Inverse problem of estimating the heat flux at the roller/workpiece interface during a rolling process

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**Article info**

**Abstract**

In this study, an inverse algorithm based on the conjugate gradient method and the discrepancy principle is applied to estimate the unknown space-dependent heat flux at the roller/workpiece interface during rolling process from the knowledge of temperature measurements taken within the roller. It is assumed that no prior information is available on the functional form of the unknown heat flux; hence the procedure is classified as the function estimation in inverse calculation. The temperature data obtained from the direct problem are used to simulate the temperature measurements, and the effect of the errors in these measurements upon the precision of the estimated results is also considered. The results show that an excellent estimation on the space-dependent heat flux can be obtained for the test cases considered in this study.

**1. Introduction**

Nowadays, rolling has been widely used in many manufacturing processes, from making steel beams with hundreds of millimeter in thickness to aluminum film thinner than a sheet of paper, that it has become one of the fundamental and irreplaceable machining methods in manufacturing industry. In this, thicker material or workpiece is made thinner by passing through a pair of counter-rotating rollers. In some circumstances, the workpiece has been heated to a high temperature to increase its plasticity before rolling to ease the rolling process. During the rolling, plastic deformation of the workpiece occurs at the bite region where additional heat is generated due to the friction occurring at the rollers/workpiece interface as well as the plastic work of the workpiece. Part of this heat is carried away by the advancing workpiece while the rest is transferred to the rollers and eventually dissipated either by forced- or natural-convection cooling. However, the heat is generated at a relatively small bite region compared to the overall volumes of rollers and workpiece, hence giving rise to very large temperature gradients at the vicinity of the roller/workpiece interface. The elevated temperature gradients cause thermal stresses inside rollers and workpiece alike and pose profound impacts to the endurance of the rollers and, more importantly, the quality of the rolled workpiece. Therefore, thermal analysis of the rolling process is very important to the design of a rolling mill, and the rolling process has been and will continue to be one of the most studied subjects because of its great importance in engineering.

Since a large portion of heat generated at the roller/workpiece interface is the direct consequence of the friction between the roller and workpiece, to establish the functional relation between the generated heat and those factors involved in friction is the primary objective of many theoretical and experimental studies, for example [1–3]. In a theoretical analysis, it is a common practice to adopt either one of the following assumptions to deal with the thermal contact in friction: (1) perfect contact where equal surface temperature of the objects involved in friction is assumed and (2) imperfect contact where a heat resistance is introduced to the contact surface of the friction objects due to the presence of the third body [4]. Considering the fact that the friction at the roller/workpiece interface is affected by a number of contact-surface factors, including surface roughness, contact pressure, rotating speed, coolant lubricant, oxide layer, and others, imperfect contact is a more reasonable approach to simulate the thermal contact in friction. Bardon [5] proposed an expression to describe the interfacial heat exchange in friction. Two contact parameters, the thermal contact resistance (TCR) and the intrinsic heat partition coefficient (IHPT), have been introduced to account for the thermal effects produced by an imperfect contact. However, to precisely quantify the two parameters with respect to the above mentioned contact-surface factors is far from a trivial task due to the complex physics involved and the difficulty in obtaining measurements at the interface vicinity [3]. To date, the precise functional relation of the two parameters versus the contact-surface factors has not been fully finalized. To tie the loose ends and settle the functional forms still requires years of theoretical and experimental research to achieve.

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doi:10.1016/j.applthermaleng.2010.02.007
In recent years, the studies of inverse heat conduction problem (IHCP) have offered convenient alternatives, which largely scale down complex experimental work, to obtain accurate thermal physical quantities such as heat sources, material’s thermal properties, and boundary temperature or heat flux distributions, in many heat conduction problems [6–9]. In these cases, the direct heat conduction problems are concerned with the determination of temperature at interior points of a region when the initial and boundary conditions, heat generation, and material properties are specified, whereas, the inverse heat conduction problem involves the determination of the surface conditions, energy generation, thermal physical properties, etc., from the knowledge of temperature measurements taken within the body. To date, a variety of analytical and numerical techniques have been developed for the solution of the inverse heat conduction problems, for example, the conjugate gradient method (CGM) [10–13], the function-specification method [14], the space-marching method [15], and the Kalman filter method [16].

