

THE CRITICAL HEAT TRANSFER CHARACTERISTICS OF AN INSULATED OVAL DUCT

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ABSTRACT

This study is to investigate the characteristics of critical and neutral thicknesses of an insulated oval duct by using computer aided two-dimensional numerical analysis and one-dimensional *PWTR* model based on accurate oval duct surfaces. While the outer radius of non-insulated situation of an insulated circular duct is less than its critical insulated radius, the critical heat transfer will occur. The heat transfer rate increases along with the increasing insulated thickness until reaches its maximum value at critical insulated thickness; then it decreases until reached the point of neutral thickness whose heat transfer rate is the same as that of non-insulated duct. Thus, there is no insulated effect before neutral insulated thickness. These characteristics are very significant for small size insulated duct, especially in situations of low ambient air/gas convective coefficients. It shows in this study that the critical phenomena of an insulated oval duct is very similar to those of an insulated circular duct, except its critical and neutral thicknesses are smaller for bigger oval axes ratio.

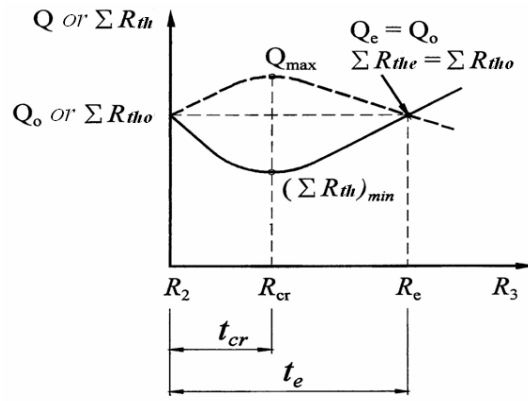
INTRODUCTION

In recent years there has been interest in heat transfer and flow characteristics of an oval duct. Recently, Chen and Fang [1] presented numerical study on the flow over a staggered oval pipe for heat-transfer enhancement; Chen et al. [2] investigated the laminar flow in an alternating horizontal or vertical oval cross-section pipe with computational fluid dynamics; Chen et al. [3-5] presented a series of heat transfer solutions of a finned oval tube with a punched longitudinal vortex generators. Leu et al. [6] studied louvered fin-and-tube heat exchangers having circular and oval tube configurations; Tiwari et al. [7] investigated heat transfer enhancement in cross-flow heat exchangers using oval tubes and multiple delta winglets; Hasan and Siren [8] presented the performance investigation of plain circular and oval tube evaporative cooled heat exchangers; Kim et al. [9] studied heat transfer and pressure drop characteristics during R22 evaporation in an oval micro-fin tube; Schulenberg [10] investigated finned elliptical tubes and their application in air-cooled heat exchanger; Ota et al. [11] studied heat transfer and flow around an elliptic cylinder; O'Brien et al. [12] investigated heat transfer and pressure drop for finned-tube heat exchangers using oval tubes and vortex generators; Chen et al. [13] developed a reliable one-dimensional approximate solution for insulated oval duct based on very accurate oval duct surface area. Meanwhile, the application of mini-scale devices becomes very important due to the advance in micro-technologies. Porter [14] has demonstrated that covering a thin layer of insulated material on a small-size duct would increase the total heat-transfer area of the duct and lead to an increase on the total heat flux. That is, an adverse effect could be resulted due to the use of insulation material, especially for low-effect insulated material. Lewins and Cockerill [15] indicated that the critical radius R_{cr} of an insulated duct is K_s/h_o , where K_s is the heat conductivity of insulated and h_o is the ambient heat convective coefficient. When one tries to cover a small-size duct with insulated material, the problem of critical heat transfer could emerge and become a major concern regarding the effect of the insulated material. Intuitively, one might think that the rate of heat transfer between a duct and its ambience begins to drop as soon as an insulated material is wrapped around the duct, and some sort of heat insulation effect can be achieved. In fact, the matter is not as simple as that. The critical heat transfer phenomena of an insulated circular duct were described more detail by Chou and Wong [16] as shown in Fig. 1.

