\frac{1}{2}k_1\left(T_4'^2-T_1^{\,2}\right)\right]\Rightarrow Q_{out}(\text{已知}T_1.T_4'\text{可求出})$$

若已知χ，可求出Q_{in}由熱力學第一定律(能量守衡)

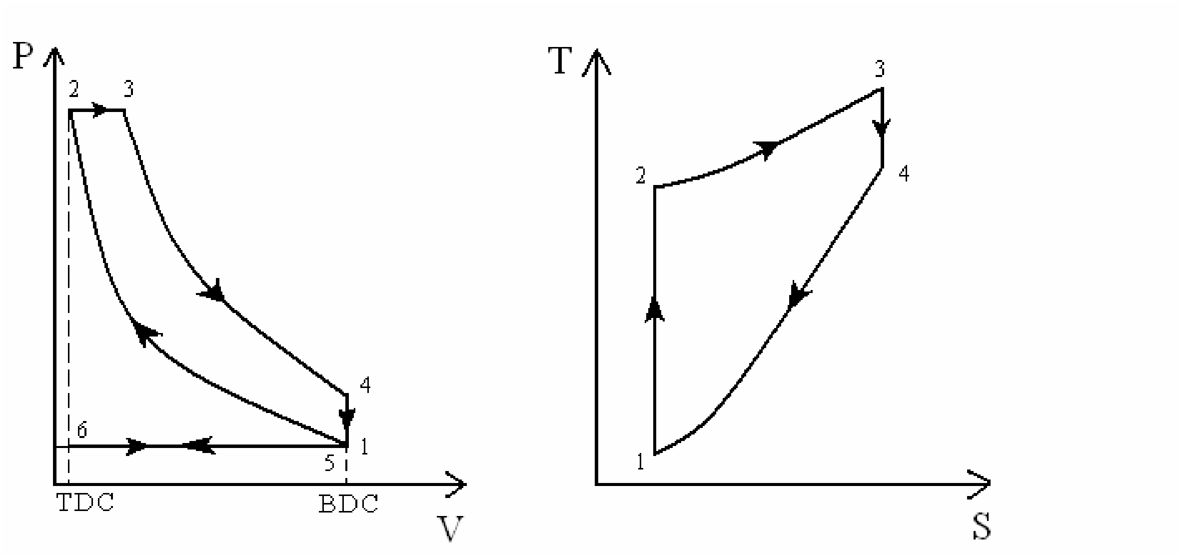
$$W=Q_{in}-Q_{out}=m_a\left[a_p\left(T_3'-T_2\right)+\frac{1}{2}k_1\left(T_3'^2-T_2^{\,2}\right)\right]-m_a\left[b_v\left(T_4'-T_1\right)+\frac{1}{2}k_1\left(T_4'^2-T_1^{\,2}\right)\right]$$

$$\eta=\frac{W}{Q_{in}}=\frac{m_a\left[a_p\left(T_3'-T_2\right)+\frac{1}{2}k_1\left(T_3'^2-T_2^{\,2}\right)\right]-m_a\left[b_v\left(T_4'-T_1\right)+\frac{1}{2}k_1\left(T_4'^2-T_1^{\,2}\right)\right]}{m_a\left[a_p\left(T_3'-T_2\right)+\frac{1}{2}k_1\left(T_3'^2-T_2^{\,2}\right)\right]}$$

$$=1-\frac{m_a\left[b_v\left(T_4'-T_1\right)+\frac{1}{2}k_1(T_4'^2-T_1^{\,2})\right]}{m_a\left[a_p\left(T_3'-T_2\right)+\frac{1}{2}k_1\left(T_3'^2-T_2^{\,2}\right)\right]}$$

$$=1-\frac{b_v\left(T_4'-T_1\right)+\frac{1}{2}k_1\left(T_4'-T_1^{\,2}\right)}{a_p\left(T_3'-T_2\right)+\frac{1}{2}k_1\left(T_3'^2-T_2^{\,2}\right)}$$

Diesel 循環之功與熱效率等方程式推導(變數)



圖(二)Diesel cycle T-S 圖與 P-V 圖

$$C_p = a_p + k_1 T$$

$$C_V = b_v + k_1 T$$

$$Q_{fuel} = m_f Q_{LHV}$$

(1)

$$Q_{\mathrm{leak}} = \chi Q_{\mathrm{fuel}}$$

$$Q_{leak} = \chi_{\mathrm{mf}} Q_{\mathrm{LHV}}$$

由封閉系統熱力學第一定律推導過程$[2 \to 3]$ $\quad Q_{in} - W_{out} = \Delta U$

$$\Rightarrow Q_{in} = m(u_3' - u_2) \Rightarrow Q_{in} = m \cdot C_p \cdot dT$$

$$\Rightarrow Q_{in} = \int_{T_2}^{T_3'} m_a (a_p + k_1 T) dT \Rightarrow Q_{in} = m_a \left(a_p \mathrm{T} + \mathrm{k}_1 \frac{1}{2} \mathrm{T}^2 \right)_{\mathrm{T2}}^{\mathrm{T3}}$$

$$\Rightarrow Q_{in} = m_a \left[a_p (T_3' - T_2) + \frac{k_1}{2} \left(T_3'^2 - T_2^{\ 2} \right) \right]$$

(2)

由$T-S$圖可知：

$$Q_{fuel}=Q_{in}+Q_{leak}\Rightarrow Q_{in}=Q_{fuel}-Q_{leak}$$

(3)

由$T-S$圖可知：$Q_{fuel}\rangle Q_{leak}$，故Q_{leak}可視為χ倍的Q_{fuel}，而χ介於$(0-1)$之間

$$\Rightarrow Q_{fuel}=\chi\cdot Q_{fuel}$$

將(1)代換Q_{fuel}後可得$\Rightarrow Q_{leak}=\chi\cdot m_f\cdot Q_{LHV}$

由(3)可知：$Q_{in}=Q_{fuel}-Q_{leak}$

(4)

將(1) (2) (3)代換後可得：

$$m_a\left[a_p(T_3'-T_2)+\frac{k_1}{2}\left(T_3'^2-T_2{}^2\right)\right]=m_f\cdot Q_{LHV}-\chi\cdot m_f\cdot Q_{LHV}$$

(5)

將(5)的m_a除到等式右邊：$a_p(T_3'-T_2)+\frac{k_1}{2}\left(T_3'^2-T_2{}^2\right)=\frac{m_f}{m_a}Q_{LHV}-\chi\frac{m_f}{m_a}Q_{LHV}$

$$\Rightarrow a_p(T_3'-T_2)+\frac{k_1}{2}\left(T_3'^2-T_2{}^2\right)=(1-\chi)\frac{m_f}{m_a}Q_{LHV}$$

$$\Rightarrow a_p(T_3'-T_2)+\frac{k_1}{2}\left(T_3'^2-T_2{}^2\right)=(1-\chi)\frac{Q_{LHV}}{\left(\frac{m_a}{m_f}\right)_{ac}}$$

而$\left(\dfrac{m_a}{m_f}\right)$=空燃比，$\lambda = \dfrac{\left(\dfrac{m_a}{m_f}\right)_{ac}}{\left(\dfrac{m_a}{m_f}\right)_s}$代入可得：

$$a_p(T_3' - T_2) + \frac{k_1}{2}\left(T_3'^2 - T_2^{\,2}\right) = (1-\chi)\frac{Q_{LHV}}{\lambda\left(\dfrac{m_a}{m_f}\right)_s} \tag{6}$$

由(6)：$a_p(T_3' - T_2) + \dfrac{k_1}{2}\left(T_3'^2 - T_2^{\,2}\right) = (1-\chi)\dfrac{Q_{LHV}}{\lambda\left(\dfrac{m_a}{m_f}\right)_s}$

$$\Rightarrow -a_p T_2 - \frac{k_1 T_2^{\,2}}{2} = (1-\chi)\frac{Q_{LHV}}{\lambda\left(\dfrac{m_a}{m_f}\right)_s} - a_p T_3' - \frac{k_1 T_3'^2}{2}$$

$$\Rightarrow a_p T_2 + \frac{k_1 T_2^{\,2}}{2} + (1-\chi)\frac{Q_{LHV}}{\lambda\left(\dfrac{m_a}{m_f}\right)_s} - a_p T_3' - \frac{k_1 T_3'^2}{2} = 0$$

由公式解$\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$\Rightarrow T_2 = \frac{-a_p \pm \sqrt{a_p^{\,2} - 2k_1\left[(1-\chi)\dfrac{Q_{LHV}}{\lambda\left(\dfrac{m_a}{m_f}\right)_s} - a_p T_3' - \dfrac{k_1 T_3'^2}{2}\right]}}{k_1} \tag{7}$$

由$T-S$圖可知，$T_2 \le (T_3' = T_{\max})$，故$(7)$的$T_2$可代換成：

$$\frac{-a_p \pm \sqrt{{a_p}^2 - 2k_1\left[(1-\chi)\dfrac{Q_{LHV}}{\lambda\left(\dfrac{m_a}{m_f}\right)_s} - a_p T_3' - \dfrac{k_1 T_3'^2}{2}\right]}}{k_1} \le T_3'$$

(8)

由$T-S$圖可知，$T_2 \ge (T_1 = T_{\min})$，故(7)的T_2代換可得：

$$\frac{-a_p \pm \sqrt{{a_p}^2 - 2k_1\left[(1-\chi)\dfrac{Q_{LHV}}{\lambda\left(\dfrac{m_a}{m_f}\right)_s} - a_p T_3' - \dfrac{k_1 T_3'^2}{2}\right]}}{k_1} \ge T_1$$