Despite the shear number of papers dedicated to the study of friction processes, there have been relatively fewer attempts to apply inverse methods to estimate thermal quantities in such processes. Huang et al. [17] used conjugate-gradient inverse method to estimate the transient heat flux on the surface of a roller in a rolling process. Although a roller is subjected to different types of heat transfer, mainly heat conduction at the bite region and heat convection at other regions cooled by fluid, along its outer surface, the study assumed a time-varying heat flux distribution around the outer surface of the roller to account for those different heat-transfer effects to simplify the boundary condition. Excellent agreement between the estimated and exact heat flux was reported. However, the authors reported that accuracy of estimation showed a certain degree of sensitivity to the location of temperature sensors, that is, the accuracy deteriorates as the temperature measurement locations moved away from the roller surface. Chen and Tuan [18] performed a least-square inverse analysis to estimate interface conductance between periodically contacting surfaces. Part from advertising the accuracy of the least-square method, the authors also showed several advantages of the proposed inverse method over the conjugate-gradient inverse method. Later, Bazuin and Laraqi [4] also used least-square inverse analysis to simultaneously estimate heat flux generated by friction, TCR, and IHPT in a dry friction between two solids. A simple transient one-dimensional model was investigated, and a sensitivity analysis was performed to realize the conditions for the three estimated quantities to be returned accurately. Some useful guidelines have been drawn, including the locations of temperature measurements, the onset of estimation, and the criterion between the time step and the density of sensors. Chen et al. [19] estimated the heat generated at the interface of two cylindrical bars during a continuous-drive welding process by using a conjugate-gradient inverse method. The results showed that excellent estimation can be achieved by placing the temperature measurement location near the interface where friction occurs.

The focus of the present study is to develop a two-dimensional inverse analysis for estimating the heat flux at the roller/workpiece interface during a rolling process using temperature measurements taken within the roller, which can offer invaluable data allowing the functional relationship of TCR and IHCP with respect to contact-surface factors to be established. In this study, different boundary conditions are applied at different regions on the roller’s outer surface to account for the effects caused by different heat transfer mechanisms. Here, we present the conjugate gradient method and the discrepancy principle [20] to estimate the heat generation at the roller/workpiece interface by using the simulated temperature measurements. Subsequently, the distributions of temperature in the roller can be determined as well. The conjugate gradient method with an adjoint equation, also called Alifanov’s iterative regularization method, belongs to a class of iterative regularization techniques, which mean the regularization procedure is performed during the iterative processes, thus the determination of optimal regularization conditions is not needed. The CGM is derived based on the perturbation principles and transforms the inverse problem into the solutions of three problems, namely, the direct, the sensitivity, and the adjoint problems, which will be discussed in detail in the following sections. On the other hand, the discrepancy principle is used to terminate the iteration process in the conjugate gradient method.

2. Analysis

2.1. Direct problem

To illustrate the methodology for developing expressions for the use in determining the unknown space-dependent heat flux entering the roller through the roller-workpiece interface during rolling process, we consider the rolling process depicted in the simplified scheme of Fig. 1. Two cylinders, with parallel axes and separated by a distance e, rotates in opposite directions. The workpiece enters the rolling machine with an initial thickness \( e_0 \) and exits with a thickness of \( e \). In this study, the model developed by Hamraoui [3] is used to describe the thermal behaviour of a single roller.
The roller receives a surface heat flux $q(\theta)$ over a portion of its outer surface ($-\beta \leq \theta \leq \beta$) and is cooled by convection ($-\pi \leq \theta \leq -\beta$ and $\beta \leq \theta \leq \pi$) with the environment. Under the condition of axisymmetry, the governing equations and the associated boundary and initial conditions for the roller can be written as [3]:

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} - \frac{\omega}{\zeta} \frac{\partial T}{\partial \theta} = 0,$$  

(1)

$$k \frac{\partial T}{\partial r} = q(\theta), \quad r = a_1, \quad -\beta \leq \theta \leq 0,$$  