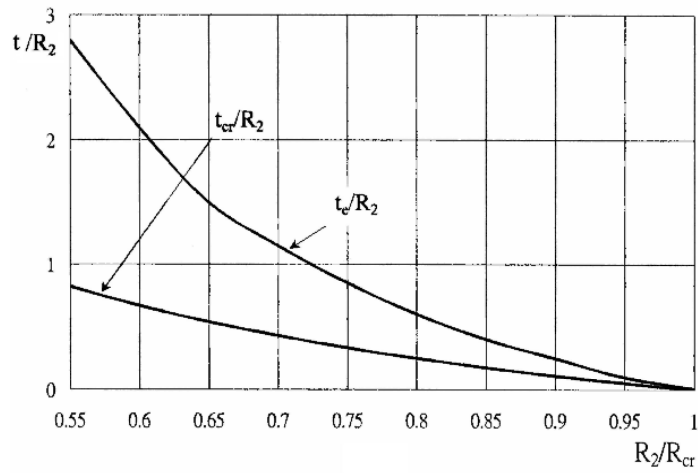
Figure 1 (a) shows that the critical heat transfer phenomena will only happen at the condition of the outer radius of non-insulated situation of an insulated circular duct is less than its critical insulated radius ($R_2 < R_{cr} = K_s/h_o$); the relationship between the heat transfer rate and the outer radius of a small-size insulated duct. As seen, the heat transfer rate of an insulated duct Q is initially increasing as the thickness of insulation material increases, and it only begins to decrease when the insulation thickness is larger than the critical thickness t_{cr} . In addition, the heat transfer rate only drops to below the bare-duct heat transfer rate Q_0 when the insulation thickness t is larger than the neutral thickness t_e . During the increasing of insulation thickness, the maximum Q (or the minimum overall heat resistance $(\Sigma R_{th})_{min}$) occurs at outer insulated radius equaling to critical radius ($R_3 = R_{cr}$). From the above, it is readily understood that the insulation of small-size duct only becomes effective when the insulation thickness is larger than the neutral thickness. The variations of t_{cr}/R_2 and t_e/R_2 versus R_2/R_{cr} are shown in Fig. 1 (b). The plot suggests that a smaller diameter duct requires thicker insulation material to achieve any heat insulation effect. Sometimes, an insulation practice has to be abandoned for a very small-size duct in situation of $R_2 < R_{cr} = K_s/h_o$, because of the big amount of insulation material required to achieve heat insulation. In the case of large-dimension duct, on the other hand, the problem of critical and neutral insulation thickness disappears as long as the bare-duct diameter R_2 is larger than R_{cr} .

Recently, heat pipe as shown in Fig.2 is a high efficient small heat exchanger and has been widely applied to many heat transfer purposes. Fig. 2 shows that there exist insulated sections in both two kinds of heat pipes with two-phase closed thermosyphon. Thus, the insulation of heat pipe is very significant design. Meanwhile, it may find small-size insulated duct in some devices such as high performance computers. Sometimes, the circular duct is pressed into oval shape for the sake of limit space inside the device. Consequently, the problem associated with critical heat transfer phenomena of an insulated oval duct, which has not yet been well addressed, also increasingly becomes an important issue related to the cooling or heating systems in small-size device. The phenomenon of critical heat transfer is present at a small-size oval duct just like its circular counterpart. Since the use of small oval duct is increasingly common in recent years, the study on the critical and neutral thicknesses of an oval duct also becomes increasingly important.

In this paper, the characteristics of critical and neutral thicknesses of an insulated oval duct are investigated in detail and compared with those of an insulated circular duct.



(a)



(b)

Figure 1 The critical heat transfer phenomena of an insulated circular duct.(a) The critical heat transfer phenomena in situation of $R_2 < R_{cr} = K_s/h_o$ (b) The relationships among t_{cr}/R_2 , t_e/R_2 and R_2/R_{cr}

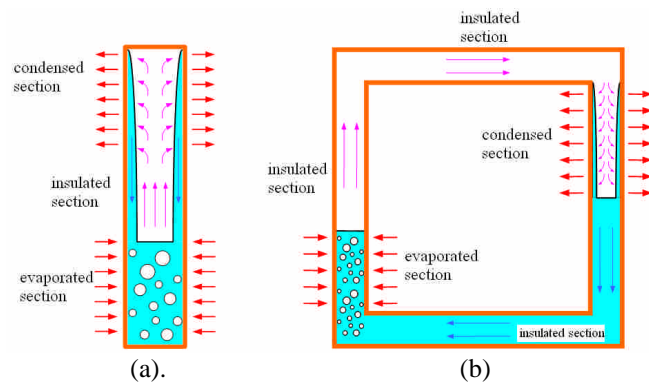


Figure 2 Heat pipe of (a)Two-phase closed thermosyphon; (b) Two-phase closed loop thermosyphon

PROBLEM STATEMENT

Figure 3 shows that an insulated oval duct with the outside half-long-axis-length a as well as half-short-axis-length b of an bare cross-sectional profile, duct thickness t_1 , duct length l , wall conductivity K , and the insulation layer with thickness t , is wrapped around the duct with conductivity K_s , exposed to internal and external fluids with convection heat transfer coefficients h_i and h_o ,

temperatures T_i and T_o , respectively. In addition, Figure 4 shows that the bare oval ducts with the various long-short-axes ratios a/b of 3/2, 2/1, 3/1, 4/1 and 6/1, transformed from a bare circular duct with the same perimeter (the radius R_2 is treated as the equivalent radius of oval duct) are analyzed in this study. The equivalent radius R_2 is also used as a base of the dimensionless insulation thickness t/R_2 and dimensionless duct size, $(R_2/R_{cr})_w = R_2 h_o / K_s$, based on *PWTR* model and equivalent circular duct based on accurate bare outside oval perimeter.