(9)

$$\Rightarrow \sqrt{{a_p}^2 - 2k_1\left[(1-\chi)\frac{Q_{LHV}}{\lambda\left(\dfrac{m_a}{m_f}\right)_s} - a_p T_3' - \frac{k_1 T_3'^2}{2}\right]} \ge k_1 T_1 + a_p$$

$$\Rightarrow {a_p}^2 - 2k_1\left[(1-\chi)\frac{Q_{LHV}}{\lambda\left(\dfrac{m_a}{m_f}\right)_s} - a_p T_3' - \frac{k_1 T_3'^2}{2}\right] \ge (k_1 T_1 + a_p)^2$$

$$\Rightarrow (1-\chi)\frac{Q_{LHV}}{\lambda\left(\dfrac{m_a}{m_f}\right)_s} - a_p T_3' - \frac{k_1 T_3'^2}{2} \le \frac{{a_p}^2 - (k_1 T_1 + a_p)^2}{2k_1}$$

$$\Rightarrow (1-\chi)\frac{Q_{LHV}}{\lambda(\frac{m_a}{m_f})_s} \le \frac{a_p{}^2-(k_1T_1+a_p)^2}{2k_1}+a_p\,T_3^{'}+\frac{1}{2}\,k_1\,T_3^{'\,2}$$

$$\Rightarrow (1-\chi) \le [\frac{a_p{}^2-(k_1T_1+a_p)^2}{2k_1}+a_p\,T_3^{'}+\frac{1}{2}\,k_1\,T_3^{'\,2}]\ \frac{\lambda(\frac{m_a}{m_f})_s}{Q_{LHV}}$$

$$\Rightarrow \chi \ge 1-[\frac{a_p{}^2-(k_1T_1+a_p)^2}{2k_1}+a_p\,T_3^{'}+\frac{1}{2}\,k_1\,T_3^{'\,2}]\ \frac{\lambda(\frac{m_a}{m_f})_s}{Q_{LHV}}$$

$$\Rightarrow \chi \ge 1-[\frac{-k_1T_1(k_1T_1+2a_p)}{2k_1}+a_p\,T_3^{'}+\frac{1}{2}\,k_1\,T_3^{'\,2}]\frac{\lambda(\frac{m_a}{m_f})_s}{Q_{LHV}}$$

$$\Rightarrow \chi \ge 1-[\frac{-T_1(k_1T_1+2a_p)}{2}+a_p\,T_3^{'}+\frac{1}{2}\,k_1\,T_3^{'\,2}]\frac{\lambda(\frac{m_a}{m_f})_s}{Q_{LHV}} \qquad (10)$$

由之前有提到$T_1=T_{\min}$ $T_3^{'}=T_{\max}$

將(10)式代換後可得

$$\chi \ge 1-\frac{\lambda(\frac{m_a}{m_f})_s}{Q_{LHV}}[\frac{-T_{\min}(k_1T_{\min}+2\mathrm{a}_\mathrm{p})}{2}+a_p\,T_{\max}+\frac{1}{2}\,k_1\,T_{\max}{}^2]$$

而χ的最小值即為只有等號成立時，故(10)式可寫成

$$\chi_{\min}=1-\frac{\lambda(\frac{m_a}{m_f})_s}{Q_{LHV}}[\frac{-T_{\min}(k_1T_{\min}+2a_p)}{2}+a_p\,T_{\max}+\frac{1}{2}\,k_1\,T_{\max}{}^2] \qquad (11)$$

由(6)式可知

$$a_p\left(T_3^{'}-T_2\right)+\ \frac{1}{2}\,k_1\left(T_3^{'2}-T_2\right)=\ (1-\chi)\,\frac{Q_{LHV}}{\lambda(\frac{m_a}{m_f})_s}$$

$$\Rightarrow (1-\chi)\ =\ \frac{\lambda(\frac{m_a}{m_f})_s}{Q_{LHV}}\,[\,a_p\left(T_3^{'}-T_2\right)+\frac{1}{2}\,k_1\left(T_3^{'2}-T_2^{\ 2}\right)]$$

$$\Rightarrow \chi=1-\frac{\lambda(\frac{m_a}{m_f})_s}{Q_{LHV}}\,[\,a_p\left(T_3^{'}-T_2\right)\ +\frac{1}{2}\,k_1\left(T_3^{'2}-T_2^{\ 2}\right)]$$

(12)

變係數推導熱效率和功：　由$T\cdot V^{\ k-1}=C$

取ln ⇒　$\ln T+\ln V^{\,k-1}=\ln C$

$\Rightarrow\ \ \ln T+(k-1)\,\ln V=\ln C$

微分⇒$\dfrac{1}{T}\,dT+(k-1)\,\dfrac{1}{V}\,dV=0$

(13)

由$k=\dfrac{C_P}{C_V}=\dfrac{C_V+R}{C_V}=\dfrac{R+(a_p+k_1T)}{(a_p+k_1T)}$

$$\Rightarrow k=1+\frac{R}{a_p+k_1T}\Rightarrow (k-1)=\frac{R}{a_p+k_1T}$$

故第(1)式可寫成：$\dfrac{dT}{T}+\dfrac{R}{a_p+k_1T}\ \ \dfrac{dV}{V}=0$

$$\Rightarrow\ (a_p+k_1\mathrm{T})\ \frac{dT}{T}+R\,\frac{dV}{V}=0$$

$$\Rightarrow \quad a_p \frac{dT}{T} + k_1\, dT + R\,\frac{dV}{V} = 0$$

$$\Rightarrow \quad a_p \frac{dT}{T} + k_1\, dT = (-R\,\frac{dV}{V})$$

積分$\Rightarrow \quad a_p \int_{Ti}^{Tj} \frac{1}{T} dT + k_1 \int_{Ti}^{Tj} dT = -R \int_{Vi}^{Vj} \frac{1}{V} dV$

$$\Rightarrow a_p \ln \frac{T_j}{T_i} + k_1 (T_j - T_i) = (-R \ln \frac{V_j}{V_i})$$

代入過程$[T_1 - T_2]$：$a_p \ln \frac{T_2}{T_1} + k_1 (T_2 - T_1) = R \ln \frac{V_1}{V_2} = R \ln r$

$\Rightarrow T_2$（給定T_1，R_r由(1)式可求出）

由(1)式求出T_2後代入(12), 而$T_3^{'} = T_{\max}$ 可求出χ

其中

$$Q_{out} = m \int_{T1}^{T4'} C_{vm} dT \quad = m_a \int_{T1}^{T4'} (b_V + k_1 T) dT$$

$$= m_a (b_V T + k_1 \cdot \tfrac{1}{2} T^2) \Big|_{T1}^{T4'} \quad = m_a \left[b_v (T_4' - T_1) + \tfrac{1}{2} k_1 (T'_4{}^2 - T_1^2) \right]$$

由熱力學第一定律(能量守衡)：

$$W = Q_{in} - Q_{out} = m_a \left[a_p \left(T_3^{'} - T_2 \right) + \frac{1}{2} k_1 \left(T'_3{}^2 - T_2^2 \right) \right] - m_a \left[b_v \left(T'_4 - T_1 \right) + \frac{1}{2} k_1 \left(T'_4{}^2 - T_1^2 \right) \right]$$

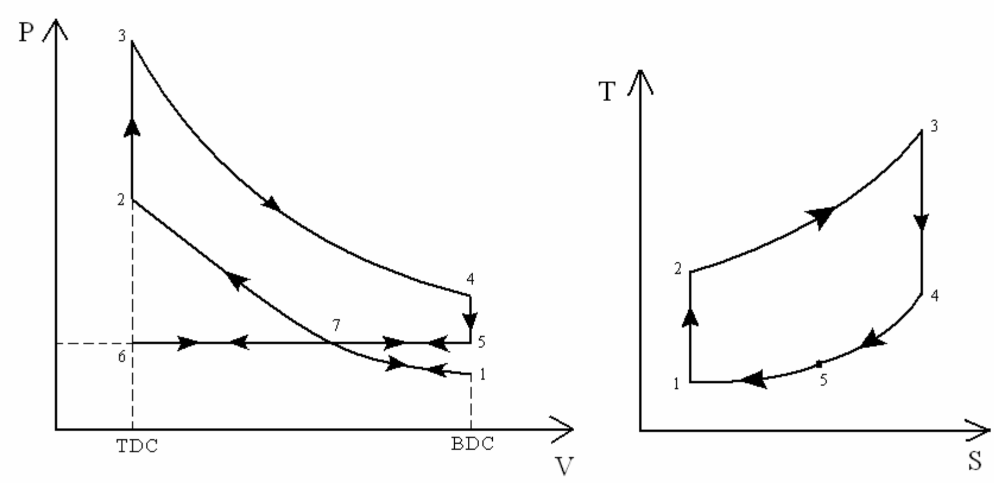
$$\eta = \frac{W}{Q_{in}} = \frac{m_a \left[a_p \left(T'_3 - T_2 \right) + \frac{1}{2} k_1 \left(T'_3{}^2 - T_2^2 \right) \right] - m_a \left[b_v \left(T'_4 - T_1 \right) + \frac{1}{2} k_1 \left(T'_4{}^2 - T_1^2 \right) \right]}{m_a \left[a_p \left(T'_3 - T_2 \right) + \frac{1}{2} k_1 \left(T'_3{}^2 - T_2^2 \right) \right]}$$

$$=1-\frac{m_a\left[b_v\left(T'_4-T_1\right)+\frac{1}{2}k_1\left(T'^{\,2}_4-T_1^{\,2}\right)\right]}{m_a\left[a_p\left(T'_3-T_2\right)+\frac{1}{2}k_1\left(T'^{\,2}_3-T_2^{\,2}\right)\right]}$$

$$=1-\frac{\left[b_v\left(T'_4-T_1\right)+\frac{1}{2}k_1\left(T'^{\,2}_4-T_1^{\,2}\right)\right]}{\left[a_p\left(T'_3-T_2\right)+\frac{1}{2}k_1\left(T'^{\,2}_3-T_2^{\,2}\right)\right]}$$

四.Miller 循環之功與熱效率等方程式推導

Miller 循環之功與熱效率等方程式推導(常數)



圖(三) Miller cycle T-S 圖與 P-V 圖

$Q_{fuel} = m_f \cdot Q_{LHV}$

(1)