(2)

$$k \frac{\partial T}{\partial r} = h_1[T - T_{amb}], \quad r = a_1, \quad -\pi \leq \theta \leq -\beta \quad \text{and} \quad \beta \leq \theta \leq \pi,$$  

(3)

$$T(r, -\pi) = T(r, \pi), \quad 0 \leq r \leq a_1,$$  

(4)

$$\frac{\partial T(r, -\pi)}{\partial \theta} = \frac{\partial T(r, \pi)}{\partial \theta}, \quad 0 \leq r \leq a_1,$$  

(5)

where $\omega$, $\zeta$, and $k$ are the constant angular speed, thermal diffusivity, and thermal conductivity of the roller, respectively, $h_1$ is the heat convection coefficient for the outer surface and $T_{amb}$ is the ambient temperature. Eqs. (4) and (5) represent the periodicity conditions of heat transfer along the angular direction. The direct problem considered here is concerned with the determination of the medium temperature when the surface heat flux $q(\theta)$, thermal physical properties of the roller, and boundary conditions are known.

### 2.2. Inverse problem

For the inverse problem, the function $q(\theta)$ is regarded as being unknown, while everything else in Eqs. (1)–(5) is known. In addition, temperature readings taken at some appropriate locations of the roller are considered available. The objective of this inverse analysis is to predict the unknown space-dependent function of heat flux, $q(\theta)$, merely from the knowledge of these temperature readings. Let the measured temperature at the measurement position $(r, \theta) = (r_m, \theta_i)$ be denoted by $Y(r_m, \theta_i)$. Here $i = 1 \sim M$, while $M$ is the total measurement points in the $\theta$ direction. In addition, $r_m$ represents the radius of which the thermocouples are located. Then this inverse problem can be stated as follows: by utilizing the above mentioned measured temperature data $Y(r_m, \theta_i)$, the unknown heat flux $q(\theta)$ is to be estimated over the specified space domain.

The solution of the present inverse problem is to be obtained in such a way that the following functional is minimized:

$$J[q(\theta)] = \sum_{i=1}^{M} |T(r_m, \theta_i) - Y(r_m, \theta_i)|^2,$$  

(6)

where $T(r_m, \theta_i)$ is the estimated (or computed) temperature at the measurement location $(r, \theta) = (r_m, \theta_i)$. In this study, $T(r_m, \theta_i)$ is determined from the solution of the direct problem given previously by using an estimated $q(\theta)$ for the exact $q(\theta)$, here $q(\theta)$ denotes the estimated quantities at the $K$th iteration. In addition, in order to develop expressions for the determination of the unknown $q(\theta)$, a “sensitivity problem” and an “adjoint problem” are constructed as described below.

### 2.3. Sensitivity problem and search step size

The sensitivity problem is obtained from the original direct problem defined by Eqs. (1)–(5) in the following manner: it is assumed that when $q(\theta)$ undergoes a variation $\Delta q(\theta)$, $T(r, \theta)$ is perturbed by $\Delta T(r, \theta)$. Then replacing in the direct problem $q$ by $q + \Delta q$ and $T$ by $T + \Delta T$, subtracting from the resulting expressions the direct problem, and neglecting the second-order terms, the following sensitivity problem for the sensitivity function $\Delta T$ can be obtained.

$$\frac{\partial^2 \Delta T}{\partial r^2} + \frac{1}{r} \frac{\partial \Delta T}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \Delta T}{\partial \theta^2} - \frac{\omega}{\zeta} \frac{\partial \Delta T}{\partial \theta} = 0,$$  

(7)

$$k \frac{\partial \Delta T}{\partial r} = \Delta q(\theta), \quad r = a_1, \quad -\beta \leq \theta \leq 0,$$  

(8)

$$k \frac{\partial \Delta T}{\partial r} = -h_1[\Delta T], \quad r = a_1, \quad -\pi \leq \theta \leq -\beta \quad \text{and} \quad \beta \leq \theta \leq \pi,$$  

(9)

$$\Delta T(r, -\pi) = \Delta T(r, \pi), \quad 0 \leq r \leq a_1,$$  

(10)

$$\frac{\partial \Delta T(r, -\pi)}{\partial \theta} = \frac{\partial \Delta T(r, \pi)}{\partial \theta}, \quad 0 \leq r \leq a_1.$$  

(11)

The sensitivity problem of Eqs. (7)–(11) can be solved by the same method as the direct problem of Eqs. (1)–(5).