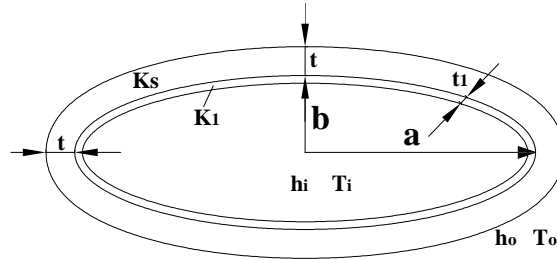


Figure 3 The parameters of an insulation oval duct

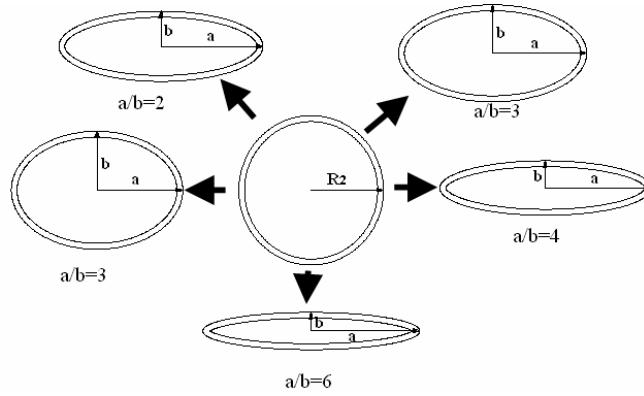


Figure 4 The relation among bare circular and oval ducts with same perimeter

THE OVAL PERIMETER

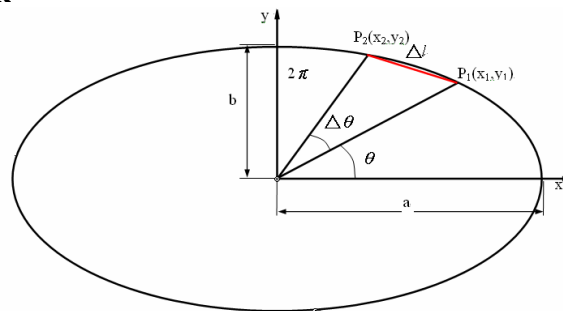


Figure 5 The cross-sectional profile of a bare oval duct and its relative parameters

The oval equation of the oval cross section profile shown in Figure 5 is:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad (1)$$

The perimeter of the oval can then be divided into n line segments. As shown in Fig. 5, the two end points for the i th line segment can be expressed as:

$$x_i = a \cos(2i\pi / n) \quad (2)$$

$$y_i = b \sin(2i\pi/n) \quad (3)$$

$$x_{i+1} = a \cos[2(i+1)\pi/n] \quad (4)$$

$$y_{i+1} = b \sin[2(i+1)\pi/n] \quad (5)$$

Subsequently, the perimeter of the oval can be approximated by the following summation:

From the definition of line length in the x - y coordinates, the accurate perimeter of an oval can be obtained as shown in following equation:

$$L_{E2} = \sum_{\theta=0}^{2\pi} \sqrt{(x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2} \quad (6)$$

The accurate inside perimeter of insulated oval duct, L_{E1} and accurate outside perimeter of insulated oval duct, L_{E3} can be obtained by substituting a and b by $(a-t_i)$ and $(b-t_i)$, as well as $(a+t)$ and $(b+t)$, respectively. The approximated perimeter would be highly accurate if the number n is very large. To find a sufficiently large number n , let $a=b=1$, which makes Eq. (1) become a unit circle. If let $n=10^6$, the error generated from Eq. (7) compared with accurate solution is about 10^{-6} ; if let $n=10^7$, the error generated from Eq. (7) is about 10^{-7} ; for obtaining more convincing results, $n=10^7$ is applied in the present investigation, it takes only about 2 seconds computing time in every result in a one-dimensional programming. Once the accurate oval perimeters are obtained, the accurate surfaces of insulated oval duct can be gotten by multiplying the duct length and accurate oval perimeters.

THE HEAT TRANSFER RATES AND ERRORS WITH PWTR MODEL

Wong et al. [17] proved that the one-dimensional **PWTR** model can apply to an insulated circular duct and obtain exact solution heat transfer rate. Chen et al. [13] obtained reliable results for insulated oval duct by using the one-dimensional **PWTR** model and basing on accurate oval perimeter while long-short-axes ratio a/b equal and less than 3/1.

The heat transfer rate of an insulated oval duct with **PWTR** model and based on accurate oval perimeter is:

$$Q_{EO} = \frac{(T_i - T_o)}{\frac{1}{h_i A_{E1}} + \frac{t_1 \ln \frac{A_{E2}}{A_{E1}}}{K(A_{E2} - A_{E1})} + \frac{t \ln \frac{A_{E3}}{A_{E2}}}{K_s(A_{E3} - A_{E2})} + \frac{1}{h_o A_{E3}}} \quad (7)$$

And its error compared with two-dimensional numerical heat transfer rate Q_n is:

$$E_{EO} = \left(\frac{Q_{EO} - Q_n}{Q_n} \right) \times 100 \% \quad (8)$$

ENERGY EQUATION AND BOUNDARY CONDITIONS

The heat conduction equation for a two-dimensional insulated oval duct is:

$$\frac{\partial}{\partial x} \left(K \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left(K \frac{\partial T}{\partial y} \right) = 0 \quad (9)$$

With the boundary conditions:

$$h_i (T_i - T_s)_{A_1} = -K \left(\frac{\partial T}{\partial N} \right)_{A_1} \quad (10)$$

$$-K_s \left(\frac{\partial T}{\partial N} \right)_{A_3} = h_o (T_s - T_o)_{A_3} \quad (11)$$

$$\left(\frac{\partial T}{\partial N} \right)_{\text{symmetrical boundary}} = 0 \quad (12)$$

Where N is the normal direction at a boundary and T_s represented the surface temperature of insulated layer.

