International Communications in Heat and Mass Transfer 36 (2009) 925–930

Contents lists available at ScienceDirect

International Communications in Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ichmt

Inverse problem of estimating time-dependent heat generation in a frictional heated strip and foundation ☆

Swun-Kwang Wang [a], Haw-Long Lee [b], Yu-Ching Yang [b,*]

[a] Department of Safety, Health, and Environmental Engineering, Chung Hwa University of Medical Technology, Rende 717, Taiwan
[b] Department of Mechanical Engineering, Kun Shan University, Tainan 710, Taiwan

ARTICLE INFO

Available online 25 July 2009

Keywords:
Inverse problem
Heat generation
Sliding-contact process
Conjugate gradient method

ABSTRACT

In this study, an inverse algorithm based on the conjugate gradient method and the discrepancy principle is applied to estimate the unknown time-dependent frictional heat generation for the tribosystem consisting of a semi-infinite foundation and a plane-parallel strip sliding over its surface, from the knowledge of temperature measurements taken within the foundation. It is assumed that no prior information is available on the functional form of the unknown heat generation; hence the procedure is classified as the function estimation in inverse calculation. Results show that an excellent estimation on the time-dependent heat generation can be obtained for the test case considered in this study. The current methodology can be applied to the prediction of heat generation in engineering problems involving sliding-contact elements.

1. Introduction

The knowledge of the flash temperatures at a sliding interface is of fundamental importance for the tribological behavior of materials and has immediate application in the fields of lubrication, metal cutting, grinding and forming tools, mechanical seals, electric contacts, etc. The determination of the flash temperatures requires the knowledge of frictional heat generated on the sliding-contact interface. The establishment of the frictional heat has never been an easy task. Theoretically, the frictional heat generation is considered broadly as a function of sliding velocity, friction coefficient, and contact pressure. In the past, there have been many investigations focusing on the flash temperature of a sliding contact [1,2]. However, most of these works were limited in a previously given heat generation at the sliding interface. Therefore, to apply these foregone investigations, it is necessary to find an effective approach to determine the heat flux at the interface.

Over the past decades, inverse analysis has become a valuable alternative when the direct measurement of data is difficult or the measuring process is very expensive, for example, the determination of heat transfer coefficients, the detection of contact resistance, the estimation of unknown thermophysical properties of new materials, the prediction of damage in the structure fields, the detection of fouling-layer profiles on the inner wall of a piping system, the optimization of geometry, the prediction of crevice and pitting in furnace wall, the determination of heat flux at the outer surface of a vehicle re-entry, and so on.

The estimation of heat source strength has been the main theme of a number of studies [3–5]. The main objective of the present study is to develop an inverse analysis to estimate the frictional heat generation for the tribosystem in Ref. [2], which consists of a sliding strip over the surface of a semi-infinite foundation. An analysis of this kind poses significant implications on the study of the problems associated with sliding-contact interface mentioned earlier. In this study, we present the conjugate gradient method and the discrepancy principle to estimate the time-varying frictional heat generation by using the simulated temperature measurements [6–9]. Subsequently, the distributions of temperature in the strip and foundation can be determined as well. The conjugate gradient method with an adjoint equation, also called Alifanov's iterative regularization method, belongs to a class of iterative regularization techniques, which means that the regularization procedure is performed during the iterative processes, thus the determination of optimal regularization conditions is not needed. No prior information is used in the functional form of the heat generation variation with time. On the other hand, the discrepancy principle is used to terminate the iteration process in the conjugate gradient method.

2. Analysis

2.1. Direct problem

To illustrate the methodology for developing expressions for the use in estimating the unknown time-dependent heat generation during transient frictional heating at the interface of a tribosystem, which consists of a sliding strip over the surface of a semi-infinite foundation, we consider the following transient heat transfer problem

☆ Communicated by W.J. Minkowycz.
* Corresponding author.
E-mail address: ycyang@mail.ksu.edu.tw (Y.-C. Yang).