### 2.4. Adjoint problem and gradient equation

To formulate the adjoint problem, Eq. (1) is multiplied by the Lagrange multiplier (or adjoint function) $\lambda$, and the resulting expression is integrated over the correspondent space domains. Then the result is added to the right hand side of Eq. (6) to yield the following expression for the functional $J[q(\theta)]$:

$$J[q(\theta)] = \sum_{i=1}^{M} |T(r_m, \theta_i) - Y(r_m, \theta_i)|^2 + \int_{\theta_1}^{\theta_2} \int_{r_1}^{r_2} \lambda(r, \theta) \left[ \frac{\partial^2 T}{\partial \theta^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} - \frac{\omega}{\zeta} \frac{\partial T}{\partial \theta} \right] \, dr \, d\theta.$$  

(12)

The variation $\Delta J$ is derived by perturbing $q(\theta)$ by $\Delta q(\theta)$, $T(r, \theta)$ is perturbed by $\Delta T(r, \theta)$ in Eq. (12). Subtracting from the resulting expression the original Eq. (12) and neglecting the second-order terms, we thus find:

$$\Delta J[q(\theta)] = 2 \int_{\theta_1}^{\theta_2} \int_{r_1}^{r_2} \sum_{i=1}^{M} [T(r, \theta) - Y(r, \theta)] \Delta T \cdot \delta(r - r_m) \cdot \delta(\theta - \theta_i) \, dr \, d\theta + \int_{\theta_1}^{\theta_2} \int_{r_1}^{r_2} \lambda(r, \theta) \cdot \left[ \frac{\partial^2 \Delta T}{\partial r^2} + \frac{1}{r} \frac{\partial \Delta T}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \Delta T}{\partial \theta^2} - \frac{\omega}{\zeta} \frac{\partial \Delta T}{\partial \theta} \right] \, dr \, d\theta.$$

(13)

where $\delta(\cdot)$ is the Dirac function. We can integrate the second double integral term in Eq. (13) by parts, utilizing the initial and boundary conditions of the sensitivity problem. The vanishing of the
integrands containing $\Delta T$ leads to the following adjoint problem for the determination of $\lambda(r, \theta)$:

$$
\frac{\partial^2 \lambda}{\partial r^2} + \frac{1}{r} \frac{\partial \lambda}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \lambda}{\partial \theta^2} = 0,
$$

(14)

$$
\frac{\partial \lambda}{\partial r} = 0, \quad r = a_1, \quad -\beta \leq \theta \leq \beta,
$$

(15)

$$
\frac{\partial \lambda}{\partial r} = -h \lambda, \quad r = a_1, \quad -\pi \leq \theta \leq -\beta \quad \text{and} \quad \beta \leq \theta \leq \pi,
$$

(16)

$$
\frac{\partial \lambda}{\partial \theta} = \frac{\partial \lambda(r, -\pi)}{\partial \theta}, \quad 0 \leq r \leq a_1,
$$

(17)

$$
\frac{\partial \lambda}{\partial \theta} = \frac{\partial \lambda(r, \pi)}{\partial \theta}, \quad 0 \leq r \leq a_1.
$$

(18)

Then the adjoint problem can be solved by the same method as the direct problem.

Finally the following integral term is left:

$$
\Delta J = \int_{\beta = 0}^{\beta = \pi} \left[ q_K(r, \theta)/K \right] \cdot \Delta q \, d\theta.
$$

(19)

From the definition used in Ref. [11], we have:

$$
\Delta J = \int_{\beta = 0}^{\beta = \pi} f(\theta) \Delta q \, d\theta,
$$

(20)

where $f(\theta)$ is the gradient of the functional $J(q)$. A comparison of Eqs. (19) and (20) leads to the following form:

$$
f(\theta) = \left[ q_K(r, \theta)/K \right]_{\beta = a_1}.
$$

(21)