NUMERICAL TWO-DIMENSIONAL HEAT TRANSFER RESULTS

The two-dimensional numerical heat transfer results of an insulated oval duct are obtained FORTRAN programming. In order to check if the numerical results are reliable, an insulated circular duct is analyzed to determine how many quadrilateral cells are needed to obtain a satisfactory result. It was found that a model of an insulated circular duct, which used 10738 quadrilateral cells, gave a

satisfactory solution of heat transfer rate within $\pm 0.01\%$ of the accurate analytic solution. Thus, using a greater number of quadrilateral cells to analyze an insulated oval duct can be expected to provide reliable results. The other judgment to prove the numerical results are very reliable by checking the errors are almost zero in the situations of $t/R_2=0$.

RESULTS AND DISCUSSIONS

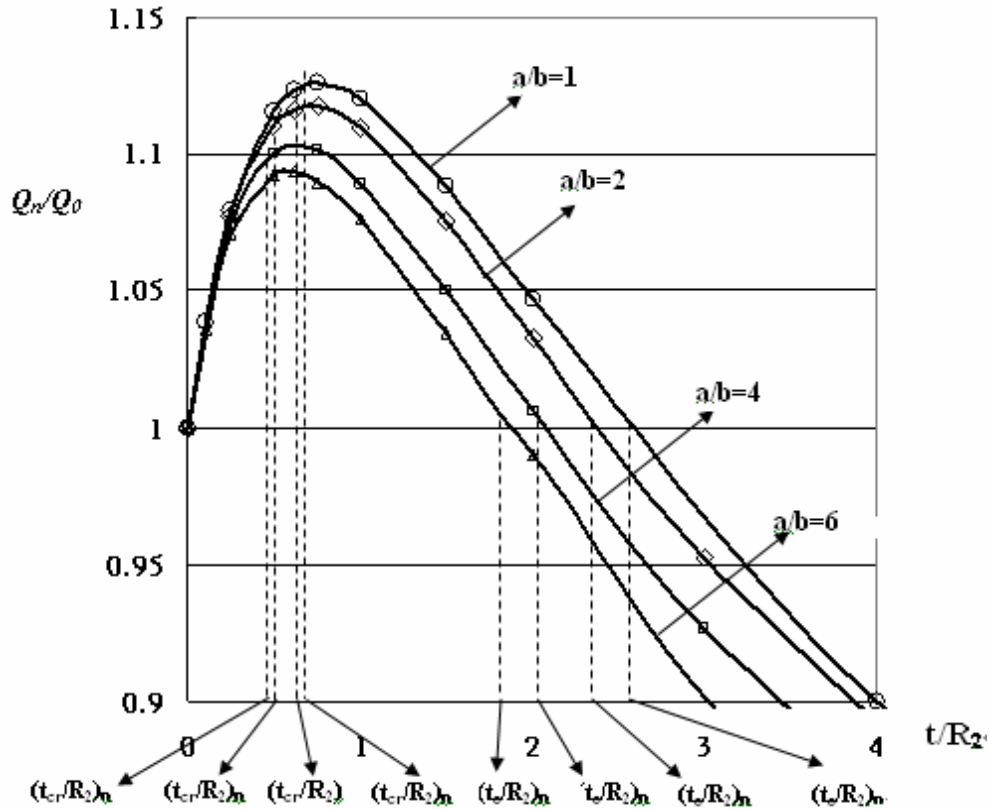


Figure. 6 The relationship among the 2-D numerical results $(Q/Q_0)_n$, $(t_{cr}/R_2)_n$, $(t_e/R_2)_n$ and a/b in situations of $(R_2/R_{cr})_w = 0.566$

Table 1. $a/b=3/2$ insulated oval with $a=0.003\text{m}$, $b=0.002\text{m}$, $R_2=0.002525\text{m}$, $t_1=0.0002\text{m}$, $K_1=77\text{wm}^{-1}\text{K}^{-1}$, $K_S=0.035\text{wm}^{-1}\text{K}^{-1}$, $h_i=10^5\text{wm}^{-2}\text{K}^{-1}$, $h_o=8.3\text{wm}^{-2}\text{K}^{-1}$, $(R_2/R_{cr})_w = 0.598801$, $T_i=100^\circ\text{C}$, $T_o=0^\circ\text{C}$

t/R_2	t	Q_{EO}	$(Q/Q_0)_{EO}$	Q_n	$(Q/Q_0)_n$	E_{EO}
	mm	Wm^{-1}		Wm^{-1}		%
0	0.0	13.167	1	13.164	1	0.0
0.24	0.6	14.052	1.067	14.074	1.069	0.1
0.63	$1.6(t_{cr})_n$	14.502	1.101	14.472	1.099	0.2
0.67	$1.6919(t_{cr})_{vw}$	14.505	1.102	14.467	1.098	0.2
0.71	1.8	14.502	1.101	14.466	1.098	0.3
0.75	1.9	14.494	1.101	14.455	1.098	0.3
1.5	3.8	13.848	1.052	13.779	1.047	0.6
2	$5.102(t_e)_n$	13.238	1.055	13.164	1.000	0.6
2.08	$5.25(t_e)_{vw}$	13.169	1.000	12.921	0.981	0.6
5	12.6	10.618	0.806	10.528	0.799	0.8
10	25.3	8.614	0.654	8.550	0.649	0.8

