Optimization of Color Rendering of Light Mixtures

Ku Chin Lin

Department of Mechanical Engineering, Kun Shan University, Taiwan.

Abstract: A method is developed for optimization of color rendering for a light mixture. No derivatives of the rendering function are required. Constraints of color correlated temperature and approximately white can be incorporated. Applicability has been illustrated by simulation and a mixture of 3 colors is demonstrated in this paper.

Keywords: color rendering, optimization, LED.

1. Introduction

Light emitted diode (LED) has been manufactured in diverse forms to replace the incandescent and fluorescent lamps and it has many merits for indoor and outdoor illumination [1]. The color rendering ability of LED light has been quantified and used to indicate the illumination quality. A review of color rendering measures can be found in [2]. Among the measures, the color rendering index (CRI) proposed by the Commission Internationale de I'Eclairage (CIE) is currently widely used although it could be questionable [3].

The CRI of light is closely related to the visible spectrum and richness of peak wavelength of the light and it is evaluated by comparing itself with a reference light. Since both daylight and thermal radiators contain rich power at all visible bands, they are officially defined as reference illuminators for correlated color temperature (CCT) above and below 5000 K, respectively. However, the double references induce singularity of the CRI evaluation at 5000 K.

Mixtures of red, green and blue (RGB) colors are commonly used to enhance the color rendering ability of light [4]. Despite reduction of luminous efficacy of radiation could occur, studies have shown that 3- or 4-color mixtures can achieve good balance between gaining the CRI and luminous efficacy [5]. However, very few literatures have studied the involved optimization. This paper presents such a study.

2. Definition of the Problem

The goal of this study is devoted to optimizing the CRI for a light mixture of colors under constraints:

$$Max\ R_a(f, \tilde{f}) \tag{1}$$

where R_a denotes the CRI function; $\tilde{f}$ the spectral power distribution (SPD) of reference illuminators; f the SPD of mixture which is assumed a linear combination of the SPDs of compound illuminators, *i.e.*

$$f(\lambda) = \sum_{i=1}^{n} w_i f_i(\lambda) \tag{2}$$

where n denotes the number of illuminators; w_i denotes the relative intensity of the i^{th} illuminator; $(w_1, w_2, \ldots, w_n)$ a point in the mixture space R^n.

The constraints under the above optimization may include the followings:

$$\sum_{i=1}^{n} w_i = 1 \tag{3a}$$

$$w_{il} \le w_i \le w_{iu},\ i = 1,2,\ldots,n \tag{3b}$$

$$T_1 \le CCT \le T_2 \tag{3c}$$

$$DC = \sqrt{(u-u_r)^2 + (v-v_r)^2} < 0.0054 \tag{3d}$$

where w_{il} and w_{iu} denote the lower and upper bounds of w_i; T_1 and T_2 the lower and upper bounds of CCT of the mixture; (u, v) the CIE 1960 UCS coordinate of the mixture; (u_r, v_r) the UCS coordinate of reference illuminators which have the same CCT as the mixture.

Note that a constraint of Eqs (3) could conflict with another if it is not properly set. For a mixture of RGB+white (RGBW), the intensity of white color must be properly bounded to achieve a designated CCT.

Mathematically, a constraint could be a hyperplane in R^n and a line segment in the CIE 1931chromaticity space C^2. A mixture space in C^2 is formed by all the chromaticity of the mixture. Then, the mixture space of 3 colors is generally a bounded triangle in C^2.

In Fig. 1, the chromaticity of individual illuminators of a RGBW mixture is denoted by p_1= (0.55,0.32), p_2=(0.27, 0.61), p_3=(0.18, 0.14) and p_4=(0.35, 0.41), respectively. A line segment between 2 colors is partitioned into 10 intervals as the ratio of relative intensities varies proportionally. For $0.3 \le w_1 \le 0.6$, the mixture space is enclosed by the pentagon of $p_{12}p_{13}p'_{13}p'_{14}p'_{12}$. Obviously, it has no intersection with the mixture space under the constraint that $CCT <$ 3000K.

3. Optimization of CRI

A point in a space has a corresponding CRI. It is named a feasible point if it satisfies constraints. The mixture space is not of convexity due to CRI singularity at 5000 K. Hence, modification of the Complex Method [6] is proposed for this application.

Since the mapping from R^2 and R^3 to C^2 is bijective and the CRI has a unique maximum in C^2, the proposed method is applicable for 2- and 3-color mixtures as well as for higher-order mixtures although the mapping is generally no longer bijective. However, optimization can be performed in hierarchy [7]. The hierarchical optimization could take computer time as the number of mixture colors is large. Fortunately, a mixture of 4 colors is generally enough for applications requiring high CRI.

4. Simulations

CRI optimization for a RGB LED mixture is demonstrated below. The SPDs of individual LEDs are given in [7] while the CRI distribution of the mixture is given in Fig. 2. Good convergent property of the proposed method is achieved despite the CRI distribution has significant non-convexity.