### 2.5. Conjugate gradient method for minimization

Assuming the functions of $T(r, \theta)$, $\Delta T(r, \theta)$, $\lambda(r, \theta)$, and $f(\theta)$ are available at the $k$th iteration, the following iteration process based on the conjugate gradient method is now used for the estimation of $q(\theta)$ by minimizing the above functional $J(q(\theta))$:

$$
q^{K+1}(\theta) = q^K(\theta) - \beta^K p^K(\theta), \quad K = 0, 1, 2, \ldots.
$$

(22)

where $q^{K+1}(\theta)$ is the value of $q(\theta)$ at the $(K+1)$th step, $\beta^K$ is the search step size in going from iteration $K$ to iteration $K+1$, and $p^K(\theta)$ is the direction of descent (i.e., search direction) given by

$$
p^K(\theta) = f^K(\theta) + \gamma^K p^{K-1}(\theta),
$$

(23)

which is conjugation of the gradient direction $f^K(\theta)$ at iteration $K$ and the direction of descent $p^{K-1}(\theta)$ at iteration $K-1$. The conjugate coefficient $\gamma^K$ is determined from

$$
\gamma^K = \frac{\sum_{i=1}^{M} [f^K(\theta) \cdot \delta(\theta - \theta_i)]^2}{\sum_{i=1}^{M} [f^{K-1}(\theta) \cdot \delta(\theta - \theta_i)]^2}
$$

with $\gamma^K = 0$.

(24)

The convergence of the above iterative procedure in minimizing the functional $J$ is proved in Ref. [11]. To perform the iterations according to Eq. (22), we need to compute the step size $\beta^K$ and the gradient of the functional $f^K(\theta)$.

The functional $J[q^{K+1}(\theta)]$ for iteration $K+1$ is obtained by rewriting Eq. (6) as:

$$
J[q^{K+1}(\theta)] = \sum_{i=1}^{M} \left[ T(q^K - \beta^K p^K) - Y(r_m, \theta_i) \right]^2.
$$

(25)

where we replace $q^{K+1}$ by the expression given by Eq. (22). If temperature $T(q^K - \beta^K p^K)$ is linearized by a Taylor expansion, Eq. (25) takes the form:

$$
J[q^{K+1}(\theta)] = \sum_{i=1}^{M} \left[ T(q^K) - \beta^K \Delta T(p^K) - Y(r_m, \theta_i) \right]^2.
$$

(26)

where $T(q^K)$ is the solution of the direct problem at $(r, \theta) = (r_m, \theta_i)$ by using estimated $q^K(\theta)$ for exact $q(\theta)$. The sensitivity function $\Delta T(p^K)$ are taken as the solution of Eqs. (7)–(11) at the measured position $(r, \theta) = (r_m, \theta_i)$ by letting $\Delta q = p^K$ [11]. The search step size $\beta^K$ is determined by minimizing the functional given by Eq. (26) with respect to $\beta^K$. The following expression can be obtained:

$$
\beta^K = \frac{\sum_{i=1}^{M} \Delta T(p^K) / |T(q^K) - Y|}{\sum_{i=1}^{M} \Delta T(p^K)^2}.
$$

(27)

### 2.6. Stopping criterion

If the problem contains no measurement errors, the traditional check condition is specified as:

$$
J(q^{K+1}) < \eta.
$$

(28)

where $\eta$ is a small specified number, can be used as the stopping criterion. However, the observed temperature data may contain measurement errors. Therefore, it is not expected that the functional in Eq. (6) will be equal to zero at the final iteration step. Computational experience has shown that it is advisable to use the discrepancy principle [20] for terminating the iteration process in the conjugate gradient method, i.e., it is assumed that the residuals for the temperatures may be approximated by $\Delta(r_m, \theta_i) = \sigma$, where $\sigma$ is the standard deviation of the temperature measurements, which is assumed to be a constant. Substituting the approximation $\Delta(r_m, \theta_i) = \sigma$ into Eq. (6), the following expression is obtained for the stopping criteria $\eta$:

$$
\eta = M \sigma^2
$$

(29)

Finally the stopping criterion is given by Eq. (28) with $\eta$ determined from Eq. (29).