Table 2. $a/b=2/1$ insulated oval with $a=0.004\text{m}$, $b=0.002\text{m}$, $R_2= 0.003084\text{m}$, $t_1=0.0002\text{m}$, $K_1=77\text{ wm}^{-1}\text{K}^{-1}$, $K_S=0.035\text{ wm}^{-1}\text{K}^{-1}$, $h_i=10^5\text{ wm}^{-2}\text{K}^{-1}$, $h_o=6.425\text{ wm}^{-2}\text{K}^{-1}$, $(R_2/R_{cr})_W = 0.566122$, $T_i=100^\circ\text{C}$, $T_o=0^\circ\text{C}$

t/R_2	t mm	Q_{EO} Wm^{-1}	$(Q/Q_0)_{EO}$	Q_n Wm^{-1}	$(Q/Q_0)_n$	E_{EO} %
0	0.00	12.45	1	12.45	1	0.00
0.25	0.77	13.38	1.074	13.41	1.077	-0.22
0.5	1.54	13.82	1.110	13.82	1.110	-0.06
0.625	1.93	13.91	1.117	13.90	1.116	0.07
0.67	2.08	13.922	1.118	13.90	1.116	0.15
0.75	<u>2.31 ($t_{cr})_n$</u>	13.934	1.119	13.91	<u>1.118</u>	0.19
0.766	<u>2.363($t_{cr})_w$</u>	13.953	<u>1.120</u>	13.90	1.117	0.3
1	3.08	13.87	1.114	13.81	1.109	0.42
1.5	4.63	13.48	1.108	13.38	1.074	0.73
2	6.17	12.97	1.041	12.86	1.032	0.92
2.2	<u>7.402 (t_e)_n</u>	12.759	1.009	12.46	<u>1.000</u>	2.3
2.51	<u>7.75 (t_e)_w</u>	12.448	<u>1.000</u>	12.35	0.990	0.8
3	9.25	11.99	0.963	11.86	0.952	1.08
5	15.42	10.51	0.844	10.39	0.834	1.15

Table 3. $a/b=3/1$ insulated oval with $a=0.003\text{m}$, $b=0.001\text{m}$, $R_2= 0.002127\text{m}$, $t_1=0.0002\text{m}$, $K_1=77\text{ wm}^{-1}\text{K}^{-1}$, $K_S=0.035\text{ wm}^{-1}\text{K}^{-1}$, $h_i=10^5\text{ wm}^{-2}\text{K}^{-1}$, $h_o=8.3\text{ wm}^{-2}\text{K}^{-1}$, $(R_2/R_{cr})_W = 0.504424$, $T_i=100^\circ\text{C}$, $T_o=0^\circ\text{C}$

t/R_2	t mm	Q_{EO} Wm^{-1}	$(Q/Q_0)_{EO}$	Q_n Wm^{-1}	$(Q/Q_0)_n$	E_{EO} %
0	0.0	11.09	1	11.09	1	0.0
0.234	0.5	11.93	1.081	12.11	10.091	-0.6
0.75	1.6	12.82	1.156	12.80	1.154	0.2
0.85	<u>1.8 ($t_{cr})_n$</u>	12.86	1.159	<u>12.82</u>	1.155	0.4
0.94	2.0	12.884	1.161	12.81	1.155	0.6
0.982	<u>2.09 ($t_{cr})_w$</u>	12.887	<u>1.162</u>	12.80	1.154	
0.99	2.1	12.886	1.161	12.79	1.153	0.7
1.27	2.7	12.83	1.156	12.70	1.145	1.1
3	6.4	11.58	1.044	11.33	1.021	2.3
3.22	<u>7.10 (t_e)_n</u>	11.338	1.022	11.09	<u>1.000</u>	2.2
3.67	<u>7.85 (t_e)_w</u>	11.109	<u>1.000</u>	10.82	0.972	2.6
4	8.5	10.88	0.981	10.63	0.958	2.5
10	21.3	8.49	0.766	8.29	0.747	2.5

Table 4. $a/b=4/1$ insulated oval with $a=0.004\text{m}$, $b=0.001\text{m}$, $R_2=0.002731\text{m}$, $t_1=0.0001\text{m}$, $K_1=77\text{wm}^{-1}\text{K}^{-1}$, $K_S=0.035\text{wm}^{-1}\text{K}^{-1}$, $h_i=10^5\text{wm}^{-2}\text{K}^{-1}$, $h_o=8.3\text{wm}^{-2}\text{K}^{-1}$, $(R_2/R_{cr})_W = 0.647541$, $T_i=100^\circ\text{C}$, $T_o=0^\circ\text{C}$

t/R_2	t mm	Q_{EO} Wm^{-1}	$(Q/Q_0)_{EO}$	Q_n Wm^{-1}	$(Q/Q_0)_n$	E_{EO} %
0	0.0	14.239	1	14.24	1	0.0
0.1	0.3	14.580	1.024	14.65	10.091	-0.6
0.26	0.7	14.875	1.044	14.99	1.137	-0.8
0.37	1.0	14.995	1.053	15.09	1.154	-0.7
0.51	$1.4 (t_{cr})_n$	15.051	1.056	15.10	1.155	-0.3
0.544	$1.486 (t_{cr})_w$	15.052	1.057	14.98	1.051	0.4
0.73	2.0	14.980	1.052	15.04	1.155	0.5
0.99	$2.7 (t_e)_n$	14.715	1.162	14.42	1.000	1.2
1.39	$3.8 (t_e)_w$	14.252	1.000	14.05	0.986	1.4
1.5	4.1	14.104	0.991	13.90	0.976	2.0
3	8.2	12.262	0.862	11.98	0.841	2.3
10	27.3	8.621	0.605	8.40	0.589	2.4