由封閉系統熱力學第一定律推導[2~3]

$Q_{in} - \Delta W = \Delta U$

$Q_{in} = mC_v (T_3' - T_2)$

$Q_{in} = m_a C_{vo} (T_3' - T_2)$

(2)

由 $T-S$ 圖可知

$Q_{fuel} = Q_{in} + Q_{leak}$

$Q_{in} = Q_{fuel} - Q_{leak}$

(3)

由 $T-S$ 圖可知

$Q_{fuel} > Q_{leak}$

故Q_{leak}可視為χ倍的Q_{fuel}，而χ介於(0~1)之間

$$Q_{leak} = \chi \cdot Q_{fuel}$$

將(1)代換Q_{fuel}後可得

$$Q_{leak} = \chi \cdot m_f \cdot Q_{LHV} \tag{4}$$

由(3)可知$Q_{in} = Q_{fuel} - Q_{leak}$

將(1)(2)(3)代換後可得

$$m_a C_{vo}(T_3' - T_2) = m_f \cdot Q_{LHV} - \chi \cdot m_f \cdot Q_{LHV} \tag{5}$$

$$\Rightarrow C_{vo}(T_3' - T_2) = \frac{m_f}{m_a} Q_{LHV} - \chi \frac{m_f}{m_a} Q_{LHV}$$

$$\Rightarrow C_{vo}(T_3' - T_2) = (1-\chi) \frac{m_f}{m_a} Q_{LHV}$$

$$\Rightarrow C_{vo}(T_3' - T_2) = (1-\chi) \frac{Q_{LHV}}{(\frac{m_a}{m_f})_{實際}} \tag{6}$$

而 $\lambda = \dfrac{(\frac{m_a}{m_f})_{實際}}{(\frac{m_a}{m_f})_{理論}}$

$$\Rightarrow (\frac{m_a}{m_f})_{理論} \cdot \lambda = (\frac{m_a}{m_f})_{實際}$$

可將(6)式寫成

$$C_{vo}(T_3' - T_2) = (1-\chi) \frac{Q_{LHV}}{\lambda(\frac{m_a}{m_f})_{理論}} \tag{7}$$

$$\Rightarrow \left(T_3'-T_2\right)= \left(1-\chi\right)\frac{Q_{LHV}}{C_{VO}\lambda(\frac{m_a}{m_f})_{理論}}$$

$$\Rightarrow T_2=T_3'-\left(1-\chi\right)\frac{Q_{LHV}}{C_{VO}\lambda(\frac{m_a}{m_f})_{理論}} \tag{8}$$

由$T-S$圖可知，$T_2 \leq T_3'$（$T_3'=T_{\max}$）
將(8)的T_2代換可得

$$T_3'-\left(1-\chi\right)\frac{Q_{LHV}}{C_{VO}\lambda(\frac{m_a}{m_f})_{理論}} \leq T_3' \tag{9}$$

由(9)可知，$\chi \leq 1$，$\chi_{\max}=1$ (10)

由$T-S$圖可知，$T_2 \geq T_1$（$T_1=T_{\min}$）
將(8)的T_2代換可得

$$T_3'-\left(1-\chi\right)\frac{Q_{LHV}}{C_{VO}\lambda(\frac{m_a}{m_f})_{理論}} \geq T_1 \tag{11}$$

$$\Rightarrow \left(1-\chi\right)\frac{Q_{LHV}}{C_{VO}\lambda(\frac{m_a}{m_f})_{理論}} \geq T_3'-T_1$$

$$\Rightarrow \left(1-\chi\right)\frac{Q_{LHV}}{\lambda(\frac{m_a}{m_f})_{理論}} \geq C_{vo}\,(T_3'-T_1)$$

$$\Rightarrow \left(1-\chi\right) \geq \frac{\lambda(\frac{m_a}{m_f})_{理論}}{Q_{LHV}}\,C_{vo}\,(T_3'-T_1)$$

$$\Rightarrow \chi \geq 1 - \frac{\lambda(\frac{m_a}{m_f})_{理論}}{Q_{LHV}} C_{vo}\ (T_3' - T_1)$$

(12)

由 $T-S$ 圖可知 $T_3'=T_{\max}$　$T_1=T_{\min}$ ，將(12)式代換後可得

$$\Rightarrow \chi \geq 1 - \frac{\lambda(\frac{m_a}{m_f})_{理論}}{Q_{LHV}} C_{vo}\ (T_{\max} - T_{\min})$$

(13)

而 χ 的最小值即為只有等號成立時，故(13)式可寫成

$$\chi_{\min} = 1 - \frac{\lambda(\frac{m_a}{m_f})_{理論}}{Q_{LHV}} C_{vo}\ (T_{\max} - T_{\min})$$

(14)

由(7)式可知

$$C_{V0}(T_3' - T_2) = (1 - \chi)\frac{Q_{LHV}}{\lambda(\frac{m_a}{m_f})_{理論}}$$

$$\Rightarrow (1 - \chi) = \frac{\lambda(\frac{m_a}{m_f})_{理論}}{Q_{LHV}} C_{V0}(T_3' - T_2)$$

$$\Rightarrow \chi = 1 - \frac{\lambda(\frac{m_a}{m_f})_{理論}}{Q_{LHV}} C_{V0}(T_3' - T_2)$$

(15)

由多變過程. T. V. 關係式: $\frac{T_2}{T_1} = (\frac{V_1}{V_2})^{n-1}$

(等熵過程多變過程 T. V. P 關係式 $\frac{T_2}{T_1} = (\frac{P_2}{P_1})^{\frac{K-1}{K}} = (\frac{V_1}{V_2})^{K-1}$)

而[1→2]為等熵過程: $\frac{T_2}{T_1}=(\frac{V_1}{V_2})^{K-1}$ ， $K=\frac{C_P}{C_V}=1.4$

定義: $\frac{V_1}{V_2}=r$ (壓縮比)代換入 T.V 關係式可得:

$\Rightarrow \frac{T_2}{T_1}=r^{K-1} \Rightarrow T_2=r^{K-1}\cdot T_1$

故可將(34)的T_2代換掉，可得:

$$\chi=1-\frac{\lambda(\frac{m_a}{m_f})_{理論}}{Q_{LHV}}C_{V0}(T_3'-r^{k-1}\cdot T_1)$$

(16)

由$T-S$圖可知$T_3'=T_{\max}$ ，$T_1=T_{\min}$故可將(16)代換成:

$$\chi=1-\frac{\lambda(\frac{m_a}{m_f})_{理論}}{Q_{LHV}}C_{V0}(T_{\max}-r^{k-1}\cdot T_{\min})$$

(17)

給定: $(\frac{m_a}{m_f})_{理論}$ ＝14.6 C_{V0}:0.72 K:1.4

Q_{LHV} ＝44000KJ/Kg $T_{\min}$:300K

故再給定不同的 λ，$T_{\max}$ ，r，即可求得不同的χ

變係數推導熱效率和功

$C_{pm} = a_p + K_1 T$

$C_{vm} = b_v + K_1 T$

$$Q_{out} = m\int_{T5}^{T4'} C_{vm} dT + m\int_{T1}^{T5} C_{pm} dT$$

$$= m_a [b_v (T_4' - T_5) + \frac{1}{2} k_1 (T_4'^2 - T_5^2)] + m_a [a_p (T_5 - T_1) + \frac{1}{2} k_1 (T_5^2 - T_1^2)]$$

$$Q_{in} = m\int_{T2}^{T3'} C_{vm} dT$$

$$= m_a [b_v (T_3' - T_2) + \frac{1}{2} k_1 (T_3'^2 - T_2^2)]$$

由多變過程$T.V$ 關係式$TV^{k-1} = C$

取 $\ln$ 所以 $\ln T + (k-1)\ln V = \ln C$

(18)

將第(18)式微分得到$\frac{1}{T} dT + (k-1)\frac{1}{V} dV = 0$

(19)

$$k = \frac{C_p}{C_v} = \frac{C_v + R}{C_v}$$

$$\Rightarrow (k-1) = \frac{R}{b_v - k_1 T}$$

(20)

將第(20)式代回第(19)式，得到

$$b_v \frac{1}{T} dT + k_1 dT = (-R\frac{d_v}{v})$$

(21)

將第(1)式積分得到$b_v \ln \frac{T_j}{T_i} + k_1 (T_j - T_i) = -R \ln \frac{V_j}{V_i}$

[T1~T2]： $b_v \ln \frac{T_2}{T_1} + k_1\left(T_2 - T_1\right) = R \ln r$

[T3~T4]： $b_v \ln \frac{T_4}{T_3} + k_1\left(T_4 - T_3\right) = R \ln \frac{1}{re}$