### 2.7. Computational procedure

The computational procedure for the solution of this inverse problem may be summarized as follows:

1. **Step 1**: Solve the direct problem given by Eqs. (1)–(5) for $T(r, \theta)$.
2. **Step 2**: Examine the stopping criterion given by Eq. (28) with $\eta$ given by Eq. (29). Continue if not satisfied.
3. **Step 3**: Solve the adjoint problem given by Eqs. (14)–(18) for $\lambda(r, \theta)$.
4. **Step 4**: Compute the gradient of the functional $J$ from Eq. (21).
5. **Step 5**: Compute the conjugate coefficient $\gamma^K$ and direction of descent $p^K$ from Eqs. (24) and (23), respectively.
6. **Step 6**: Set $\Delta q(\theta) = p^K(\theta)$ and solve the sensitivity problem given by Eqs. (7)–(11) for $\Delta T(r, \theta)$.
7. **Step 7**: Compute the search step size $\beta^K$ from Eq. (27).
8. **Step 8**: Compute the new estimation for $q^{K+1}(\theta)$ from Eq. (22) and return to Step 1.

### 3. Results and discussion

The numerical procedure in this study is based on the unstructured-mesh, fully collocated, finite-volume code, ‘STREAM’ developed by the first named author. This is the descendent of the structured-mesh, multi-block code of ‘STREAM’ [21]. Hamraoui [3] conducted a grid-dependence test on the direct problem and concluded that a 60 × 120 mesh respectively in the $r$- and $\theta$-direction is fine enough to secure a grid-independent solution. Given in Fig. 2a, the computational mesh in this study is 70 × 130 in size, which is even finer than the mesh used in [3]. In this mesh, cells have been distributed non-uniformly to enhance numerical resolution in some critical regions. Here, the outer surface ($r = a_1$) and the bite
region, where the temperature gradient is largest, are the most important parts in this problem, and fine cells are used toward the outer surface and in the vicinity of the bite region. On the other hand, coarser cells are applied toward the center of the roller where temperature gradient becomes much gentler. To versify the present code for the use in this problem, a test case is conducted with the geometrical and thermal parameters identical to the benchmark test case in [3]. These parameters are:

\[
k = 10 \text{ W m}^{-1} \text{K}^{-1}, \quad \alpha = 2.67 \times 10^{-6} \text{ m}^2 \text{s}^{-1}, \quad a_1 = 0.01 \text{ m}, \quad h_1 = 100 \text{ W m}^{-2} \text{K}^{-1}, \quad q = 10^5 \text{ W m}^{-2}, \quad \omega = 0.5 \text{ rads}^{-1}, \text{ and } \beta = 0.16 \text{ rad}.
\]

The temperature contours of the roller are illustrated in Fig. 2b, whereas the comparison of the outer-surface temperature distributions obtained by the present code and that in [3] is given in Fig. 3. Fig. 2b confirms that temperature gradient is particularly large at the vicinity of the bite region where severe thermal stresses could develop as a result. The excellent agreement between the present study and Hamraoui’s demonstrated in Fig. 3 verifies the correctness of the USTREAM code used for this study.

The objective of this article is to validate the present approach when used in estimating the unknown space-dependent heat generation at the roller/workpiece interface during a rolling process accurately with no prior information on the functional form of the unknown quantities, a procedure called function estimation. However, we did not have a real experimental setup to measure temperature. Instead, we use USTREAM code to solve the direct problem first to obtain the so-called “simulated temperature measurement”, which serves as our temperature measurement to be used in the inverse calculation. In order to illustrate the accuracy of the present inverse analysis, we consider the simulated exact value of \( q(\theta) \) as:

\[
q(\theta) = 10^5 \left[ 1 + 0.3 \sin \left( \frac{\theta + \beta}{2\beta} \pi \right) \right] \text{ W m}^{-2}.
\]

The function \( q(\theta) \) is so chosen that, inside the bite region, the heat flux is at its maximum at the center and decreases towards the two edges. In the current paper, four test cases are conducted to, first, verify the correctness of the inverse method and second, to investigate the effects of two important parameters, namely, the rotational speed \( \omega \) of the roller and the locations of the temperature measurement \( r_m \), on the accuracy of the inverse method. The geometry and thermal properties of the four test cases are mostly identical to those in the validation test case except \( q(\theta) \) being in the functional form of Eq. (30) and \( \omega \) being either 0.5 rads\(^{-1}\) or 5 rads\(^{-1}\). The values of \( \omega \) and \( r_m \) of the four test cases are given in Table 1.