Table 5. $a/b=4/1$ insulated oval with $a=0.004\text{m}$, $b=0.001\text{m}$, $R_2=0.002731\text{m}$, $t_1=0.0001\text{m}$, $K_1=77\text{wm}^{-1}\text{K}^{-1}$, $K_S=0.035\text{wm}^{-1}\text{K}^{-1}$, $h_i=10^5\text{wm}^{-2}\text{K}^{-1}$, $h_o=7.257\text{wm}^{-2}\text{K}^{-1}$, $(R_2/R_{cr})_W = 0.56617$, $T_i=100^\circ\text{C}$, $T_o=0^\circ\text{C}$

t/R_2	t mm	Q_{EO} Wm^{-1}	$(Q/Q_0)_{EO}$	Q_n Wm^{-1}	$(Q/Q_0)_n$	E_{EO} %
0	0.0	12.450	1	12.45	1	0.0
0.26	0.7	13.265	1.065	13.37	1.078	-0.9
0.51	1.4	13.652	1.097	13.70	1.105	-0.4
0.58	$1.6 (t_{cr})_n$	13.704	1.101	13.80	1.107	-0.6
0.73	2.0	13.752	1.104	13.74	1.105	0.3
0.766	$2.092 (t_{cr})_w$	13.754	1.105	13.70	1.100	0.7
1	2.7	13.702	1.100	13.56	1.093	1.0
1.5	4.1	13.329	1.070	13.07	1.054	2.0
2	$5.5 (t_e)_n$	12.830	1.030	12.45	1.000	3.0
2.4	$6.55 (t_e)_w$	12.450	1.000	12.20	0.98	-2.0
3	8.2	11.888	0.954	11.54	0.930	-3.1
5	13.7	10.443	0.839	10.23	0.821	-2.3

The relationship among the 2-D numerical results $(Q/Q_0)_n$, $(t_{cr}/R_2)_n$, $(t_e/R_2)_n$ and a/b in situations of $(R_2/R_{cr})_W = 0.566$ is shown in Figure 6. It can be seen from Figure. 6 that insulated oval duct tends reducing the degree of critical heat transfer phenomena; the greater the a/b is, the smaller the $(t_{cr}/R_2)_n$ and $(t_e/R_2)_n$ will be. The more detail data can be found in Tables 1-6. Tables 1-6 show that all the $(t_{cr}/R_2)_w$ of $a/b=1$ are greater than $(t_{cr}/R_2)_n$ of $a/b>1$; all $(t_e/R_2)_w$ are greater than $(t_e/R_2)_n$. And Figures 7-9 show that for the same a/b , the smaller the $(R_2/R_{cr})_W$ is, the greater the $(t_{cr}/R_2)_n$ and $(t_e/R_2)_n$ will be; it means that a smaller oval duct requires thicker insulation material to insure a positive insulation effect; the errors of *PWTR* model E_{EO} , all are quite small, thus *PWTR* model can be used to predicate the reliable approximate results. Another interesting observation from the results is that the value of $(R_{cr})_n/(R_{cr})_W$ is always less than 1 regardless the value of (a/b) is. This suggests that an oval duct has a smaller critical radius than its equilibrant circular duct, hence would require less insulation material to

achieve the same insulation effect. However, this also implies that the *PWTR* model would overestimate the critical and neutral thickness of an oval duct. It can be seen that the predictive accuracy of *PWTR* model deteriorates as the value of (a/b) increases. The discrepancy already reaches 10% when (a/b) is 4. Tables 1-7 also show that the errors of *PWTR* model E_{EO} , all are quite small and acceptable, thus one-dimensional *PWTR* model with accurate oval surface is still can be used to predict the critical heat transfer since it is very convenient used. It is suggested the following rules could be used: Firstly, it may cause the critical heat transfer phenomena if the value of $(R_2/R_{cr})_w < 1$. Secondly, if $(R_2/R_{cr})_w > 0.9$, Figure 1 shows that $(t_e/R_2)_w < 0.3$; but for $(a/b) > 4$, $(t_e/R_2)_n < 0.15$, it may worth to make insulation other than give up in situation of insulation does necessary.