0735-1933/$ – see front matter © 2009 Elsevier Ltd. All rights reserved.
doi:10.1016/j.icheatmasstransfer.2009.07.001

Nomenclature

d	thickness of the strip (m)
h	convection heat transfer coefficient (W m^{-2} K^{-1})
J	functional
J'	gradient of functional
k	thermal conductivity (W m^{-1} K^{-1})
P	compressive pressure (N m^{-2})
p	direction of descent
q	intensity of the frictional heat flux (W m^{-2})
T	temperature (K)
T_0	initial temperature of the system (K)
t	time coordinate (s)
V	sliding velocity (m s^{-1})
Y	measured temperature (K)
x	spatial coordinate (m)
y	spatial coordinate (m)
z	spatial coordinate (m)

Greek symbols

Bi	Biot's number ($=dh/k_s$)
Δ	small variation quality
α	thermal diffusivity (m^2 s^{-1})
β	step size
γ	conjugate coefficient
η	very small value
λ	variable used in adjoint problem
σ	standard deviation
τ	transformed time coordinate
ϖ	random variable

Superscripts/subscripts

K	iterative number
m	measurement position
$*$	dimensionless quantity

of friction process as shown in Fig. 1. Here, the perfect heat contact between the strip and the foundation is assumed. It is supposed that the compressive pressures $P(t)$ are applied to the upper surface of the strip and to the infinity of the foundation. The strip slides with velocity $V(t)$ in the direction of y-axis on the semi-infinite surface. Then, the dimensionless governing equations and the associated boundary and initial conditions for the system can be written as [2]:

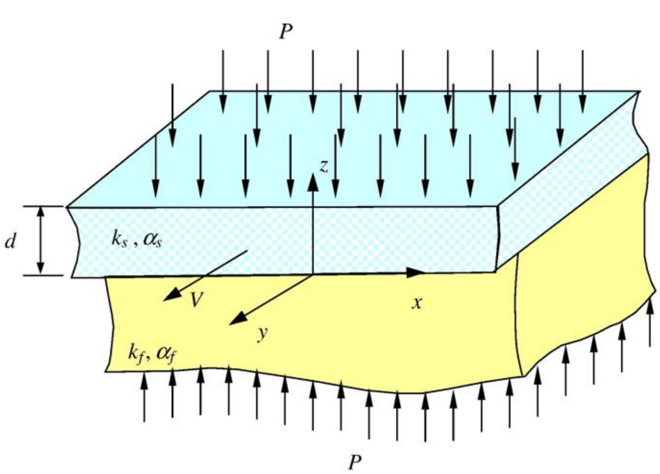

Fig. 1. Geometry and coordinate system.

$$\frac{\partial^2 T_s^*(z^*,t^*)}{\partial z^{*2}} = \frac{\partial T_s^*(z^*,t^*)}{\partial t^*}, \; 0 \le z^* \le 1, \; t^* > 0, \tag{1}$$

$$\frac{\partial^2 T_f^*(z^*,t^*)}{\partial z^{*2}} = \frac{1}{\alpha_{fs}} \frac{\partial T_f^*(z^*,t^*)}{\partial t^*}, \; -\infty < z^* \le 0, \; t^* > 0, \tag{2}$$

$$\frac{\partial T_s^*(1,t^*)}{\partial z^*} = Bi \cdot [T_s^*(1,t^*) - T_\infty^*], \; z^* = 1, \; t^* > 0, \tag{3}$$

$$T_f^*(z^*,t^*) \to 0, \; z^* \to -\infty, \; t^* > 0, \tag{4}$$

$$T_s^*(0,t^*) = T_f^*(0,t^*), \; z^* = 0, \; t^* > 0, \tag{5}$$

$$\frac{\partial T_s^*(0,t^*)}{\partial z^*} - k_{fs} \frac{\partial T_f^*(0,t^*)}{\partial z^*} = q^*(t^*), \; z^* = 0, \; t^* > 0, \tag{6}$$

$$T_s^*(z^*,0) = 0, \; 0 \le z^* \le 1, \; t^* = 0, \tag{7}$$

$$T_f^*(z^*,0) = 0, \; -\infty < z^* \le 0, \; t^* = 0, \tag{8}$$

where the subscripts s and f refer to the regions of strip and foundation, respectively. $q^*(t^*)$ in Eq. (6) is the dimensionless frictional heat generation. In general, the frictional heat output should be a function of the compressive pressure, friction coefficient, and sliding velocity.

The dimensionless variables used in the above formulation are defined as follows:

$$z^* = z/d, \quad t^* = \alpha_s t / d^2, \quad q^* = dq / k_s T_0, \quad T_s^* = (T_s - T_0)/T_0,$$
$$T_f^* = (T_f - T_0)/T_0, \quad \alpha_{fs} = \alpha_f / \alpha_s, \quad k_{fs} = k_f / k_s, \quad Bi = dh / k_s, \tag{9}$$

where d is the thickness of the strip, q is the frictional heat generation, and T_0 is the temperature of the system at $t=0$. k and α are the thermal conductivity and thermal diffusivity, respectively. The direct problem considered here is concerned with the determination of the medium temperature when the frictional heat output $q^*(t^*)$, thermo-physical properties of the system, and initial and boundary conditions are known.