W=$Q_{in} - Qout$

$$= m_a\left[b_v\left(T'_3 - T_2\right) + \frac{1}{2}k_1\left(T'^{\,2}_3 - T_2^{\,2}\right)\right] - m_a\left[b_v\left(T'_4 - T_5\right) + \frac{1}{2}k_1\left(T'^{\,2}_4 - T_5^{\,2}\right)\right]$$

$$+ m_a\left[a_p\left(T_5 - T_1\right) + \frac{1}{2}k_1\left(T_5^{\,2} - T_1^{\,2}\right)\right]$$

$$\eta = \frac{W}{Q_{in}}$$

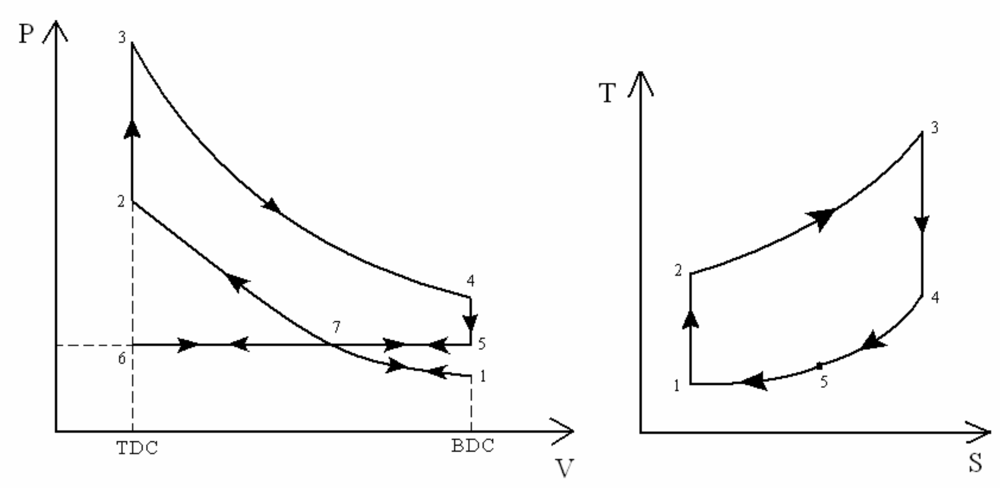
$$= \frac{m_a\left[b_v(T'_3 - T_2) + \frac{1}{2}K_1\left(T'^2_3 - T_2^{\,2}\right)\right] - \left\{m_a\left[b_v(T'_4 - T_5) + \frac{1}{2}K_1\left(T'^2_4 - T_5^{\,2}\right)\right] + m_a\left[a_p(T_5 - T_1) + \frac{1}{2}K_1\left(T_5^{\,2} - T_1^{\,2}\right)\right]\right\}}{m_a\left[b_v(T'_3 - T_2) + \frac{1}{2}K_1\left(T'^2_3 - T_2^{\,2}\right)\right]}$$

$$= 1 - \frac{m_a\left[bv(T'_4 - T_5) + \frac{1}{2}K_1(T'^2_4 - T_5^{\,2})\right] + m_a\left[ap(T_5 - T_1) + \frac{1}{2}K_1(T_5^{\,2} - T_1^{\,2})\right]}{m_a\left[bv(T'_3 - T_2) + \frac{1}{2}K_1(T'^2_3 - T_2^{\,2})\right]}$$

同除m_a

$$= 1 - \frac{b_v\left(T'_4 - T_5\right) + \frac{1}{2}k_1\left(T'^{\,2}_4 - T_5^{\,2}\right) + a_p\left(T_5 - T_1\right) + \frac{1}{2}k_1\left(T_5^{\,2} - T_1^{\,2}\right)}{b_v\left(T'_3 - T_2\right) + \frac{1}{2}k_1\left(T'^{\,2}_3 - T_2^{\,2}\right)}$$

Miller 循環之功與熱效率等方程式推導(變數)



圖(三) Miller cycle T-S 圖與 P-V 圖

$$C_{pm} = a_p + k_1 T$$

$$C_{vm} = b_v + k_1 T$$

$$Q_{fuel} = m_f Q_{LHV}$$

(1)

由封閉系統熱力學第一定律推導 $[2 \rightarrow 3]$

$$Q_{in} - W_{out} = \Delta U \Rightarrow Q_{in} = m(u_3' - u_2) \Rightarrow Q_{in} = m \cdot C_v \cdot dT$$

$$\Rightarrow Q_{in} = \int_{T_2}^{T_3'} m_a (b_v + k_1 T) dT \Rightarrow Q_{in} = m_a \left(b_v T + \frac{k_1 T^2}{2} \right)$$

$$\Rightarrow Q_{in} = m_a \left[b_v (T_3' - T_2) + \frac{k_1}{2} \left(T_3'^2 - T_2^2 \right) \right]$$

(2)

由 $T-S$ 圖可知:

$$Q_{fuel} = Q_{in} + Q_{leak} \Rightarrow Q_{in} = Q_{fuel} - Q_{leak}$$

(3)

由$T-S$圖可知：$Q_{fuel}\rangle Q_{leak}$，故Q_{leak}可視為χ倍的Q_{fuel}，而χ介於$(0-1)$之間

$$Q_{leak}=\chi\, Q_{fuel}$$

將(1)代換Q_{fuel}後可得$\Rightarrow Q_{leak}=\chi\cdot m_f\cdot Q_{LHV}$

(4)

由(3)可知：$Q_{in}=Q_{fuel}-Q_{leak}$

將(1).(2).(4)代換後可得：

$$m_a\left[b_v\left(T_3'-T_2\right)+\frac{k_1}{2}\left(T_3'^2-T_2{}^2\right)\right]=m_f Q_{LHV}-\chi\cdot m_f\cdot Q_{LHV}$$

(5)

將(5)的m_a除到等式右邊：

$$b_v\left(T_3'-T_2\right)+\frac{k_1}{2}\left(T_3'^2-T_2{}^2\right)=\frac{m_f}{m_a}Q_{LHV}-\chi\cdot\frac{m_f}{m_a}Q_{LHV}$$

$$\Rightarrow b_v\left(T_3'-T_2\right)+\frac{k_1}{2}\left(T_3'^2-T_2{}^2\right)=(1-\chi)\frac{m_f}{m_a}Q_{LHV}$$

$$\Rightarrow b_v\left(T_3'-T_2\right)+\frac{k_1}{2}\left(T_3'^2-T_2{}^2\right)=(1-\chi)\frac{Q_{LHV}}{\left(\frac{m_a}{m_f}\right)_{實際}}$$

而$\left(\frac{m_a}{m_f}\right)=$空燃比，$\lambda=\frac{\left(\frac{m_a}{m_f}\right)_{實際}}{\left(\frac{m_a}{m_f}\right)_{理論}}$代入可得：

$$b_v\left(T_3'-T_2\right)+\frac{k_1}{2}\left(T_3'^2-T_2{}^2\right)=(1-\chi)\frac{Q_{LHV}}{\lambda\left(\frac{m_a}{m_f}\right)_{理論}}$$

(6)

由(6)： $b_v\left(T_3'-T_2\right)+\frac{k_1}{2}\left(T_3'^2-T_2{}^2\right)=(1-\chi)\frac{Q_{LHV}}{\lambda\left(\frac{m_a}{m_f}\right)_{\text{理論}}}$

$$\Rightarrow b_vT_3'-b_vT_2+\frac{k_1T_3'^2}{2}-\frac{k_1T_2{}^2}{2}=(1-\chi)\frac{Q_{LHV}}{\lambda\left(\frac{m_a}{m_f}\right)_{\text{理論}}}$$

$$\Rightarrow -b_vT_2-\frac{k_1T_2{}^2}{2}=(1-\chi)\frac{Q_{LHV}}{\lambda\left(\frac{m_a}{m_f}\right)_{\text{理論}}}\quad -b_vT_3'-\frac{k_1T_3'^2}{2}$$

$$\Rightarrow b_vT_2+\frac{k_1T_2{}^2}{2}+(1-\chi)\frac{Q_{LHV}}{\lambda\left(\frac{m_a}{m_f}\right)_{\text{理論}}}\quad -b_vT_3'-\frac{k_1T_3'^2}{2}=0$$

由公式解 $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$$\Rightarrow T_2=\frac{-b_v\pm\sqrt{b_v{}^2-2k_1\left[(1-\chi)\frac{Q_{LHV}}{\lambda\left(\frac{m_a}{m_f}\right)_{\text{裡論}}}-b_vT_3'-\frac{k_1T_3'^2}{2}\right]}}{k_1}$$

(7)

由 $T-S$ 圖可知，$T_2\le\left(T_3'=T_{\max}\right)$，故(7)的 T_2 可代換成：

$$\frac{-b_v\pm\sqrt{b_v{}^2-2k_1\left[(1-\chi)\frac{Q_{LHV}}{\lambda\left(\frac{m_a}{m_f}\right)_{\text{裡論}}}-b_vT_3'-\frac{k_1T_3'^2}{2}\right]}}{k_1}\le T_3'$$

(8)

由 T-S 圖可知，$T_2 \geq (T_1 = T_{\min})$，故(7)的$T_2$代換可得:

$$\frac{-b_v \pm \sqrt{{b_v}^2 - 2k_1\left[(1-\chi)\dfrac{Q_{LHV}}{\lambda\left(\dfrac{m_a}{m_f}\right)_{\text{裡論}}} - b_v T_3' - \dfrac{k_1 T_3'^2}{2}\right]}}{k_1} \geq T_1$$

(9)

$$\Rightarrow \sqrt{{b_v}^2 - 2k_1\left[(1-\chi)\frac{Q_{LHV}}{\lambda\left(\dfrac{m_a}{m_f}\right)_{\text{裡論}}} - b_v T_3' - \frac{k_1 T_3'^2}{2}\right]} \geq k_1 T_1 + b_v$$

$$\Rightarrow {b_v}^2 - 2k_1\left[(1-\chi)\frac{Q_{LHV}}{\lambda\left(\dfrac{m_a}{m_f}\right)_{\text{裡論}}} - b_v T_3' - \frac{k_1 T_3'^2}{2}\right] \geq (k_1 T_1 + b_v)^2$$

$$\Rightarrow 2k_1\left[(1-\chi)\frac{Q_{LHV}}{\lambda\left(\dfrac{m_a}{m_f}\right)_{\text{裡論}}} - b_v T_3' - \frac{k_1 T_3'^2}{2}\right] \leq {b_v}^2 - (k_1 T_1 + b_v)^2$$

$$\Rightarrow (1-\chi)\frac{Q_{LHV}}{\lambda\left(\dfrac{m_a}{m_f}\right)_{\text{裡論}}} - b_v T_3' - \frac{k_1 T_3'^2}{2} \leq \frac{{b_v}^2 - (k_1 T_1 + b_v)^2}{2k_1}$$

$$\Rightarrow \ (1-\chi)\frac{Q_{LHV}}{\lambda(\dfrac{m_a}{m_f})_{\text{理論}}} \leq \ \frac{{b_v}^2 - (k_1 T_1 + b_v)^2}{2k_1} + b_v\ T_3' + \frac{1}{2}\ k_1 T_3'^2$$

$$\Rightarrow \ (1-\chi) \leq \ [\frac{b_v{}^2-(k_1T_1+b_v)^2}{2k_1}+b_v\,T_3'+\frac{1}{2}\,k_1T_3'^2\,]\ \ \frac{\lambda(\frac{m_a}{m_f})_{理論}}{Q_{LHV}}$$

$$\Rightarrow \ \chi \geq \ 1-[\frac{b_v{}^2-(k_1T_1+b_v)^2}{2k_1}+b_v\,T_3'+\frac{1}{2}\,k_1T_3'^2\,]\ \ \frac{\lambda(\frac{m_a}{m_f})_{理論}}{Q_{LHV}}$$

$$\Rightarrow \ \chi \geq \ 1-[\frac{-k_1T_1(k_1T_1+2b_v)}{2k_1}+b_v\,T_3'+\frac{1}{2}\,k_1T_3'^2\,]\ \ \frac{\lambda(\frac{m_a}{m_f})_{理論}}{Q_{LHV}}$$

$$\Rightarrow \ \chi \geq \ 1-[\frac{-T_1(k_1T_1+2b_v)}{2}+b_v\,T_3'+\frac{1}{2}\,k_1T_3'^2\,]\ \ \frac{\lambda(\frac{m_a}{m_f})_{理論}}{Q_{LHV}} \tag{10}$$