Table 6. $a/b=6/1$ insulated oval with $a=0.006\text{m}$, $b=0.001\text{m}$, $R_2=0.003963\text{m}$, $t_1=0.0001\text{m}$, $K_1=77\text{Wm}^{-1}\text{K}^{-1}$, $K_S=0.035\text{Wm}^{-1}\text{K}^{-1}$, $h_i=10^5\text{Wm}^{-2}\text{K}^{-1}$, $h_o=8.3\text{Wm}^{-2}\text{K}^{-1}$, $(R_2/R_{cr})_w=0.9398$, $T_i=100^\circ\text{C}$, $T_o=0^\circ\text{C}$

t/R_2	t mm	Q_{EO} Wm^{-1}	$(Q/Q_0)_{EO}$	Q_n Wm^{-1}	$(Q/Q_0)_n$	E_{EO} %
0	0.0	20.665	1	20.667	1	0.0
0.05	0.2 ($t_{cr})_n$	20.571	0.995	20.683	1.002	-0.5
0.06	0.25	20.545	0.944	20.681	1.001	-0.6
0.07	0.27 ($t_e)_n$	20.534	0.933	20.667	1.000	
0.1	0.4	20.5	0.990	20.646	0.998	-0.9
0.25	1.0	20.1	0.970	20.260	0.980	-1.0
0.5	2.0	19.2	0.930	19.227	0.930	0.1
1	4.0	17.5	0.846	17.140	0.829	2.1
2	7.9	14.8	0.718	14.257	0.689	3.8

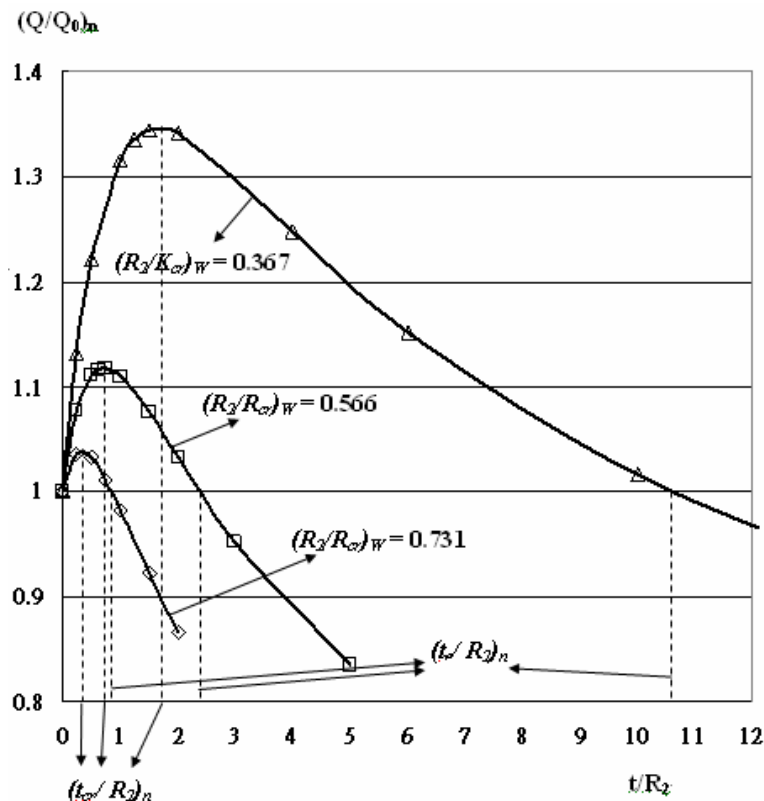


Figure 7 The relationship among the 2-D numerical results $(Q/Q_0)_n$, $(t_{cr}/R_2)_n$, $(t_e/R_2)_n$

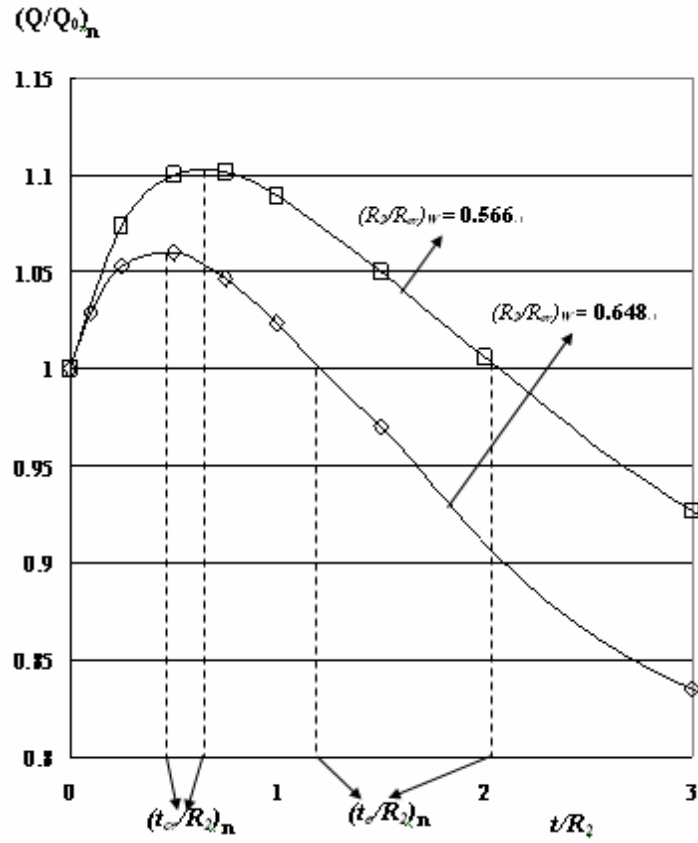


Fig. 8 The relationship among the 2-D numerical results $(Q/Q_0)_n$, $(t_{cr}/R_2)_n$, $(t_e/R_2)_n$ and $(R_2/R_{cr})_w$ in situations of $a/b=4$