2.2. Inverse problem

For the inverse problem, the function $q^*(t^*)$ is regarded as being unknown, while everything else in Eqs. (1)–(8) is known. In addition, temperature readings taken at $z=z_m$ in the foundation region are considered available. The objective of the inverse analysis is to predict the unknown time-dependent function of intensity of the frictional heat generation, $q^*(t^*)$, merely from the knowledge of these temperature readings. Let the measured temperature at the measurement position $z=z_m$ and time t be denoted by $Y(z_m,t)$. Then this inverse problem can be stated as follows: by utilizing the above mentioned measured temperature data $Y(z_m,t)$, the unknown $q^*(t^*)$ is to be estimated over the specified time domain.

The solution of the present inverse problem is to be obtained in such a way that the following functional is minimized:

$$J[q^*(t^*)] = \int_{t^*=0}^{t_f^*} [T_s^*(z_m^*,t^*) - Y^*(z_m^*,t^*)]^2 \mathrm{d}t^*, \tag{10}$$

where $Y^*(z_m, t^*) = [Y(z_m,t) - T_0]/T_0$, and $Y(z_m,t)$ is the estimated (or computed) temperature at the measurement location $z=z_m$. In this study, $Y^*(z_m,t^*)$ are determined from the solution of the direct problem given previously by using an estimated $q^{*K}(t^*)$ for the exact $q^*(t^*)$, here $q^{*K}(t^*)$ denotes the estimated quantities at the Kth iteration. t_f^* is the final time of the measurement. In addition, in order to develop expressions for the determination of the unknown $q^*(t^*)$, a "sensitivity problem" and an "adjoint problem" are constructed as described below.

2.3. Sensitivity problem and search step size

The sensitivity problem is obtained from the original direct problem defined by Eqs. (1)–(8) in the following manner: It is assumed that when $q^*(t^*)$ undergoes a variation $\Delta q^*(t^*)$, $T_s^*(z^*,t^*)$ and $T_f^*(z^*,t^*)$ are perturbed by $T_s+\Delta T_s^*$ and $T_f+\Delta T_f^*$, respectively. Then replacing in the direct problem $q^*(t^*)$ by $q^*(t^*)+\Delta q^*(t^*)$, T_s^* by $T_s+\Delta T_s$, and T_f by $T_f+\Delta T_f$, subtracting from the resulting expressions the direct problem, and neglecting the second-order terms, the following sensitivity problem for the sensitivity function ΔT_s and ΔT_f^* can be obtained:

$$\frac{\partial^2 \Delta T_s^*(z^*,t^*)}{\partial z^{*2}} = \frac{\partial \Delta T_s^*(z^*,t^*)}{\partial t^*},\ 0\le z^*\le 1,\ t^*>0, \tag{11}$$

$$\frac{\partial^2 \Delta T_f^*(z^*,t^*)}{\partial z^{*2}} = \frac{1}{\alpha_{fs}}\frac{\partial \Delta T_f^*(z^*,t^*)}{\partial t^*},\ -\infty<z^*\le 0,\ t^*>0, \tag{12}$$

$$\frac{\partial \Delta T_s^*(1,t^*)}{\partial z^*} = Bi\cdot \Delta T_s^*(1,t^*),\ z^*=1,\ t^*>0, \tag{13}$$

$$\Delta T_f^*(z^*,t^*)\to 0,\ z^*\to -\infty,\ t^*>0, \tag{14}$$

$$\Delta T_s^*(0,t^*) = \Delta T_f^*(0,t^*),\ z^*=0,\ t^*>0, \tag{15}$$

$$\frac{\partial \Delta T_s^*(0,t^*)}{\partial z^*} - k_{fs}\frac{\partial \Delta T_f^*(0,t^*)}{\partial z^*} = \Delta q^*(t^*),\ z^*=0,\ t^*>0, \tag{16}$$

$$\Delta T_s^*(z^*,0)=0,\ 0\le z^*\le 1,\ t^*=0, \tag{17}$$

$$\Delta T_f^*(z^*,0)=0,\ -\infty<z^*\le 0,\ t^*=0. \tag{18}$$

The sensitivity problem of Eqs. (11)–(18) can be solved by the same method as the direct problem of Eqs. (1)–(8).