As seen in the above table, \( \omega \) at the values of 0.5 rads\(^{-1}\) and 5 rads\(^{-1}\) signify the roller operating at low and high speed, respectively. Meanwhile, \( r_m = 0.0099 \) and 0.0085 m represent the temperature measurement locations being almost on the bite region and a distance away from the bite region, respectively. In the above two locations, the temperature sensors are assumed to be arranged from \( \theta = -\beta \) to \( \beta \), corresponding to the range of bite region in the \( \theta \)-direction.

In the analysis, we do not have a real experimental set up to measure the temperature \( Y(r_m, \theta) \) in Eq. (6). Instead, we assume

<table>
<thead>
<tr>
<th>Case</th>
<th>( \omega ) (rads(^{-1}))</th>
<th>( r_m ) (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.5</td>
<td>0.0099</td>
</tr>
<tr>
<td>2</td>
<td>5.0</td>
<td>0.0099</td>
</tr>
<tr>
<td>3</td>
<td>0.5</td>
<td>0.0085</td>
</tr>
<tr>
<td>4</td>
<td>5.0</td>
<td>0.0085</td>
</tr>
</tbody>
</table>
a real heat flux \(q(\theta)\), of Eq. (30), and substitute the exact \(q(\theta)\) into the direct problem of Eqs. (1)–(5) to calculate the temperatures at the locations where the temperature sensors are placed. Here, we assume that there are 20 sensors, that is, \(M = 20\). The results are taken as the computed temperature \(Y_{\text{exact}}(r_m, \theta_i)\). Nevertheless, in reality, the temperature measurements always contain some degree of error, whose magnitude depends upon the particular measuring method employed. In order to take the measurement errors into account, a random error noise is added to the above computed temperature \(Y_{\text{exact}}(r_m, \theta_i)\) to obtain the measured temperature \(Y(r_m, \theta_i)\).

Hence, the measured temperature \(Y(r_m, \theta_i)\) is expressed as

\[
Y(r_m, \theta_i) = Y_{\text{exact}}(r_m, \theta_i) + \sigma \epsilon,
\]

where \(\epsilon\) is a random variable within \([-2.576, 2.576]\) for a 99% confidence bounds, and \(\sigma\) is the standard deviation of the measurement. The measured temperature \(Y(r_m, \theta_i)\) generated in such way is the so-called simulated measurement temperature.

The estimated values of the unknown function \(q(\theta)\), obtained with the initial guesses \(q^0 = 0.0\) and measurement error of deviation \(\sigma = 0.0\), and the temperature at the measurement locations for cases 1 and 2 are shown in Fig. 4a and b, respectively. These results in Fig. 4a confirm that the estimated results are in very good agreement with those of the exact values, with maximum errors being around 1% for both cases. The results also demonstrate that the variation of roller’s rotational speed \(\omega\) has little effect on the accuracy of the estimation, that is, the accuracy of the inverse method is independent of \(\omega\). The measurement temperature distributions in Fig. 4b show an even better agreement between the inverse and exact values, with maximum errors well below 1%. Since the correction of the estimated quantity is driven by the magnitude of difference between the estimated and measured quantities at the measurement location, the excellent agreement in Fig. 4b serves as an indicator of the inverse procedure being converged.

The estimated \(q(\theta)\), again obtained with the initial guesses \(q^0 = 0.0\) and measurement error of deviation \(\sigma = 0.0\), and the temperature at the measurement locations for cases 3 and 4 are shown in Fig. 5a and b, respectively. In these two cases, the temperature sensors have been moved a distance away from the bite region.

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**Fig. 4.** Estimated \(q(\theta)\) and the temperature distributions at the measurement location of cases 1 and 2 with the initial guesses \(q^0 = 0.0\) and measurement error of deviation \(\sigma = 0.0\); (a) \(q(\theta)\) distributions and (b) temperature distributions at the measurement location.