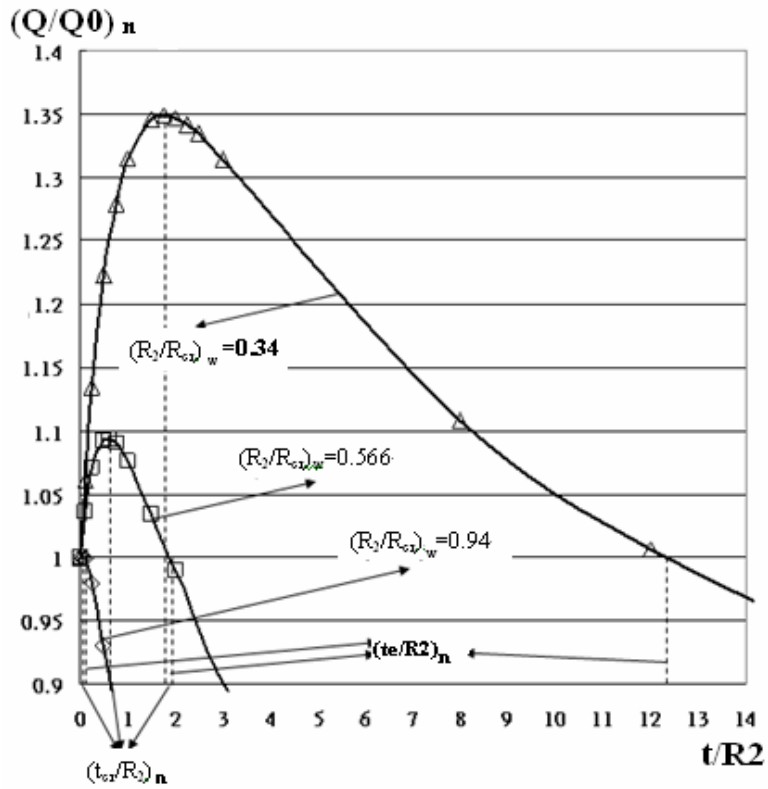


Fig. 9 The relationship among the 2-D numerical results $(Q/Q_0)_n$, $(t_{cr}/R_2)_n$, $(t_e/R_2)_n$ and $(R_2/R_{cr})_w$ in situations of $a/b=6$

CONCLUSIONS

It is well known for a circular duct that when $(R_2/R_{cr})_W < 1$, covering insulation material on the duct only results in an increase on the total heat transfer rate unless the insulation material is thicker than the neutral thickness. This phenomenon, termed as “critical heat transfer”, also takes place in the case of a small-size oval duct, which has been proved by the 2D numerical simulation using USTREAM code. While the value of (a/b) is fixed, the magnitudes of $(t_{cr}/R_2)_n$ and $(t_e/R_2)_n$ increase as the value of $(R_{cr})_n/(R_{cr})_W$ decreases. That is, the smaller the dimension of an oval duct is, the thicker the required insulation material becomes to achieve positive insulation effect. This is similar to a small-size circular duct. On the other hand, the magnitudes of $(t_{cr}/R_2)_n$ and $(t_e/R_2)_n$ decrease as the value of (a/b) increases, meaning less insulation material would be required for a ‘flat’ oval duct to achieve positive insulation.

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NOMENCLATURE

a =half-long-axis length of oval duct

b =half-short-axis length of oval duct

A_1 = inner surface area of a duct

A_2 = outer surface area of a bare duct

A_3 = outer surface area of an insulated duct

$A_{E1}=(L_{E1}l)$ inside surface area of a oval duct based on accurate oval perimeter;

$A_{E2}=(L_{E2}l)$ outside surface area of a bare oval duct based on accurate oval perimeter

$A_{E3}=(L_{E3}l)$ outside surface area of an insulated oval duct based on accurate oval perimeter

E_{EO} = heat transfer rate error of an insulated oval duct with PWTR model and based on accurate oval perimeter

h_i = inner heat convection coefficient

h_o = outer heat convection coefficient

K = conductivity of duct

K_s = conductivity of insulation layer

l = length of oval duct

L_{E1} = accurate inside perimeter of insulated oval duct

L_{E2} = accurate outside perimeter of bare oval duct

L_{E3} = accurate outside perimeter of insulated oval duct

n =the divided section numbers of oval perimeter

N = the normal direction of adiabatic boundary

PWTR= Plane Wedge Thermal Resistance

Q_{EO} =heat transfer rate of insulated oval duct with PWTR model and based on accurate oval perimeter

Q_n =accurate 2D numerical heat transfer rate of insulated oval duct based on accurate oval perimeter

R_2 =radius of equivalent circular duct based on accurate bare outside oval perimeter

$(R_2/R_{cr})_w=R_2h_o/K_s$ = dimensionless duct size based on **PWTR** model or equivalent circular duct based on accurate bare outside oval perimeter

t = thickness of insulation layer

t_f = thickness of duct

T_i = temperature of the fluid inside the duct

T_o = temperature of the fluid outside the duct

x = the horizontal coordinate of a rectangular coordinates

y = the vertical coordinate of a rectangular coordinates