2.4. Adjoint problem and gradient equation

To formulate the adjoint problem, Eqs. (1) to (2) are multiplied by the Lagrange multipliers (or adjoint functions) λ_s^* and λ_f^*, respectively, and the resulting expressions are integrated over the time and correspondent space domains. Then the results are added to the right hand side of Eq. (10) to yield the following expression for the functional $J[q^*(t^*)]$:

$$\begin{aligned} J[q^*(t^*)] = & \int_{t^*=0}^{t_f^*}\int_{z^*=0}^{1}[T_s^*(z^*,t^*) \\ & -Y^*(z^*,t^*)]^2\cdot\delta(z^*-z_m^*)dz^*dt^* \\ & + \int_{t^*=0}^{t_f^*}\int_{z^*=0}^{1}\lambda_s^*(z^*,t^*)\cdot\left[\frac{\partial^2 T_s^*}{\partial z^{*2}}-\frac{\partial T_s^*}{\partial t^*}\right]dz^*dt^* \\ & + \int_{t^*=0}^{t_f^*}\int_{z^*=-\infty}^{0}\lambda_f^*(z^*,t^*)\cdot\left[\frac{\partial^2 T_f^*}{\partial z^{*2}}-\frac{1}{\alpha_{fs}}\frac{\partial T_f^*}{\partial t^*}\right]dz^*dt^*. \end{aligned} \tag{19}$$

The variation ΔJ is derived after $q^*(t^*)$ is perturbed by $\Delta q^*(t^*)$, T_s^* and T_f^* are perturbed by $T_s^*+\Delta T_s^*$ and $T_f^*+\Delta T_f^*$, respectively, in Eq. (19). Subtracting from the resulting expression the original equation (Eq. (19)) and neglecting the second-order terms, we thus find:

$$\begin{aligned} \Delta J[q^*(t^*)] = & \int_{t^*=0}^{t_f^*}\int_{z^*=0}^{1}2[T_s^*(z^*,t^*) \\ & -Y^*(z^*,t^*)]\cdot\Delta T_s^*\cdot\delta(z^*-z_m^*)dz^*dt^* \\ & + \int_{t^*=0}^{t_f^*}\int_{z^*=0}^{1}\lambda_s^*(z^*,t^*)\cdot\left[\frac{\partial^2 \Delta T_s^*}{\partial z^{*2}}-\frac{\partial \Delta T_s^*}{\partial t^*}\right]dz^*dt^* \\ & + \int_{t^*=0}^{t_f^*}\int_{z^*=-\infty}^{0}\lambda_f^*(z^*,t^*)\cdot\left[\frac{\partial^2 \Delta T_f^*}{\partial z^{*2}}-\frac{1}{\alpha_{fs}}\frac{\partial \Delta T_f^*}{\partial t^*}\right]dz^*dt^*, \end{aligned} \tag{20}$$

where $\delta(\cdot)$ is the Dirac function. We can integrate the second and the third double integral terms in Eq. (20) by parts, utilizing the initial and boundary conditions of the sensitivity problem. The vanishing of the integrands containing ΔT_s and ΔT_f^* leads to the following adjoint problem for the determination of λ_s^* and λ_f^*:

$$\begin{aligned} & \frac{\partial^2 \lambda_s^*(z^*,t^*)}{\partial z^{*2}} + \frac{\partial \lambda_s^*(z^*,t^*)}{\partial t^*} + 2[T_s^*(z^*,t^*)-Y^*(z_m^*,t^*)]\cdot\delta(z^*-z_m^*) \\ & = 0,\ 0\le z^*\le 1,\ t^*>0, \end{aligned} \tag{21}$$

$$\frac{\partial^2 \lambda_f^*(z^*,t^*)}{\partial z^{*2}} + \frac{1}{\alpha_{fs}}\frac{\partial \lambda_f^*(z^*,t^*)}{\partial t^*} = 0,\ -\infty<z^*\le 0,\ t^*>0, \tag{22}$$

$$-\frac{\partial \lambda_s^*(1,t^*)}{\partial z^*} = Bi\cdot\lambda_s^*(1,t^*),\ z^*=1,\ t^*>0, \tag{23}$$