**Fig. 5.** Estimated \(q(\theta)\) and the temperature distributions at the measurement location of cases 3 and 4 with the initial guesses \(q^0 = 0.0\) and measurement error of deviation \(\sigma = 0.0\); (a) \(q(\theta)\) distributions and (b) temperature distributions at the measurement location.
to investigate the impact of measurement locations on the accuracy of inverse method. Huang et al. [17] reported that the estimation accuracy exhibits a certain degree of sensitivity on the measurement location in their transient inverse problem. However, this may not be the case in the present steady inverse problem. Unlike the parabolic time-marching governing equation in a transient problem, the governing equation in the present study is steady-state. The characteristic of time-marching introduces a ‘lag effect’ which presents a challenge for any inverse method because any disturbance of estimated quantity needs a certain period of time to propagate to the measurement quantity if they are not located at the same place. This lag effect poses an impact on the accuracy of inverse method, and the impact becomes greater if the distance between the estimated and measurement quantities is longer [22]. However, such impact does not appear in the current steady-state problem where estimated and measurement quantities are strongly dependent, and the accuracy of the inverse method is less sensitive to their relative positions inside the domain. The results shown in Fig. 5 indicate that estimated \( q(\theta) \) and the temperatures at measurement location are all in very good agreement with the exact values, verifying that by moving the measurement location a distance away from the bite region does not compromise the accuracy of the inverse method. This test poses significant implication that in practice, temperature sensors can be deployed at convenient locations without sacrificing accuracy.

With the heat flux at the bite region well estimated, the temperature distributions within the domain can be calculated accurately. This is demonstrated in Fig. 6 where the comparison of circumferential temperature distributions, at \( r = 0.001, 0.005, \) and 0.009 m, respectively, between the exact and inverse solutions for case 1 is given. The plots show that the field temperature distributions of the inverse solution at the above three positions are almost identical to those of the exact solution.

Finally, Fig. 7 illustrates the inverse solutions of \( q(\theta) \) and measurement temperature distributions, obtained with the initial guess values \( q_0 = 0 \), measurement error of deviation \( \sigma = 0.01 \), and temperature measurement taken at \( r_m = 0.0099 \) m. For a temperature of unity and 99% confidence, that standard deviation, \( \sigma = 0.01 \), corresponds to measurement error of 2.58%, which can be seen in Fig. 7a. In the meantime, Fig. 7b reveals that the maximum error of the estimated \( q(\theta) \) in this case is about 3%. This shows that for the cases considered in this study, the involvement of measurement error does not cause obvious deterioration on the accuracy of the inverse solution.

\[ \text{Fig. 6. Comparison of field temperature distributions of case 1.} \]

\[ \text{Fig. 7. Estimated } q(\theta) \text{ and the temperature distributions at the measurement location of case 1 with the initial guesses } q_0 = 0.0 \text{ and measurement error } \sigma = 0.001; \]

(a) temperature distributions at the measurement location and (b) \( q(\theta) \) distributions.

4. Conclusion

An inverse algorithm based on the conjugate gradient method and the discrepancy principle was successfully applied for the solution of the inverse problem to determine the unknown space-dependent heat flux at the roller/workpiece interface during a rolling process, while knowing the temperature distributions at some measurement locations. Subsequently, the temperature variations in the system can be calculated. Numerical results show that the method proposed herein can accurately estimate the space-dependent heat flux and temperature distributions for those problems in two different roller rotational speeds and the two temperature measurement locations investigated in this paper. Even when the inevitable measurement errors are involved, the estimation accuracy has not been seriously compromised. The proposed inverse algorithm does not require prior information for the functional form of the unknown quantities to perform the inverse calculation, and excellent estimated values can be obtained for the
considered problem. The methodology of the present study can serve as a useful tool to establish the complex functional relationship of TCR and IHPC with respect to contact-surface factors.

Acknowledgements

This work was supported by the National Science Council, Taiwan, Republic of China, under the Grant Numbers NSC 97-2221-E-168-041 and NSC 97-2221-E-168-039. The authors are also grateful for the help from Mrs. Shin-Liang Chen in Fooyin University.

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