由之前有提到 $T_1=T_{\min}$ $T_3'=T_{\max}$

將(10)式代換後可得

$$\chi \geq \ 1-\frac{\lambda(\frac{m_a}{m_f})_{理論}}{Q_{LHV}}\ [\frac{-T_{\min}(k_1T_{\min}+2b_v)}{2}+b_v\,T_{\max}+\frac{1}{2}\,K_1\,T_{\max}{}^2\,]$$

而χ的最小值即為只有等號成立時，故(10)式可寫成

$$\chi_{\min}=1-\frac{\lambda(\frac{m_a}{m_f})_{理論}}{Q_{LHV}}\ [\frac{-T_{\min}(k_1T_{\min}+2b_v)}{2}+b_v\,T_{\max}+\frac{1}{2}\,K_1\,T_{\max}{}^2\,] \tag{11}$$

由(6)式可知

$$b_v\,(T_3'-T_2)+\ \frac{1}{2}\,k_1\left(T_3'^2-T_2{}^2\right)\ =\ (1-\chi)\,\frac{Q_{LHV}}{\lambda(\frac{m_a}{m_f})_{理論}}$$

$$\Rightarrow (1-\chi)=\ \frac{\lambda(\frac{m_a}{m_f})_{理論}}{Q_{LHV}}\ [b_v\,(T_3'-T_2)+\ \frac{1}{2}\,k_1\left(T_3'^2-T_2{}^2\right)]$$

$$\Rightarrow \chi = 1 - \frac{\lambda(\frac{m_a}{m_f})_{理論}}{Q_{LHV}} \left[b_v \left(T_3' - T_2\right) + \frac{1}{2} k_1 \left(T_3'^2 - T_2^2\right)\right]$$

(12)

由 $TV^{k-1} = C$

取ln　$\ln T + (k-1)\ln V = \ln C$

微分得到 $\Rightarrow \frac{1}{T} dT + (k-1)\frac{1}{V} dV = 0$

(13)

由 $k = \frac{C_p}{C_v} = \frac{C_v + R}{C_v} = \frac{R + (b_v + k_1 T)}{(b_v + k_1 T)}$

$$\Rightarrow k = 1 + \frac{R}{b_v + k_1 T}$$

$$\Rightarrow (k-1) = \frac{R}{b_v + k_1 T}$$

故第(1)式可寫成:

$$\frac{dT}{T} + \frac{R}{b_v + k_1 T} \frac{dV}{V} = 0$$

$$\Rightarrow (b_v + k_1 T) \frac{dT}{T} + R \frac{dV}{V} = 0$$

$$\Rightarrow b_v \frac{dT}{T} + k_1 dT + R \frac{dV}{V} = 0$$

$$\Rightarrow b_v \frac{dT}{T} + k_1 dT = (-R \frac{dV}{V})$$

積分⇒ $b_v \int_{Ti}^{Tj} \frac{1}{T} dT + k_1 \int_{Ti}^{Tj} dT = -R \int_{Vi}^{Vj} \frac{1}{V} dV$

⇒ $b_v \ln\frac{T_j}{T_i} + k_1 \left(T_j - T_i\right) = (-R \ln\frac{V_j}{V_i})$

[T1~T2]： $b_v \ln\frac{T_2}{T_1} + k_1\left(T_2 - T_1\right) = R \ln\frac{V_1}{V_2} = R\ln r$

⇒T2(給定T_1, R, r由(1)式可求出)

由(1)式求出T_2後代入(12),而$T_3' = T_{\max}$,可求出χ

變係數推導熱效率和功

$C_{pm} = a_p + k_1 T$

$C_{vm} = b_v + k_1 T$

$$Q_{out} = m\int_{T5}^{T4'} C_{vm} dT + m\int_{T1}^{T5} C_{pm} dT$$

$$= m_a\,[\,b_v\,(T_4' - T_5) + \frac{1}{2}\,k_1\,(T_4'^2 - T_5^{\,2})\,] + m_a\,[\,a_p\,(T_5 - T_1) + \frac{1}{2}\,k_1\,(T_5^{\,2} - T_1^{\,2})]$$

$$Q_{in} = m\int_{T2}^{T3'} C_{vm} dT$$

$$= m_a\,[\,b_v\,(T_3' - T_2) + \frac{1}{2}\,k_1\,(T_3'^2 - T_2^{\,2})$$

$TV^{k-1} = C$

取 ln 所以 $\ln T + (k-1)\ln V = \ln C$ (14)

將第(14)式微分得到 $\frac{1}{T}\,dT + (k-1)\frac{1}{V}\,dV = 0$ (15)

$$k = \frac{C_p}{C_v} = \frac{C_v + R}{C_v}$$

$$\Rightarrow (k-1) = \frac{R}{b_v - k_1 T}$$ (16)

將第(16)式代回第(15)式，得到

$$b_v + k_1 dT = (-R\,\frac{d_v}{v})$$

將第(1)式積分得到 $b_v\,\ln\frac{T_j}{T_i} + k_1(T_j - T_i) = -R\ln\frac{V_j}{V_i}$

[T1~T2]：$b_v \ln\frac{T_2}{T_1}+k_1(T_2-T_1)=R\ln r$

[T3~T4]：$b_v \ln\frac{T_4}{T_3}+k_1(T_4-T_3)=R\ln\frac{1}{re}$

W=$Q_{in}-Qout$

$$=m_a\left[b_v(T'_3-T_2)+\frac{1}{2}k_1\left(T'^{\,2}_3-T_2^{\,2}\right)\right]-m_a\left[b_v(T'_4-T_5)+\frac{1}{2}k_1\left(T'^{\,2}_4-T_5^{\,2}\right)\right]$$
$$+m_a\left[a_p(T_5-T_1)+\frac{1}{2}k_1\left(T_5^{\,2}-T_1^{\,2}\right)\right]$$

$$\eta=\frac{W}{Q_{in}}$$

$$=\frac{m_a\left[b_v(T'_3-T_2)+\frac{1}{2}K_1\left(T'^2_3-T_2^2\right)\right]-\left\{m_a\left[b_v(T'_4-T_5)+\frac{1}{2}K_1\left(T'^2_4-T_5^2\right)\right]+m_a\left[a_p(T_5-T_1)+\frac{1}{2}K_1\left(T_5^2-T_1^2\right)\right]\right\}}{m_a\left[b_v(T'_3-T_2)+\frac{1}{2}K_1\left(T'^2_3-T_2^2\right)\right]}$$

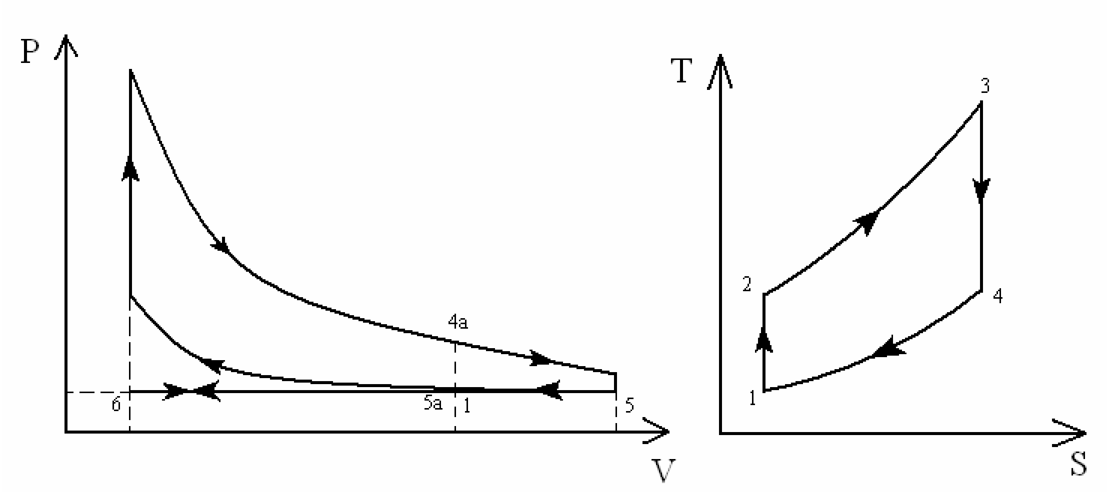
$$=1-\frac{m_a\left[bv(T'_4-T_5)+\frac{1}{2}K_1(T'^2_4-T_5^{\,2})\right]+m_a\left[ap(T_5-T_1)+\frac{1}{2}K_1(T_5^{\,2}-T_1^{\,2})\right]}{m_a\left[bv(T'_3-T_2)+\frac{1}{2}K_1(T'^2_3-T_2^{\,2})\right]}$$