$$\lambda_f^*(z^*,t^*)\to 0,\ z^*\to -\infty,\ t^*>0, \tag{24}$$

$$k_{fs}\lambda_s^*(0,t^*) = \lambda_f^*(0,t^*),\ z^*=0,\ t^*>0, \tag{25}$$

$$\frac{\partial \lambda_s^*(0,t^*)}{\partial z^*} = \frac{\partial \lambda_f^*(0,t^*)}{\partial z^*},\ z^*=0,\ t^*>0, \tag{26}$$

$$\lambda_s^*(z^*,0)=0,\ 0\le z^*\le 1,\ t^*=t_f^*, \tag{27}$$

$$\lambda_f^*(z^*,0)=0,\ -\infty<z^*\le 0,\ t^*=t_f^*. \tag{28}$$

The adjoint problem is different from the standard initial value problem in that the final time condition at time $t^*=t_f^*$ is specified instead of the customary initial condition at time $t^*=0$. However, this problem can be transformed to an initial value problem by the transformation of the time variable as $\tau^*=t_f^*-t^*$. Then the adjoint problem can be solved by the same method as the direct problem.

Finally the following integral term is left:

$$\Delta J = \int_{t^*=0}^{t_f^*}\lambda_f^*(0,t^*)\Delta q^*(t^*)dt^*. \tag{29}$$

From the definition used in Ref. [9], we have:

$$\Delta J = \int_{t^*=0}^{t_f^*}J'(t^*)\Delta q^*(t^*)dt^*, \tag{30}$$

where $J'(t^*)$ is the gradient of the functional $J(q^*)$. A comparison of Eqs. (29) and (30) leads to the following form:

$$J'(t^*) = \lambda_f^*(0,t^*). \tag{31}$$

2.5. Conjugate gradient method for minimization

The following iteration process based on the conjugate gradient method is now used for the estimation of $q^*(t^*)$ by minimizing the above functional $J[q^*(t^*)]$:

$$q^{*K+1}(t^*) = q^{*K}(t^*) - \beta^K p^{*K}(t^*), K = 0, 1, 2, ..., \tag{32}$$

where β^K is the search step size in going from iteration K to iteration $K+1$, and $p^{*K}(t^*)$ is the direction of descent (i.e., search direction) given by:

$$p^{*K}(t^*) = J'^K(t^*) + \gamma^K p^{*K-1}(t^*), \tag{33}$$

which is the conjugation of the gradient direction $J'^K(t^*)$ at iteration K and the direction of descent $p^{*K-1}(t^*)$ at iteration $K-1$. The conjugate coefficient γ^K is determined from:

$$\gamma^K = \frac{\int_{t^*=0}^{t_f^*}[J'^K(t^*)]^2 dt^*}{\int_{t^*=0}^{t_f^*}[J'^{K-1}(t^*)]^2 dt^*}, \text{with } \gamma^0 = 0. \tag{34}$$

The convergence of the above iterative procedure in minimizing the functional J is proved in Ref. [6]. To perform the iterations according to Eq. (32), we need to compute the step size β^K and the gradient of the functional $J'^K(t^*)$.

The functional $J[q^{*K+1}(t^*)]$ for iteration $K+1$ is obtained by rewriting Eq. (10)as:

$$J[q^{*K+1}(t^*)] = \int_{t^*=0}^{t_f^*}[T_s^*(q^{*K} - \beta^K p^{*K}) - Y^*(z_m^*, t^*)]^2 dt^*, \tag{35}$$

where we replace q^{*K+1} by the expression given by Eq. (32). If temperature $T_s^*(q^{*K} - \beta^K p^{*K})$ is linearized by a Taylor expansion, Eq. (35) takes the form:

$$J[q^{*K+1}(t^*)] = \int_{t^*=0}^{t_f^*}[T_s^*(q^{*K}) - \beta^K \Delta T_s^*(p^{*K}) - Y^*(z_m^*, t^*)]^2 dt^*, \tag{36}$$

where $T_s^*(q^{*K})$ is the solution of the direct problem at $z^* = z_m^*$ by using estimated $q^{*K}(t^*)$ for exact $q^*(t^*)$ at time t^*. The sensitivity function $\Delta T_s^*(p^{*K})$ are taken as the solution of Eqs. (11)–(18) at the measured position $z^* = z_m^*$ by letting $\Delta q^* = p^{*K}$ [6]. The search step size β^K is determined by minimizing the functional given by Eq. (36) with respect to β^K. The following expression can be obtained:

$$\beta^K = \frac{\int_{t^*=0}^{t_f^*}\Delta T_s^*(p^{*K})[T_s^*(q^{*K}) - Y^*]dt^*}{\int_{t^*=0}^{t_f^*}[\Delta T_s^*(p^{*K})]^2 dt^*}. \tag{37}$$