With the constraints 4000K $\leq$ CCT $\leq$ 6000K and DC < 0.0054, the convergent states of 50 simulations are shown in chromaticity in Fig. 3. The convergence spans within Δx = 0.0009 and Δy = 0.0012 at the intersection of CCT= 6000K and DC=0.0054 that agrees with the CRI distribution in Fig. 2. Without the constraints, the convergent states of 50 simulations are shown in chromaticity in Fig. 4. The convergence spans within Δx = 0.0007 and Δy = 0.0009. At convergence, correlation between x and y is nearly linear. This also agrees with the CRI distribution in Fig. 2 as the correlation line coincides with the major axis of the elliptic vertex of the highest CRI.

5. Conclusions

A feasible method for the CRI optimization for a light mixture is proposed. The method requires taking no derivatives of the rendering function so it is adequate for the application of CRI optimization. The adequacy of the method has been extensively verified by simulation for 3- and 4-color mixtures.

Enhancement of CRI may suffer from trading off luminous efficacy in practice. Further studies of applying the above method for optimization of luminous efficacy with constraints in CRI are currently undertaken.

6. Acknowledgements

This work is supported by Ministry of Economics, Taiwan, R.O.C., under the Grand Number 97-EC-17-A-07-S1-105.

7. References

1) Y. Ohno, "Optical metrology for LEDs and solid state lighting," Fifth Symposium Optics in Industry, Proceedings of SPIE, **6046**, 604625 (2006).
2) X. Guo, K. W. Houser, "A review of colour rendering indices and their application to commercial light sources," Lighting Research and Technology, **36** (2004) 183–197.
3) S. Boissard and M. Fontoynont, "Optimization of LED-based lighting blendings for object presentation," Color Research and Application, **34**, Issue 4 (2009) 310–320.
4) R. Mirhosseini, M. F. Schubert, S. Chhajed, J. Cho, J. K. Kim and E. F. Schubert, "Improved color rendering and luminous efficacy in phosphor-converted white light emitting diodes by use of dual-blue emitting active regions," OPTICS EXPRESS, **17**, No. 13 (2009).
5) A. Zukauskas, R. Vaicekauskas, F. Ivanauskas, R. Gaska and M. S. Shur, "Optimization of white polychromatic semiconductor lamps," Appl. Phys. Lett., **80(2)** (2002) 234-237.
6) M. J. Box, "A new method of constrained optimization and a comparison with other methods," Computer J., **8** (1965) 42-45.
7) K.C. Lin, "An Approach for Optimization of the Color Rendering Index of Light Mixtures," the Journal of the Optical Society of America A, Manuscript ID: 117616, (2009). (has been submitted)

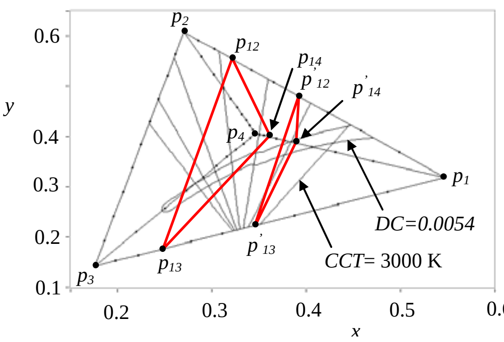

Fig. 1 Illustration of confliction between constraints.

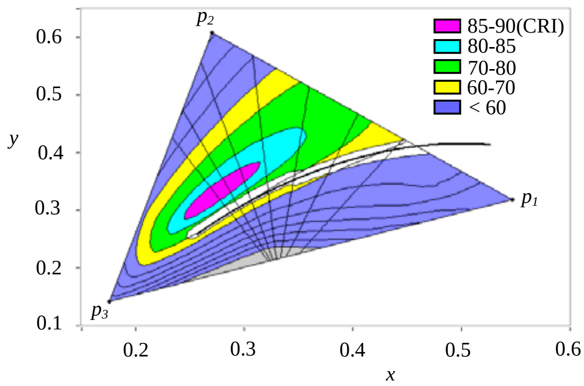

Fig. 2 CRI distribution of an RGB mixture.

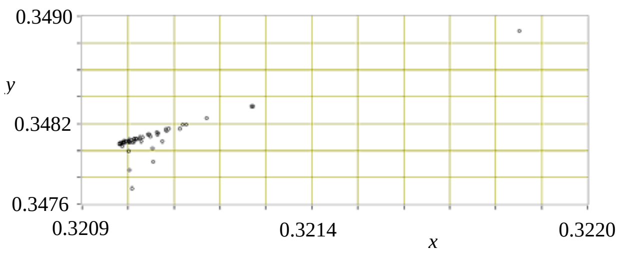

Fig. 3 Convergent states, with the constraints.

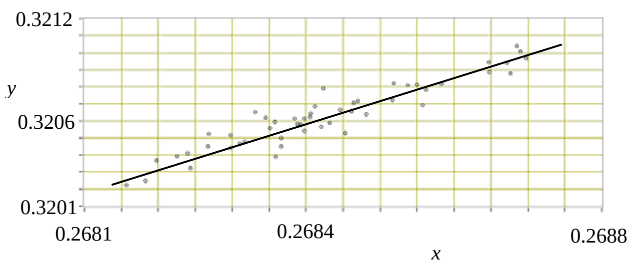

Fig. 4 Convergent states, without the constraints.