同除m_a

$$=1-\frac{b_v(T'_4-T_5)+\frac{1}{2}k_1\left(T'^{\,2}_4-T_5^{\,2}\right)+a_p(T_5-T_1)+\frac{1}{2}k_1\left(T_5^{\,2}-T_1^{\,2}\right)}{b_v(T'_3-T_2)+\frac{1}{2}k_1\left(T'^{\,2}_3-T_2^{\,2}\right)}$$

五. Atkinson 循環之功與熱效率等方程式推導

Atkinson 循環之功與熱效率等方程式推導(常數)



圖(四) Atkinson cycle P-V 圖與 T-S 圖

$Q_{fuel} = m_f \cdot Q_{LHV}$

(1)

$Q_{in} - W_{out} = \Delta U$

$Q_{in} = m\ C_V\ (T_3' - T_2)$

$Q_{in} = m_a\ C_{V0}\ (T_3' - T_2)$

(2)

由 $T-S$ 圖可知

$Q_{fuel} = Q_{in} + Q_{leak}$

$Q_{in} = Q_{fuel} - Q_{leak}$

(3)

由 $T-S$ 圖可知

$Q_{fuel} > Q_{leak}$

故 Q_{leak} 可視為 χ 倍的 Q_{fuel} ，而 χ 介於(0~1)之間

$Q_{leak} = \chi\ Q_{fuel}$

將(1)代換Q_{fuel}後可得

$$Q_{leak} = \chi\ m_f\ Q_{LHV} \tag{4}$$

由(3)可知$Q_{in} = Q_{fuel} - Q_{leak}$

將(1)(2)(4)代換後可得

$$m_a\ C_{V0}\ (T_3' - T_2) = m_f\ Q_{LHV} - \chi\ m_f\ Q_{LHV} \tag{5}$$

$$C_{V0}\ (T_3' - T_2) = \frac{(1-\chi)Q_{LHV}}{(\frac{m_a}{m_f})_{實際}} \tag{6}$$

$$\lambda = \frac{(\frac{m_a}{m_f})_{實際}}{(\frac{m_a}{m_f})_{理論}} \quad \Rightarrow (\frac{m_a}{m_f})_{理論}\ \lambda = (\frac{m_a}{m_f})_{實際}$$

可將(6)寫成

$$C_{V0}\ (T_3' - T_2) = \frac{(1-\chi)Q_{LHV}}{\lambda(\frac{m_a}{m_f})_{理論}} \tag{7}$$

$$T_2 = T_3' - \frac{(1-\chi)Q_{LHV}}{C_{VO}\lambda(\frac{m_a}{m_f})_{理論}} \tag{8}$$

由$T-S$圖可知，$T_2 \le T_3'$（$T_3' = T_{\max}$）

將(8)的T_2代換可得:

$$T_3' - \frac{(1-\chi)Q_{LHV}}{C_{VO}\lambda(\frac{m_a}{m_f})_{理論}} \le T_3'$$

(9)

由(9)可知，$\chi \le 1$，$\chi_{\max} = 1$

(10)

由$T-S$圖可知，$T_2 \ge T_1$（$T_1 = T_{\min}$）
將(8)的T_2代換可得:

$$T_3' - \frac{(1-\chi)Q_{LHV}}{C_{VO}\lambda(\frac{m_a}{m_f})_{理論}} \ge T_1$$

(11)

$$\chi \ge 1 - \frac{\lambda(\frac{m_a}{m_f})_{理論} C_{VO}(T_3' - T_1)}{Q_{LHV}}$$

(12)

由$T-S$圖可知$T_3' = T_{\max}$　$T_1 = T_{\min}$，將(30)式代換後可得

$$\chi \ge 1 - \frac{\lambda(\frac{m_a}{m_f})_{理論} C_{VO}(T_{\max} - T_{\min})}{Q_{LHV}}$$

(13)

而χ的最小值即為只有等號成立時，故(31)式可寫成

$$\chi_{\min} = 1 - \frac{\lambda(\frac{m_a}{m_f})_{理論} C_{VO}(T_{\max} - T_{\min})}{Q_{LHV}}$$

(14)

由(7)：

$$C_{V0}\,(T_3'-T_2)=\frac{(1-\chi)Q_{LHV}}{\lambda(\frac{m_a}{m_f})_{理論}}$$

$$\Rightarrow (1-\chi)=\frac{\lambda(\frac{m_a}{m_f})_{理論}C_{VO}(T_3'-T_2)}{Q_{LHV}}$$

$$\Rightarrow \chi=1-\frac{\lambda(\frac{m_a}{m_f})_{理論}C_{VO}(T_3'-T_2)}{Q_{LHV}}$$

(15)

由多變過程. T. V. 關係式: $\frac{T_2}{T_1}=(\frac{V_1}{V_2})^{n-1}$

而[1→2]為等熵過程: $\frac{T_2}{T_1}=(\frac{V_1}{V_2})^{k-1}$

定義: $\frac{T_2}{T_1}=(\frac{V_1}{V_2})^{k-1}=r^{k-1}\Rightarrow T_2=r^{k-1}\,T_1$

故可將(15)的T_2代換掉，可得:

$$\chi=1-\frac{\lambda(\frac{m_a}{m_f})_{理論}C_{VO}(T_3'-r^{k-1}T_1)}{Q_{LHV}}$$

(16)

由$T-S$圖可知$T_3'=T_{\max}$ ，$T_1=T_{\min}$故可將(16)代換成:

$$\chi=1-\frac{\lambda(\frac{m_a}{m_f})_{理論}C_{VO}(T_{\max}-T_{\min}r^{k-1})}{Q_{LHV}}$$

(17)

給定：$(\frac{m_a}{m_f})_{理論}$ ＝14.6　　C_{V0} ＝0.72　 k＝1.4

Q_{LHV} ＝44000KJ/Kg　　$T_{\min}$ ＝300K

故再給定不同的 λ，$T_{\min}$ ，r，即可求得不同的χ

$$C_{pm} = a_p + k_1 T \qquad C_{vm} = b_v + k_1 T$$

$$Q_{out} = m \int_{T_1}^{T_4'} C_{vm} dT$$

$$= m_a \left[b_v (T_4' - T_1) + \frac{1}{2} k_1 (T_4'^2 - T_1^2) \right]$$

$$Q_{in} = m \int_{T_2}^{T_3'} C_{vm} dT$$

$$= m_a \left[b_v (T_3' - T_2) + \frac{1}{2} k_1 (T_3'^2 - T_2^2) \right]$$

$$T \cdot V^{k-1} = C$$

取$\ln \Rightarrow \ln T + (k-1) \ln V = \ln C$

(18)

將第(18)式微分得到 $\frac{1}{T} dT + (k-1) \frac{1}{V} dV = 0$

(19)

$$k = \frac{Cp}{Cv} = \frac{Cv + R}{Cv}$$

$$\Rightarrow (k-1) = \frac{R}{b_v + k_1 T}$$

(20)

將第(20)式代回第(19)式，得到

$$b_v \frac{1}{T} dT + k_1 dT = (-R\frac{dv}{v})$$

將第(18)式積分得到 $b_v \ln\frac{Tj}{Ti} + k_1 (T_j - T_i) = -R \ln\frac{Vj}{Vi}$

$$[\ T_1 \rightarrow T_2\ ]: b_v \ln\frac{T_2}{T_1} + k_1(T_2 - T_1) = R\ln\frac{V_1}{V_2} = R\ln r \Rightarrow T_2$$

$$[\ T_3 \rightarrow T_4\ ]:\ b_v \ln\frac{T_4}{T_3} + k_1(T_4 - T_3) = R\ln\frac{1}{r} \Rightarrow T_4'$$

$$Q_{in} = Q_{fuel} - Q_{leak}$$

$$Q_{fuel} = m_f\ Q_{LHV}$$

$$Q_{leak} = \chi\ m_f\ Q_{LHV}$$

$$\Rightarrow m_a\left[b_v(T_3' - T_2) + \frac{1}{2}k_1(T_3'^2 - T_2^2)\right] = m_f\ Q_{LHV} - \chi\ m_f\ Q_{LHV}$$

$$\lambda = \frac{(\frac{m_a}{m_f})_{實際}}{(\frac{m_a}{m_f})_{理論}}$$

$$\Rightarrow b_v(T_3' - T_2) + \frac{1}{2}k_1(T_3'^2 - T_2^2) = \frac{(1-\chi)Q_{LHV}}{\lambda(\frac{m_a}{m_f})_{理論}} \Rightarrow T_3'$$

由熱力學第一定律(能量守衡)　W＝$Q_{in}-Q_{out}$

$$W=(m_f\ Q_{LHV}-\chi\ m_f\ Q_{LHV})-m_a\left[b_v(T_4'-T_1)+\frac{1}{2}k_1(T_4'^2-T_1^2)\right]$$

$$\Rightarrow\ W=[(\frac{m_f}{m_a})Q_{LHV}-(\frac{m_f}{m_a})\chi\ Q_{LHV}]-\left[b_v(T_4'-T_1)+\frac{1}{2}k_1(T_4'^2-T_1^2)\right]$$

$$\Rightarrow\ W=\frac{(1-\chi)Q_{LHV}}{\lambda(\frac{m_a}{m_f})_{\text{理論}}}-\left[b_v(T_4'-T_1)+\frac{1}{2}k_1(T_4'^2-T_1^2)\right]$$