2.6. Stopping criterion

If the problem contains no measurement errors, the traditional check condition specified as:

$$J(q^{*K+1}) < \eta, \tag{38}$$

where η is a small specified number, can be used as the stopping criterion. However, the observed temperature data contains measurement errors; as a result, the inverse solution will tend to approach the perturbed input data, and the solution will exhibit oscillatory behavior as the number of iteration is increased [10]. Computational experience has shown that it is advisable to use the discrepancy principle for terminating the iteration process in the conjugate gradient method. Assuming $T_s^*(z_m^*, t^*) - Y^*(z_m^*, t^*) \cong \sigma$, the stopping criteria η by the discrepancy principle can be obtained from Eq. (10) as:

$$\eta = \sigma^2 t_f^* \tag{39}$$

where σ is the standard deviation of the measurement error. Then the stopping criterion is given by Eq. (38) with η determined from Eq. (39).

3. Results and discussion

The objective of this article is to validate the present approach when used in estimating the unknown time-dependent frictional heat generation at the interface of a strip and a semi-infinite foundation during a sliding contact accurately with no prior information on the functional form of the unknown quantities, a procedure called function estimation. In the present study, we consider the simulated exact value of $q^*(t^*)$ as a simple sinusoidal variation over the time period $t^* = 0$ to 1:

$$q^*(t^*) = 0.2\sin(t^*\pi). \tag{40}$$

The material of the foundation is assumed to be steel ($k_f = 42$ W m^{-1} K^{-1} and $\alpha_f = 1.2 \times 10^{-5}$ m^2 s^{-1}), while the material of the strip is aluminum ($k_s = 209$ W m^{-1} K^{-1} and $\alpha_s = 8.6 \times 10^{-5}$ m^2 s^{-1}). The Biot's number is taken as $Bi = 1.4$ in this study [2]. A single thermocouple is assumed to be located at the interface ($z_m^* = 0$). In terms of the time domain, the total dimensionless measurement time is chosen as $t_f^* = 1.0$ and measurement time step is taken to be 0.04. Besides, the same computational procedure as in Ref. [5] is used in the numerical calculations.

In the analysis, we do not have a real experimental set up to measure the temperature $Y^*(z_m^*, t^*)$ in Eq. (10). Instead, we assume a real heat flux, $q^*(t^*)$ of Eq. (40), and substitute the exact $q^*(t^*)$ into the direct problem of Eqs. (1)–(8) to calculate the temperatures at the location where the thermocouple is placed. The results are taken as the computed temperature $Y_{\text{exact}}^*(z_m^*, t^*)$. Nevertheless, in reality, the temperature measurements always contain some degree of error, whose magnitude depends upon the particular measuring method employed. In order to consider the situation of measurement errors, a random error noise is added to the above computed temperature $Y_{\text{exact}}^*(z_m^*, t^*)$ to obtain the measured temperature $Y^*(z_m^*, t^*)$. Hence, the measured temperature $Y^*(z_m^*, t^*)$ is expressed as

$$Y^*(z_m^*, t^*) = Y_{\text{exact}}^*(z_m^*, t^*) + \varpi\sigma, \tag{41}$$

where ϖ is a random variable within -2.576 to 2.576 for a 99% confidence bounds, and σ is the standard deviation of the measurement. The measured temperature $Y^*(z_m^*, t^*)$ generated in such way is the so-called simulated measurement temperature.

Fig. 2 shows the estimated values of the unknown function $q^*(t^*)$, obtained with the initial guesses $q^{*0} = 0.0$, temperature measurement taken at $z_m^* = 0.0$, and measurement error of deviation $\sigma = 0.0$ and 0.01, respectively. These results confirm that the estimated results are in very good agreement with those of the exact values. For a temperature of unity and 99% confidence, that standard deviation, $\sigma = 0.01$, corresponds to a measurement error of 2.58%. The results in Fig. 2 demonstrate that, for the cases considered in this study, an increase in the measurement error does not cause obvious deterioration on the accuracy of the inverse solution. Meanwhile, in order to

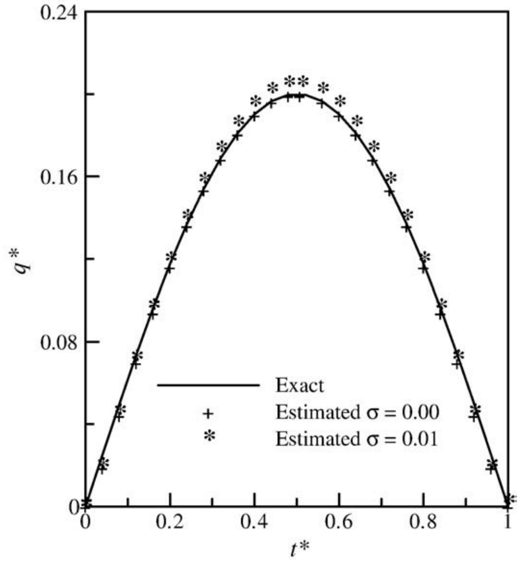

Fig. 2. Estimated heat generation at 5th iteration with initial guesses $q^{*0}(t^*)=0.0$, $z_m^*=0.0$, and $\sigma=0.0$ and 0.01, respectively.

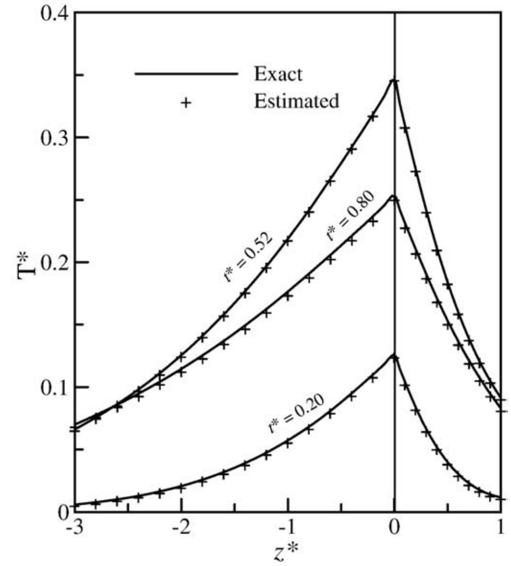

Fig. 4. Estimated temperature distributions at 5th iteration with initial guesses $q^{*0}(t^*)=0.0$, $z_m^*=0.0$, and $\sigma=0.0$.

investigate the influence of measurement location upon the estimated results, Fig. 3 illustrates the estimated unknown function $q^*(t^*)$, with temperature measurement taken at $z_m^*=-0.1$. Here, the initial guesses $q^{*0}=0.0$ and measurement error $\sigma=0.01$, and satisfactory results are still returned which has proved that different measurement locations pose no influence on the accuracy of the present inverse method.

The estimated temperature distributions in the strip and semi-infinite foundation for $t^*=0.20$, 0.52, and 0.80, respectively, are demonstrated in Fig. 4. The results in Fig. 4 are obtained with the initial guesses $q^{*0}=0.0$, temperature measurement taken at $z_m^*=0.0$, and measurement error of deviation $\sigma=0.0$. These results confirm that the estimated temperature values are in very good agreement with those of the exact values for the case considered in this study. It can be found in Fig. 4 that overall, the temperature rises rapidly at the interface as a consequence of the rapid rise of its internal energy by heat generation, but it drops sharply as the distance from the interface increases.

In order to demonstrate the capability of the presented methodology in obtaining an accurate estimation no matter how complex the unknown function is, we consider another case of $q(t^*)$ with the following form:

$$q(t^*) = 0.2\times[0.3\times\sin(2\pi t^*)+0.25\times\sin(4\pi t^*)+3t^*\times(1.1-t^*)]. \tag{42}$$

Fig. 5 shows the estimated results of $q^*(t^*)$, obtained with the initial guesses $q^{*0}=0.0$, temperature measurement taken at $z_m^*=0.0$, and measurement error of deviation $\sigma=0.01$. It can be found in Fig. 5 that an excellent estimation still can be obtained with this complex unknown function.