$$\eta=\frac{W}{Q_{in}}$$

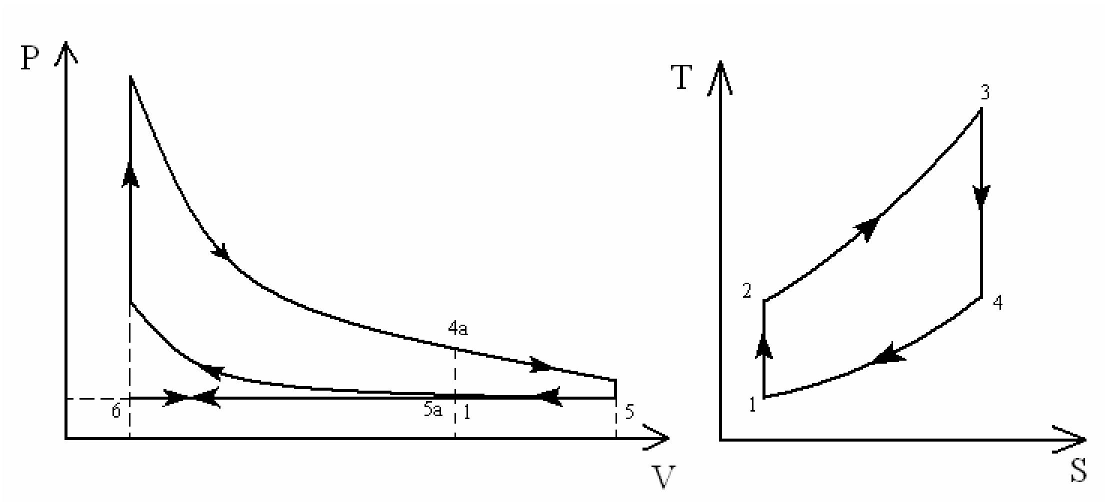
$$=\frac{(m_fQ_{LHV}-\chi m_fQ_{LHV})-m_a[b_v(T_4'-T_1)+\frac{1}{2}k_1(T_4'^2-T_1^2)]}{(m_fQ_{LHV}-\chi m_fQ_{LHV})}$$

$$=1-\frac{m_a[b_v(T_4'-T_1)+\frac{1}{2}k_1(T_4'^2-T_1^2)]}{(m_fQ_{LHV}-\chi m_fQ_{LHV})}$$

$$=1-\frac{b_v(T_4'-T_1)+\frac{1}{2}k_1(T_4'^2-T_1^2)}{(\frac{m_f}{m_a})Q_{LHV}-\chi(\frac{m_f}{m_a})Q_{LHV}}$$

$$=1-\frac{b_v(T_4'-T_1)+\frac{1}{2}k_1(T_4'^2-T_1^2)}{(1-\chi)\frac{Q_{LHV}}{(\frac{m_a}{m_f})_{ac}}}$$

Atkinson 循環之功與熱效率等方程式推導(變數)



圖(四) Atkinson cycle P-V 圖與 T-S 圖

$C_P = a_p + k_1 T$

$C_V = b_v + k_1 T$

$Q_{fuel} = m_f \cdot Q_{LHV}$

(1)

由封閉系統熱力學第一定律推導$[2 \rightarrow 3]$

$Q_{in} - W_{out} = \Delta U$

$\Rightarrow Q_{in} = (U_3 - U_2)$

$\Rightarrow Q_{in} = m\ C_V\ dT$

$\Rightarrow Q_{in} = \int_{T_2}^{T_3'} m_a (b_v + k_1 T) dT$

$\Rightarrow Q_{in} = m_a (b_v T + k_1 \frac{1}{2} T^2) \Big|_{T_2}^{T_3}$

$$\Rightarrow Q_{in} = m_a \left[b_v (T_3' - T_2) + \frac{1}{2} k_1 (T_3'^2 - T_2^2) \right]$$

(2)

由$T-S$圖可知:

$$Q_{fuel} = Q_{in} + Q_{leak}$$

$$\Rightarrow Q_{in} = Q_{fuel} - Q_{leak}$$

(3)

由$T-S$圖可知$Q_{fuel} > Q_{leak}$

故Q_{leak}可視為χ倍的Q_{fuel} ，而χ介於(0~1)之間

$$\Rightarrow Q_{leak} = \chi\, Q_{fuel}$$

將(1)代換Q_{fuel}後可得:

$$\Rightarrow Q_{leak} = \chi\, m_f \cdot Q_{LHV}$$

(4)

由(3)可知：$Q_{in} = Q_{fuel} - Q_{leak}$

將(1)(2)(4)代換後可得：

$$m_a \left[b_v (T_3' - T_2) + \frac{1}{2} k_1 (T_3'^2 - T_2^2) \right] = m_f \cdot Q_{LHV} - \chi\, m_f \cdot Q_{LHV}$$

(5)

將(5)的m_a除到等式右邊：

$$\left[b_v (T_3' - T_2) + \frac{1}{2} k_1 (T_3'^2 - T_2^2) \right] = \frac{m_f}{m_a} Q_{LHV} - \chi \frac{m_f}{m_a} Q_{LHV}$$

$$\Rightarrow b_v(T_3' - T_2) + \frac{1}{2}k_1(T_3'^2 - T_2^2) = (1-\chi)\frac{m_f}{m_a}Q_{LHV}$$

而$(\frac{m_a}{m_f})$：空燃比　$\lambda = \frac{(\frac{m_a}{m_f})_{實際}}{(\frac{m_a}{m_f})_{理論}}$

代入可得$b_v(T_3' - T_2) + \frac{1}{2}k_1(T_3'^2 - T_2^2) = \frac{(1-\chi)Q_{LHV}}{\lambda(\frac{m_a}{m_f})_{理論}}$

(6)

由(6)：$b_v(T_3' - T_2) + \frac{1}{2}k_1(T_3'^2 - T_2^2) = \frac{(1-\chi)Q_{LHV}}{\lambda(\frac{m_a}{m_f})_{理論}}$

$$\Rightarrow b_vT_3' - b_vT_2 + \frac{1}{2}k_1T_3'^2 - \frac{1}{2}k_1T_2^2 = \frac{(1-\chi)Q_{LHV}}{\lambda(\frac{m_a}{m_f})_{理論}}$$

$$\Rightarrow -b_vT_2 - \frac{1}{2}k_1T_2^2 = \frac{(1-\chi)Q_{LHV}}{\lambda(\frac{m_a}{m_f})_{理論}} - b_vT_3' - \frac{1}{2}k_1T_3'^2$$

$$\Rightarrow \frac{1}{2}k_1T_2^2 + b_vT_2 + [\frac{(1-\chi)Q_{LHV}}{\lambda(\frac{m_a}{m_f})_{理論}} - b_vT_3' - \frac{1}{2}k_1T_3'^2] = 0$$

由公式解：$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$\therefore T_2 = \frac{-b_v \pm \sqrt{b_v^2 - 2k_1[(1-\chi)\frac{Q_{LHV}}{\lambda(\frac{m_a}{m_f})_{理論}} - b_vT_3' - \frac{1}{2}k_1T_3'^2]}}{k_1}$$

(7)

由$T-S$圖可知：$T_2 \leq (T_3' = T_{MAX})$，故(7)的$T_2$可代換成：

$$\frac{-b_v \pm \sqrt{b_v^2 - 2k_1[(1-\chi)\dfrac{Q_{LHV}}{\lambda(\dfrac{m_a}{m_f})_{理論}} - b_v T_3' - \dfrac{1}{2}k_1 T_3'^2]}}{k_1} \leq T_3' \tag{8}$$

由$T-S$圖可知：$T_2 \geq (T_1 = T_{\min})$，將(7)的$T_2$代換可得：

$$\frac{-b_v \pm \sqrt{b_v^2 - 2k_1[(1-\chi)\dfrac{Q_{LHV}}{\lambda(\dfrac{m_a}{m_f})_{理論}} - b_v T_3' - \dfrac{1}{2}k_1 T_3'^2]}}{k_1} \geq T_1 \tag{9}$$

由(9)可知：$-b_v \pm \sqrt{b_v^2 - 2k_1[(1-\chi)\dfrac{Q_{LHV}}{\lambda(\dfrac{m_a}{m_f})_{理論}} - b_v T_3' - \dfrac{1}{2}k_1 T_3'^2]} \geq k_1 T_1$

$$\Rightarrow \sqrt{b_v^2 - 2k_1[(1-\chi)\dfrac{Q_{LHV}}{\lambda(\dfrac{m_a}{m_f})_{理論}} - b_v T_3' - \dfrac{1}{2}k_1 T_3'^2]} \geq k_1 T_1 + b_v$$

$$\Rightarrow b_v^2 - 2k_1[\frac{(1-\chi)Q_{LHV}}{\lambda(\dfrac{m_a}{m_f})_{理論}} - b_v T_3' - k_1 T_3'^2] \geq (k_1 T_1 + b_v)^2$$

$$\Rightarrow -2k_1[\frac{(1-\chi)Q_{LHV}}{\lambda(\dfrac{m_a}{m_f})_{理論}} - b_v T_3' - k_1 T_3'^2] \geq (k_1 T_1 + b_v)^2 - b_v^2$$

$$\Rightarrow 2k_1\left[\frac{(1-\chi)Q_{LHV}}{\lambda(\frac{m_a}{m_f})_{理論}}-b_v\,T_3'-k_1\,T_3'^2\right]\leq b_v^2-(k_1\,T_1+b_v)^2$$

$$\Rightarrow \frac{(1-\chi)Q_{LHV}}{\lambda(\frac{m_a}{m_f})_{理論}}-b_v\,T_3'-k_1\,T_3'^2\leq\frac{b_v^2-(k_1T_1+b_v)^2}{2k_1}$$

$$\Rightarrow \frac{(1-\chi)Q_{LHV}}{\lambda(\frac{m_a}{m_f})_{理論}}\leq\frac{b_v^2-(k_1T_1+b_v)^2}{2k_1}+b_v\,T_3'-k_1\,T_3'^2$$

$$\Rightarrow (1-\chi)\leq\left[\frac{b_v^2-(k_1T_1+b_v)^2}{2k_1}+b_v\,T_3'-k_1\,T_3'^2\right]\frac{\lambda(\frac{m_a}{m_f})_{理論}}{Q_{LHV}}$$

$$\Rightarrow -\chi\leq\left[\frac{b_v^2-(k_1T_1+b_v)^2}{2k_1}+b_v\,T_3'-k_1\,T_3'^2\right]\frac{\lambda(\frac{m_a}{m_f})_{理論}}{Q_{LHV}}-1$$

$$\Rightarrow \chi\geq 1-\left[\frac{b_v^2-(k_1T_1+b_v)^2}{2k_1}+b_v\,T_3'-k_1\,T_3'^2\right]\frac{\lambda(\frac{m_a}{m_f})_{理論}}{Q_{LHV}}$$

$$\Rightarrow \chi\geq 1-\left[\frac{-k_1T_1(k_1T_1+2b_v)}{2k_1}+b_v\,T_3'-k_1\,T_3'^2\right]\frac{\lambda(\frac{m_a}{m_f})_{理論}}{Q_{LHV}}$$

$$\Rightarrow \chi\geq 1-\left[\frac{-T_1(k_1T_1+2b_V)}{2}+b_v\,T_3'-k_1\,T_3'^2\right]\frac{\lambda(\frac{m_a}{m_f})_{理論}}{Q_{LHV}}$$