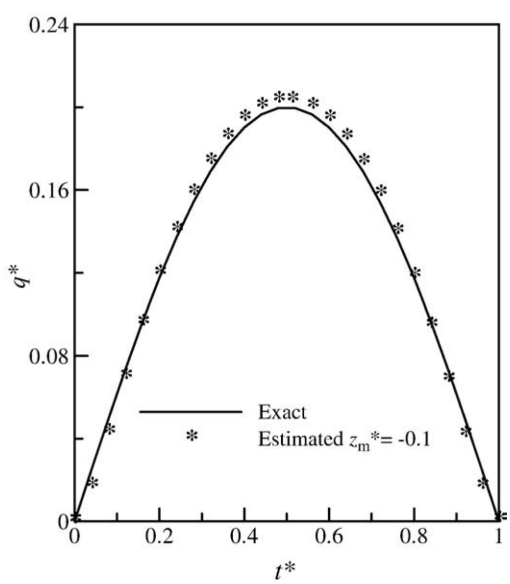

Fig. 3. Estimated heat generation at 5th iteration with initial guesses $q^{*0}(t^*)=0.0$, $z_m^*=-0.1$, and $\sigma=0.01$.

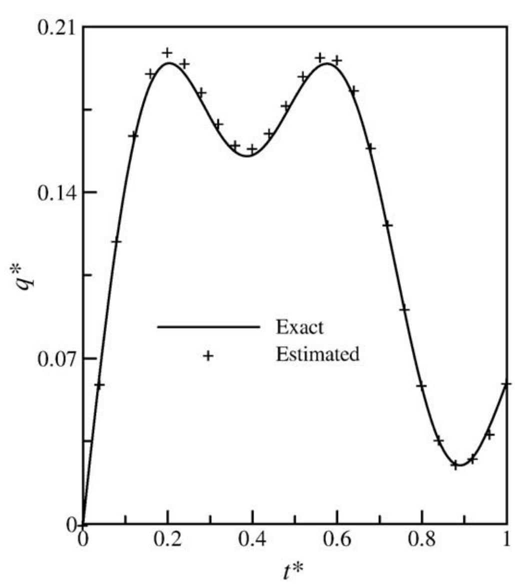

Fig. 5. Estimated heat generation at 5th iteration with initial guesses $q^{*0}(t^*)=0.0$, $z_m^*=0.0$, and $\sigma=0.01$.

4. Conclusion

An inverse algorithm based on the conjugate gradient method and the discrepancy principle was successfully applied to estimate the unknown time-dependent frictional heat generation for the tribosystem consisting of a semi-infinite foundation and a plane-parallel strip sliding over its surface, while knowing the temperature history at some measurement locations. Subsequently, the temperature distributions in the system can be calculated. Numerical results confirm that the method proposed herein can accurately estimate the time-dependent heat generation and temperature distributions for the problem even involving the inevitable measurement errors.

References

[1] A. Evtushenko, M. Kutsei, Non-stationary frictional heat problem for plane-parallel half-space system, J. Friction Wear 28 (2007) 246–259.
[2] A.A. Yevtushenko, M. Kuciej, Influence of convective cooling on the temperature in a frictionally heated strip and foundation, Int. Com. Heat Mass Transfer 36 (2009) 129–136.
[3] C. Le Niliot, F. Lefevre, A parameter estimation approach to solve the inverse problem of point heat sources identification, Int. J. Heat Mass Transfer 47 (2004) 827–841.
[4] H.L. Lee, W.J. Chang, W.L. Chen, Y.C. Yang, An inverse problem of estimating the heat source in tapered optical fibers for scanning near-field optical microscopy, Ultramicroscopy 107 (2007) 656–662.
[5] W.L. Chen, Y.C. Yang, S.S. Chu, Estimation of heat generation at the interface of cylindrical bars during friction process, Appl. Thermal Eng. 29 (2009) 351–357.
[6] O.M. Alifanov, Inverse Heat Transfer Problem, Springer-Verlag, New York, 1994.
[7] Y.C. Yang, Simultaneously estimating the contact heat and mass transfer coefficients in a double-layer hollow cylinder with interface resistance, Appl. Thermal Eng. 27 (2007) 501–508.
[8] W.L. Chen, Y.C. Yang, On the inverse heat convection problem of the flow over a cascade of rectangular blades, Int. J. Heat Mass Transfer 51 (2008) 4184–4194.
[9] Y.C. Yang, W.L. Chen, An iterative regularization method in simultaneously estimating the inlet temperature and heat-transfer rate in a forced-convection pipe, Int. J. Heat Mass Transfer 52 (2009) 1928–1937.
[10] O.M. Alifanov, Application of the regularization principle to the formulation of approximate solution of inverse heat conduction problem, J. Eng. Phys. 23 (1972) 1566–1571.