(10)

由之前有提到：$T_3'=T_{\max}$ ，$T_1=T_{\min}$

將(10)代換後可得：$\chi \geq 1 - \dfrac{\lambda(\frac{m_a}{m_f})_{理論}}{Q_{LHV}}[\dfrac{-T_{\min}(k_1 T_{\min} + 2b_v)}{2} + b_v T_{\max} + \dfrac{1}{2}k_1 T_{\max}{}^2]$

而χ的最小值即為只有等號成立時，故(10)可變為：

$$\chi_{\min} = 1 - \frac{\lambda(\frac{m_a}{m_f})_{理論}}{Q_{LHV}}[\frac{-T_{\min}(k_1 T_{\min} + 2b_v)}{2} + b_v T_{\max} + \frac{1}{2}k_1 T_{\max}{}^2]$$

(11)

由第(6)式可知：$b_v(T_3' - T_2) + \dfrac{1}{2}k_1(T_3'^2 - T_2^2) = \dfrac{(1-\chi)Q_{LHV}}{\lambda(\frac{m_a}{m_f})_{理論}}$

$$\Rightarrow (1-\chi) = \frac{\lambda(\frac{m_a}{m_f})_{理論}}{Q_{LHV}}\left[b_v(T_3' - T_2) + \frac{1}{2}k_1(T_3'^2 - T_2^2)\right]$$

$$\Rightarrow \chi = 1 - \frac{\lambda(\frac{m_a}{m_f})_{理論}}{Q_{LHV}}\left[b_v(T_3' - T_2) + \frac{1}{2}k_1(T_3'^2 - T_2^2)\right]$$

(12)

由$TV^{k-1} = C$取ln

$$\Rightarrow \ln T + \ln V^{k-1} = \ln C$$

$$\Rightarrow \ln T + (k-1)\ln V = \ln C$$

$$\Rightarrow \frac{1}{T}dT + (k-1)\frac{1}{V}dV = 0$$

(13)

由$k = \dfrac{C_P}{C_V} = \dfrac{C_V + R}{C_V} = \dfrac{R + (b_v + k_1 T)}{(b_v + k_1 T)}$

$$\Rightarrow k = 1 + \frac{R}{b_v + k_1 T}$$

$$\Rightarrow (k-1) = \frac{R}{b_v + k_1 T}$$

故(13)可變為：$\frac{1}{T}\,dT + \frac{R}{b_v + k_1 T}\,\frac{1}{V}\,dV = 0$

$$\Rightarrow (b_v + k_1 T)\,\frac{1}{T}\,dT + \mathrm{R}\,\frac{1}{V}\,dV = 0$$

$$\Rightarrow b_v\,\frac{1}{T}\,dT + k_1\,dT + \mathrm{R}\,\frac{1}{V}\,dV = 0$$

$$\Rightarrow b_v\,\frac{1}{T}\,dT + k_1\,dT = -\ \mathrm{R}\,\frac{1}{V}\,dV$$

積分可得：$b_v \int_{T_I}^{T_J} \frac{1}{T}\,dT + k_1 \int_{T_I}^{T_J} dT = -\mathrm{R} \int_{T_I}^{T_J} \frac{1}{V}\,dV$

$$\Rightarrow b_v \ln \frac{T_J}{T_I} + k_1 (T_J - T_I) = -\mathrm{R} \ln \frac{V_J}{V_I}$$

代入過程$[1 \rightarrow 2]$：$b_v \ln \frac{T_2}{T_1} + k_1 (T_2 - T_1) = \mathrm{R} \ln \frac{V_1}{V_2} = \mathrm{R} \ln \mathrm{r} \Rightarrow T_2$（給定$T_1$，r，R 可求出）

(14)

由(14)求出T_2後代入(12)而$T_3' = T_{\max}$，可求出χ

$$C_{pm} = a_p + k_1 T \qquad C_{vm} = b_v + k_1 T$$

$$Q_{out} = m \int_{T_1}^{T_4'} C_{vm}\,dT$$

$$= m_a \left[b_v (T_4' - T_1) + \frac{1}{2} k_1 (T_4'^2 - T_1^2) \right]$$

$$Q_{in} = m\int_{T_2}^{T_3'} C_{vm} dT$$

$$= m_a\left[b_v (T_3' - T_2) + \frac{1}{2} k_1 (T_3'^2 - T_2^2) \right]$$

$$T \cdot V^{k-1} = C$$

取 $\ln \Rightarrow \ln T + (k-1)\ln V = \ln C$ (18)

將第(18)式微分得到 $\frac{1}{T} dT + (k-1)\frac{1}{V} dV = 0$ (19)

$$k = \frac{Cp}{Cv} = \frac{Cv + R}{Cv}$$

$$\Rightarrow (k-1) = \frac{R}{b_v + k_1 T}$$ (20)

將第(20)式代回第(19)式，得到

$$b_v \frac{1}{T} dT + k_1 dT = (-\mathrm{R}\frac{dv}{v})$$

將第(18)式積分得到 $b_v \ln\frac{Tj}{Ti} + k_1 (T_j - T_i) = -\mathrm{R}\ln\frac{Vj}{Vi}$

$$[\ T_1 \to T_2\ ]: b_v \ln\frac{T_2}{T_1} + k_1(T_2 - T_1) = R\ln\frac{V_1}{V_2} = R\ln r \Rightarrow T_2$$

$$[\ T_3 \to T_4\ ]:\ b_v \ln\frac{T_4}{T_3} + k_1(T_4 - T_3) = R\ln\frac{1}{r} \Rightarrow T_4'$$

$$Q_{in} = Q_{fuel} - Q_{leak}$$

$$Q_{fuel} = m_f \, Q_{LHV}$$

$$Q_{leak} = \chi \, m_f \, Q_{LHV}$$

$$\Rightarrow m_a \left[b_v (T_3' - T_2) + \frac{1}{2} k_1 (T_3'^2 - T_2^2) \right] = m_f \, Q_{LHV} - \chi \, m_f \, Q_{LHV}$$

$$\lambda = \frac{(\frac{m_a}{m_f})_{實際}}{(\frac{m_a}{m_f})_{理論}}$$

$$\Rightarrow b_v (T_3' - T_2) + \frac{1}{2} k_1 (T_3'^2 - T_2^2) = \frac{(1-\chi) Q_{LHV}}{\lambda (\frac{m_a}{m_f})_{理論}} \Rightarrow T_3'$$

由熱力學第一定律(能量守衡)　W＝$Q_{in} - Q_{out}$

$$W = (m_f \, Q_{LHV} - \chi \, m_f \, Q_{LHV}) - m_a \left[b_v (T_4' - T_1) + \frac{1}{2} k_1 (T_4'^2 - T_1^2) \right]$$

$$\Rightarrow \; W = [(\frac{m_f}{m_a}) Q_{LHV} - (\frac{m_f}{m_a}) \chi \, Q_{LHV}] - \left[b_v (T_4' - T_1) + \frac{1}{2} k_1 (T_4'^2 - T_1^2) \right]$$

$$\Rightarrow \; W = \frac{(1-\chi) Q_{LHV}}{\lambda (\frac{m_a}{m_f})_{理論}} - \left[b_v (T_4' - T_1) + \frac{1}{2} k_1 (T_4'^2 - T_1^2) \right]$$

$$\eta = \frac{W}{Q_{in}}$$

$$= \frac{(m_f Q_{LHV} - \chi m_f Q_{LHV}) - m_a[b_v(T_4' - T_1) + \frac{1}{2}k_1(T_4'^2 - T_1^2)]}{(m_f Q_{LHV} - \chi m_f Q_{LHV})}$$

$$= 1 - \frac{m_a[b_v(T_4' - T_1) + \frac{1}{2}k_1(T_4'^2 - T_1^2)]}{(m_f Q_{LHV} - \chi m_f Q_{LHV})}$$

$$= 1 - \frac{b_v(T_4' - T_1) + \frac{1}{2}k_1(T_4'^2 - T_1^2)}{(\frac{m_f}{m_a})Q_{LHV} - \chi(\frac{m_f}{m_a})Q_{LHV}}$$

$$= 1 - \frac{b_v(T_4' - T_1) + \frac{1}{2}k_1(T_4'^2 - T_1^2)}{(1-\chi)\frac{Q_{LHV}}{(\frac{m_a}{m_f})_{ac}}}$$

六. 結論:

在這篇文章中，出現了一個新的適用於現實的且比之前更精確的關係介於燃料化學能量和穿過汽缸壁的熱損失之間，而在之前曾介紹過燃料化學能量和穿過汽缸壁的熱損失是發生在某段有效溫度之間。在文章中有提到熱損失參數和被運用的燃油能量是相輔相成的，而且他們是不能被任意選擇的。熱損失量如果是被考慮成一個傳送燃料能量的百分比。很顯然的，當燃料的總能量被運用，其循環的最大溫度將達到一個我們所不希望達到的過高標準。假設任何熱損失都是發生在穿越冷卻水的狀況，其預期的最大循環溫度將會比無熱損失時的狀況還低一些，這篇文章主要注重在溫度上，這次僅完成推導的範圍，希望在未來時間接下來研究者可以將數據導入公式中，以求實際與理論是相合的，且可以再做進一步的分析。

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